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Mathematical Modeling of PV based Energy Harvesting System Using Multi-Staged Power Conditioning Unit

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ABSTRACT
This paper provides mathematical modeling of multi-staged power conditioning unit (PCU) used for harvesting electrical energy from solar power. Mathematical modeling of PV panel, PV fed boost converter, DC-AC inverter and filters has been provided in this paper. Simulation results have been provided considering a design case study.
Keywords - DC-AC Inverter, DC-DC Converter, PCU, PV Panel

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I. INTRODUCTION

The world's energy consumption has skyrocketed after years of industrial revolution. The energy need has increased because of growing population and higher standard of living. The current trend is that the energy demand is only going up. Globally in 2010, around 87% of the consumed energy was produced with fossil fuels i.e. coal (28%), natural gas (21%) and oil (38%). 6% came from nuclear plants and remaining 7% came from renewable energy sources. Burning of fossil fuels generates pollutant gases, notably CO2 which is the main contributor to greenhouse effect i.e. global warming. Another problem is, that the reserve of fossil fuels is limited, and it is estimated that e.g. oil would run out in 50 to 100 years. The global warming has serious effect on the world's ecological structure in the long run, such as severe droughts near the equator and the rise in sea level, which would ultimately drive 100 million people away from their homes. The need for renewable and clean energy is real, which is why many governments have started to invest on them, and new laws are legislated in order to cut down emissions.

Solar energy is one of the abundant form of energy and solar energy harvesting is one of the most profitable way of generating electricity. Many researchers and power electronic design professional have worked a lot in designing efficient solar energy system and connecting the solar system to the conventional grids. In [1], the author proposes a method of modeling and simulation of photovoltaic arrays. The main objective is to find the parameters of the nonlinear I–V equation by adjusting the curve at three points: open circuit, maximum power, and short circuit. In [2], the authors investigated the effect of climatic conditions i.e. variable irradiation level, wind speed, temperature, and humidity level and dust accumulation in the modeling of a realistic PV system. In [3], the authors have provided a detailed interpretation of solar PV cell characteristics. There are several maximum power point tracking algorithms reported in literature and implemented in commercial PV based system. A proper classification and comparative analysis of different MPPT algorithms have been studied in [4]–[9]. A detailed investigation of different design aspects of DC-DC converter in renewable generation concept has been investigated by [10]. Design of stand-alone PV system for DC micro-grid has been reported in [11]. Grid connected PV system has been reported in [12].

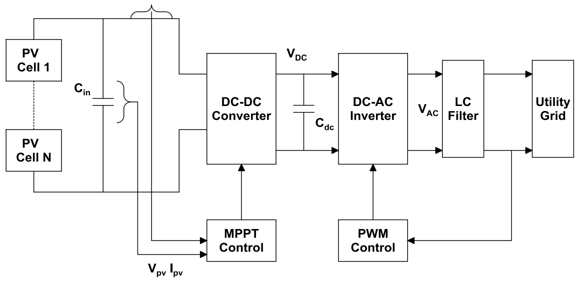
This paper provides a detailed mathematical modeling and analysis of PV based distributed generator. The distributed generation system comprises of multi-staged power conditioning unit (PCU). Multi-staged PCU comprises of multiple power electronic converter (PEC). Detailed modeling and simulation has been provided.

This paper comprises of five sections. Section II discusses details of power conditioning unit and Section III gives detailed mathematical modeling of each and every component and sub-system of PCU. Section IV provides simulation results and Section V concludes the paper.

II. POWER CONDITIONING UNIT

Figure 1 shows the multi-staged PCU used in this thesis. The multi-staged PCU comprises of a

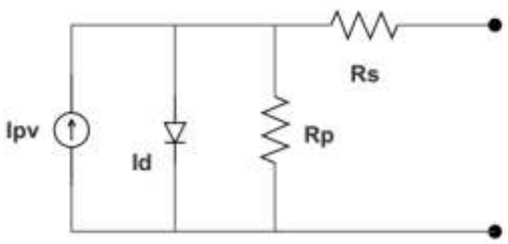
DC-DC boost converter and DC-AC utility inverter. Separate control for converter as well as inverter have been utilized for proper working of distributed generation system. In [13], the authors provide a review of different type of PCU and high-gain DC-DC converter used in renewable energy sources.



