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Mean Field Game Linear Quadratic-Gaussian Control in Leader-Follower Stochatic Dynamic Game Model and Its Apllication

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ABSTRACT
In this paper we consider large population Leader –follower (L-F) stochastic multi-agent systems where the agents have linear stochastic dynamics and are coupled via their quadratic cost functions. The cost of each leader is based on a trade-off between moving toward a certain reference trajectory which is unknown to the followers and staying near the centroid. On the other hand, followers react by tracking convex combinations of centroid and the centroid of the leaders. Here, as in most practical leader-follower modelling of multi-agent systems, the leaders ignore the followers, but the followers' behaviours are influenced by the leaders. The article is generalised that the cost of collective and the case of collective dynamics which include leaders, followers and an unknown (to the followers) reference trajectory for the leaders.
Key words: Mean field game, Leader- Follower, Linear Quadratic Gaussian (LQG) stochastic dynamic game, Likelihood ratio, Nash equilibrium for infinite population system.

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I. INTRODUCTION

Decision making and collective behaviour often involve some forms of leader-follower behaviours. These behaviours are observed in humans [4] and many other species in nature [3, 6] and are studied in variety of disciplines such as game theory [9], distributed networks [10], crowd flow dynamics [2] and biology among others. Such behaviours in nature are often attributed to the fact that there exist some individuals in the group which have more information than others, for instance the location of resources or migratory routes.

We approach the large population L-F model by mean field linear quadratic Gaussian (MF LQG) stochastic control theory. In this frame work, the computation of the followers control laws requires knowledge of the complete reference trajectory of the leaders which is generally not known to the followers, hence a likelihood ratio based adaptation scheme is proposed. The main contributions of this chapter are as follows:

(i) A likelihood ratio based adaptation algorithm (on a simple population of the leader's trajectories) is employed by the adaptive followers to the member of a given finite class of models which is generating the reference trajectory of the leaders. Under appropriate conditions, it is shown that the true reference trajectory model is identified by finite time with probability 1 for each follower as the leaders population goes to infinity.

(ii) (ii) A demonstration that the use of the resulting mean field control laws yields a set of leaders and adaptive followers, the control laws possessing an almost sure (a.s) ϵ_N-Nash equilibrium property, where ϵ_N goes to zero as the population size N goes to infinity.

The implementation of the overall MF control laws for the leaders and followers assumes the following forms:

(i) Each leader enacts an adaptive MF control law which consists of the feedback of its own local stochastic state and the pre-computed leaders' deterministic mass effects.

(ii) Each follower enacts an adaptive MF control law which consists of the feedback of its own local stochastic state and the estimation based mass effects of the leaders and followers.

We first developed a non-adaptive but general model with weighed couplings in the leaders and the followers cost functions (which depended on the locality parameters of the agents). As is shown in [9] main adaptation result of the uniform cost coupling model in the case that followers' only track the centroid of the leaders. Subsequently, it is evident [8], the optimality properties of the adaptive followers MF control laws are studied. Here we complete the analysis of a more general (and realistic) scenario where the followers are tracking convex combinations of their own centroid and the centroid of their leaders [7].

Hence, we have an $\in_N$- Nash equilibrium property for the adaptive followers of MF control laws.

II. PROBLEM FORMATION, TERMINOLOGY AND APPLICATIONS:

Let the following notation be used to represent the integer valued subscript as the label for a certain agent of the population and superscript L and F for leader and follower agent respectively. In addition the expected value of a random variable, i.e., $\bar{z}(t) = E_z(t)$ is computed.. Again $\|.\|$ denotes the 2-norm of vector and $\|.\|_\infty$ denotes the infinity or sup norm. Then $\|z\|_Q = (Z^T QZ)^{\frac{1}{2}}$ stands for any appropriate dimension vector z and matrix $Q \geq 0$. A^T denotes the transpose of vector or matrix A and the trace of a square matrix A. Let C_n be the family of all n-dimensional continuous functions on $\mathbb{R}^+$ and

$$C_n{}^b = \{f \in C_n : \|f\|_\infty = sup_{t\geq 0}\|f(t)\| < \infty\}.$$

Note that $C_n{}^b$ is a Banach space under the norm $\|.\|_\infty$.

Let L denotes the countably infinite set of leaders,

$$\{L_{1,2}, \ \ldots \ . \ . \ . \ , L_{NL}\} \subset L \quad 1$$

of cardinality N_L and similarly for set of countably infinite followers F and the subset

$$\{F_{1,2}, \ \ldots \ . \ . \ . \ , F_{NF}\} \subset F \quad 2$$

of cardinality N_F . We assume that $L \cap F = \emptyset$.

1.1 Leader Stochastic LQG dynamic game model

The dynamics for the N_L leaders are given by

$$dz_i^L = (A_i z_i^L + B_i u_i^L)dt + C_i\, dw_i^L, \quad t \geq 0, \ 1 \leq i \leq N_L \quad 3$$

where