Grid integration of PV cell

III. MATHEMATICAL MODELING OF MULTI STAGED PCU

The circuit diagram of a single diode model of PV cell is illustrated in Fig. 2. The ideal PV cell consists of a constant current source and a diode whereas the practical PV cell consists of additional series R_s and parallel resistance R_p



Equivalent circuit diagram of PV cell

The basic equation which describes the I-V characteristics of an ideal PV cell can be represented as $I = I_{pv} - I_d$ (1)

Where diode current can be represented as

$$I_d = I_o\left[\exp\left(\frac{qV}{akT}\right)-1\right] \quad (2)$$

Substituting Eq(2) in Eq(1), we get

$$I = I_{pv} - I_o\left[\exp\left(\frac{qV}{aKT}\right)-1\right] \quad (3)$$

Where q is the electron charge, K is Boltzmann constant, a is the ideality constant, T is the ambient temperature. Practical PV panels comprises of series or parallel connected PV cell. The I-V characteristics of practical PV panel can be represented as

$$I = I_{pv} - I_o\left[\exp\left(\frac{V+R_sI}{V_ta}\right)-1\right]-\frac{V+R_sI}{R_p} \quad (4)$$

Here, I_{pv} is PV current and I_o is saturation current. R_p is equivalent parallel resistance, R_s is the equivalent series resistance and V_t is the thermal voltage of PV cell with N_s cells connected in series connection.

The current of the PV cell is dependent on solar irradiance and temperature. The relation between the PV current and temperature can be represented as

$$I_{pv} = \left(I_{pv,n} + K_I\Delta_T\right)\frac{G}{G_n} \quad (5)$$

where $I_{pv,n}$ is light generated current at nominal operating condition (25 degreeC, 1000W/sq.m), Δ_T is the difference of temperature (Actual and nominal temperature), G is the irradiance of the surface and G_n is the nominal irradiance The relationship of diode saturation current with temperature can be represented as

$$I_o = I_{o,n}\left(\frac{T_n}{T}\right)^3 \exp\left[\frac{qE_g}{aK}\left(\frac{1}{T_n}-\frac{1}{T}\right)\right] \quad (6)$$

Here, E_g is the bandgap of semiconductor

Substituting three significant-points of the I–V characteristic of PV module, namely: the short-circuit point, the maximum power point, and the open-circuit point in Eq.4

$$I_{sc} = I_{pv} - I_o\left(\exp\left(\frac{I_{sc}R_s}{N_sV_t}\right)\right)-\frac{I_{sc}R_s}{R_p} \quad (7)$$

$$I_{mpp} = I_{pv} - I_o\left(\exp\left(\frac{V_{mpp}+I_{mpp}R_s}{N_sV_t}\right)\right)-\frac{V_{mpp}+I_{mpp}R_s}{R_p} \quad (8)$$

$$I_{pv} = I_o\left(\exp\left(\frac{V_{oc}}{N_sV_t}\right)\right)+\frac{V_{oc}}{R_p} \quad (9)$$

At MPP, the derivative of power with respect to voltage is zero and also short circuit condition [14]

$$\left.\frac{dP}{dV}\right|_{MPP} = 0 \qquad \left.\frac{dI}{dV}\right|_{I=I_{sc}} = -\frac{1}{R_p} \quad (10)$$

$$I_o = \left(I_{sc} - \frac{V_{oc}-I_{sc}R_s}{R_p}\right)\exp\left(\frac{-V_{oc}}{N_sV_t}\right) \quad (11)$$

B. PV fed boost converter

The boost converters are commonly used to extract the maximum power from the PV array. These converters produce an output voltage which is higher than the input voltage.
The duty cycle of boost converter is represented as

$$\frac{V_o}{V_{in}} = \frac{1}{1-d} \quad (12)$$

In this work, state space average model of boost converter is used to model the boost converter. In state space average model, the dynamics of the converter changes with the instantaneous change in status of the switch i.e S = ON or S = OFF. The state space representation of system when S = ON is represented by

$$\begin{cases} \frac{d\mathbf{x}_1}{dt} = \mathbf{A}_1\mathbf{x}_1 + \mathbf{B}_1 u_1 \\ y_1 = \mathbf{C}_1\mathbf{x}_1 + D_1 u_1 \end{cases} \quad (13)$$

$$\mathbf{A}_1 = \begin{bmatrix} 0 & 0 \\ 0 & \frac{-1}{RC} \end{bmatrix} \mathbf{B}_1 = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} \mathbf{C}_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

The state space representation of system when S = OFF is represented by

$$\begin{cases} \frac{d\mathbf{x}_2}{dt} = \mathbf{A}_2\mathbf{x}_2 + \mathbf{B}_2 u_2 \\ y_2 = \mathbf{C}_2\mathbf{x}_2 + D_2 u_2 \end{cases} \quad (14)$$

$$\mathbf{A}_2 = \begin{bmatrix} 0 & \frac{-1}{L} \\ \frac{1}{C} & \frac{-1}{RC} \end{bmatrix} \mathbf{B}_2 = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} \mathbf{C}_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

The state-space representation of boost converter is obtained by combining the two stages (ON and OFF) and averaging it with duty cycle d. It can be represented as

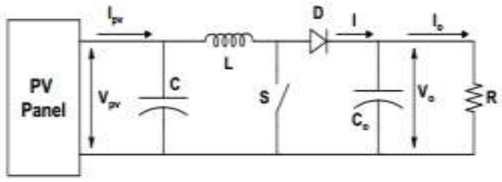
$$\begin{cases} \frac{d\mathbf{x}}{dt} = \mathbf{A}\mathbf{x} + \mathbf{B}u \\ y = \mathbf{C}x + Du \end{cases} \quad (15)$$

$$\begin{cases} \mathbf{A} = \mathbf{A}_1 d + (1-d)\mathbf{A}_2 \\ \mathbf{B} = \mathbf{B}_1 d + (1-d)\mathbf{B}_2 \\ \mathbf{C} = \mathbf{C}_1 d + (1-d)\mathbf{C}_2 \\ \mathbf{D} = \mathbf{D}_1 d + (1-d)\mathbf{D}_2 \end{cases} \quad (16)$$

$$\frac{d}{dt}\begin{bmatrix} I_L \\ V_c \end{bmatrix} = \begin{bmatrix} 0 & \frac{-(1-d)}{L} \\ \frac{1-d}{C} & \frac{-1}{RC} \end{bmatrix} \begin{bmatrix} I_L \\ V_c \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} V_{in}$$

(17)

When the PV voltage is insufficient, the PV voltage can be boosted with the use of the boost converter. The circuit diagram of boost converter integrated with PV panel has been shown in Figure 3. Here, the PV panel is represented as V_{eq} and R_{eq} which represent the equivalent voltage and equivalent resistance respectively. The state-space modeling of PV fed boost converter has been reported in [15].



Circuit diagram of boost converter integrated with PV panel

During ON state of the switch, the state-space representation of PV-fed boost converter can be represented as

$$\begin{bmatrix} \dot{i}_L \\ \dot{v}_{c_1} \\ \dot{v}_{c_2} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{L} & 0 \\ -\frac{1}{C_1} & -\frac{1}{R_{eq}C_1} & 0 \\ 0 & 0 & -\frac{1}{RC_2} \end{bmatrix} \begin{bmatrix} i_L \\ v_{c_1} \\ v_{c_2} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{R_{eq}C_1} \\ 0 \end{bmatrix} v_{eq}$$

(18)
During OFF state of the switch, the state-space representation of PV-fed boost converter can be represented as

$$\begin{bmatrix} \dot{i}_L \\ \dot{v}_{c_1} \\ \dot{v}_{c_2} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{L} & -\frac{1}{L} \\ -\frac{1}{C_1} & -\frac{1}{R_{eq}C_1} & 0 \\ \frac{1}{C_2} & 0 & -\frac{1}{RC_2} \end{bmatrix} \begin{bmatrix} i_L \\ v_{c_1} \\ v_{c_2} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{R_{eq}C_1} \\ 0 \end{bmatrix} v_{eq}$$

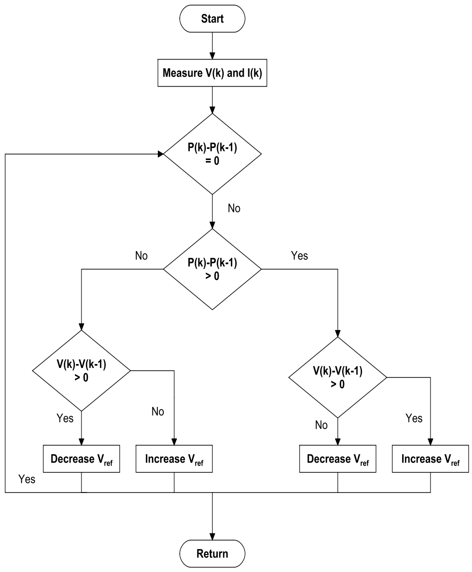
(19)
The small signal modeling of PV-fed boost converter can be represented as

$$\begin{bmatrix} \dot{i}_L \\ \dot{v}_{c_1} \\ \dot{v}_{c_2} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{L} & -\frac{1-d}{L} \\ -\frac{1}{C_1} & -\frac{1}{R_{eq}C_1} & 0 \\ \frac{1-d}{C_2} & 0 & -\frac{1}{RC_2} \end{bmatrix} \begin{bmatrix} i_L \\ v_{c_1} \\ v_{c_2} \end{bmatrix} + \begin{bmatrix} -\frac{V_{c_2}}{L} & 0 \\ 0 & \frac{1}{R_{eq}C_1} \\ \frac{-i_L}{C_2} & 0 \end{bmatrix} dv_{eq}$$

(20)

C. Maximum power point tracking

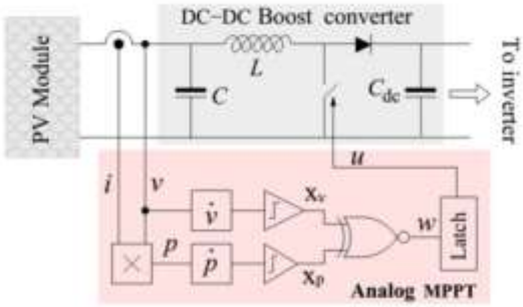
In this method, the voltage of the PV system is considered as a reference signal. The target of this method is to force the reference voltage of the PV system to V_{mpp}, which causes the instantaneous PV voltage to track V_{mpp}. This is done by applying small and constant perturbation to the PV voltage a step-by-step. After each perturbation the variation in output power dp is measured. A positive dP indicates that output power will approach MPP. Therefore, a perturbation of positive sign is applied to the PV voltage in the next stage. On the other hand, if dP is negative, a negative sign perturbation is applied.



Flow chart of P&O MPPT method

Figure 4 provides the flow chart of perturb and observe MPPT algorithm. Figure 5 presents the circuit diagram of DC-DC converter integrated with PV panel and perturb and observe MPPT control logic.

This system consists of a PV module, a boost converter comprising of an input filter capacitor C, an inductor L, output filter capacitor, a diode and a controllable switch regulated by the analog MPPT controller. Here PV module is connected to boost converter along with the analog MPPT controller to regulate the voltage and current of the PV module such that it will operate at MPP by continuously adjusting converter duty cycle. Solar cells show a unique global point at which power is maximum. Denoting v, i as the module voltage and current respectively, the array's v − p characteristics exhibits a unique global maximum at the point denoted by V_{mpp} and $P_{\max}$ where $v = V_{mpp}$. So the power P will be maximum at the operating array voltage $v = V_{mpp}$.



Circuit diagram of boost converter integrated with PV panel and MPPT control

Practically it is more convenient to use the boolean 0/1 instead of -1/+1 for representing the sign of a value. Thus, comparators are then used for evaluating its sign by producing binary signals xp and xv for power and voltage respectively. And finally an XOR gate is used to multiply these two signs, which are now expressed as booleans or binary form. Then the exclusive-ORed output is sampled by a SR-latch with a constant clock frequency. Now whether the PV voltage should be increased or decreased to reach MPP is indicated by the signal w which drives the switch through latching circuit. When its value is 0, v should be increased and when it is 1, v should be decreased.

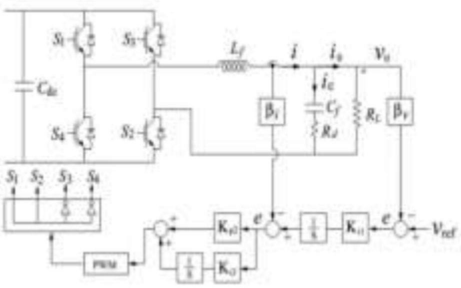
D. DC-AC VSI

The mathematical model of DC-AC VSI can be represented as

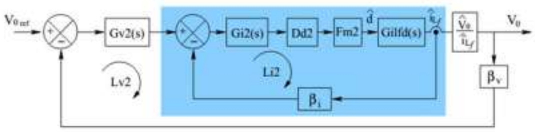
$$\begin{bmatrix} \dot{i}_L \\ \dot{v}_o \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} i_L \\ v_c \end{bmatrix} + \begin{bmatrix} V_{dc} \\ 0 \end{bmatrix} u \qquad (21)$$

E. Control of DC-AC VSI

In this system the output of the inverter is connected with the local load, producing a pure sinusoidal load voltage. Maintaining good voltage regulation and achieving fast dynamic response under sudden load fluctuation are extremely important in stand-alone PV systems. The two loops (inner inductor current loop and outer output voltage loop) control strategy is adopted for achieving the above power quality issues. Figure 6 presents the control scheme of stand-alone VSI. Figure 7 presents the block diagram of control of standalone VSI.



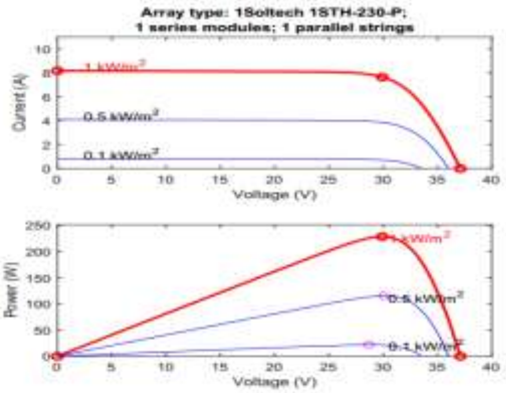
Control Scheme for Stand-alone VSI



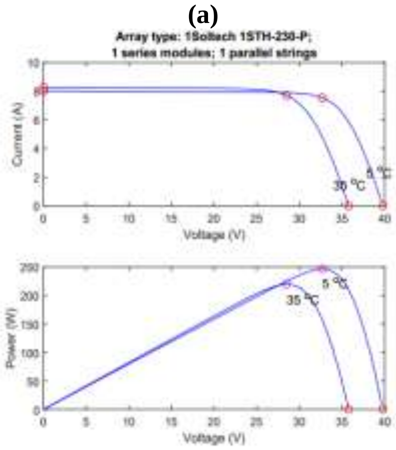
Block diagram for control of stand-alone VSI

IV. SIMULATION RESULTS

Table I provides the simulation parameters for multi-staged PCU for PV based distribution generation system.



(a)



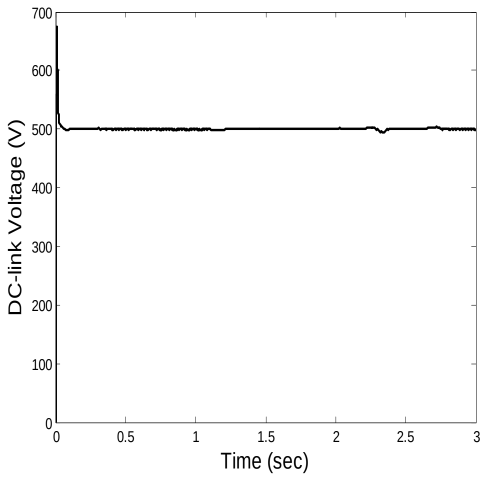
(b)

(a) P-V and V-I charactersitics of PV array with variable irradiance, (b) P-V and V-I characteristics of PV array with variable temperature

MATLAB-Simulink has been used to simulate the PV based DC drive system. The P–V and V–I characteristics of the PV module with variable irradiance and variable temperature are illustrated in Figure 8(a) and Figure 8(b) respectively. The V–I and P–V characteristics are highly nonlinear in nature and the MPPT algorithm finds the maximum power point.

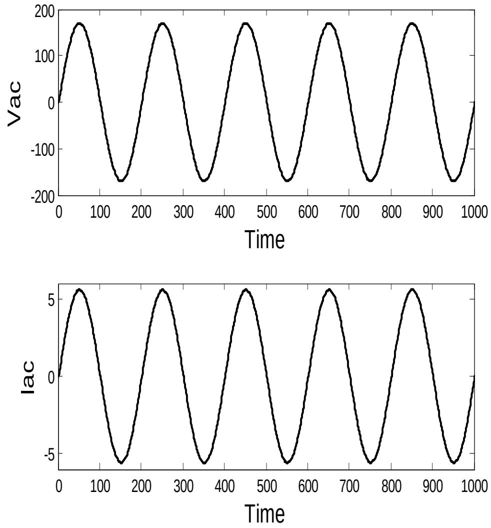
The inductor L and capacitor C values of boost converter can be derived using the maximum allowable inductor ripple current Δi_L , maximum allowable ripple voltage Δv_c and f is the switching frequency of boost converter. The inductor value can be calculated as $L = \dfrac{V_{in}D}{f\Delta i_L}$ The capacitor value can be calculated as $C = \dfrac{I_o D}{f\Delta v_c}$

Figure 9 shows the DC-link voltage of PV based system at uniform irradiance and constant temperature.

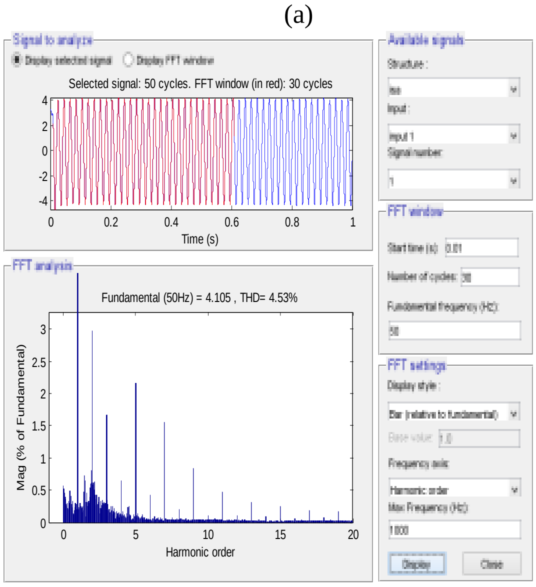


DC-link voltage in uniform irradiance and temperature

Figure 10(a) presents the output voltage of DC-AC inverter and Figure 10(b) presents the total harmonic distortion of the inverter current which is 4.53% which is less than 5% as prescribed by IEEE-1547 standards.



(a)



(b)

(a) Inverter output voltage and current (b) THD

V. CONCLUSION

This paper provides the detailed mathematical modeling of PV based distributed generation system which comprises of PV panel, DC-DC converter and DC-AC inverter along with LCL filter and load or grid. State-space modeling of different converter has been provided. Detailed mathematicalmodeling
of PV panel has been provided. Perturb and observe MPPT algorithm is used to extract maximum power from the PV panel. Simulation analysis has been provided to substantiate the theoretical approach.

REFERENCES

[1]. M. G. Villalva, J. R. Gazoli, and E. Ruppert Filho, “Comprehensive approach to modeling and simulation of photovoltaic arrays,” IEEE Transactions on power electronics, vol. 24, no. 5, pp. 1198–1208, 2009.

[2]. A. Gupta and Y. K. Chauhan, “Detailed performance analysis of realistic solar photovoltaic systems at extensive climatic conditions,” Energy, vol. 116, pp. 716–734, 2016.

[3]. M. Wolf, G. Noel, and R. J. Stirn, “Investigation of the double exponential in the current voltage characteristics of silicon solar cells,” IEEE Transactions on electron Devices, vol. 24, no. 4, pp. 419–428, 1977.

A. Gupta, Y. K. Chauhan, and R. K. Pachauri, “A comparative investigation of maximum power point tracking methods for solar pv system,” Solar energy, vol. 136, pp. 236–253, 2016.

B. Bendib, H. Belmili, and F. Krim, “A survey of the most used mppt methods: Conventional and advanced algorithms applied for photovoltaic systems,” Renewable and Sustainable Energy Reviews, vol. 45, pp. 637– 648, 2015.

C. Subudhi and R. Pradhan, “A comparative study on maximum power point tracking techniques for photovoltaic power systems,” IEEE Transactions on sustainable energy, vol. 4, no. 1, pp. 89–98, 2013. [7] H. Rezk and A. M. Eltamaly, “A comprehensive comparison of different mppt techniques for photovoltaic systems,” Solar energy, vol. 112, pp. 1–11, 2015.

[4]. T. Esram and P. L. Chapman, “Comparison of photovoltaic array maximum power point tracking techniques,” IEEE transactions on energy conversion, vol. 22, no. 2, pp. 439–449, 2007. [9] R. Ahmad, A. F. Murtaza, and H. A. Sher, “Power tracking techniques for efficient operation of photovoltaic array in solar applications–a review,” Renewable and Sustainable Energy Reviews, vol. 101, pp. 82– 102, 2019.

[5]. S. Sivakumar, M. J. Sathik, P. Manoj, and G. Sundararajan, “An assessment on performance of dc–dc converters for renewable energy applications,” Renewable and Sustainable Energy Reviews, vol. 58, pp. 1475–1485, 2016.

[6]. G. B. Ingale, S. Padhee, and U. C. Pati, “Design of stand alone pv system for dc-micro grid,” in 2016 International Conference on Energy Efficient Technologies for Sustainability (ICEETS). IEEE, 2016, pp. 775–780.

[7]. A. Panda, M. Pathak, and S. Srivastava, “A single phase photovoltaic inverter control for

grid connected system," Sadhana, vol. 41, no. 1, pp. 15–30, 2016.
[8]. S. Padhee, U. C. Pati, and K. Mahapatra, "Overview of high-step-up dc–dc converters for renewable energy sources," IETE Technical Review, vol. 35, no. 1, pp. 99–115, 2018.
[9]. D. Sera, R. Teodorescu, and P. Rodriguez, "Pv panel model based on datasheet values," in 2007 IEEE international symposium on industrial electronics. IEEE, 2007, pp. 2392–2396.
[10]. S. Selvakumar, M. Madhusmita, C. Koodalsamy, S. P. Simon, and Y. R. Sood, "High-speed maximum power point tracking module for pv systems," IEEE Transactions on Industrial Electronics, vol. 66, no. 2, pp. 1119–1129, 2018.

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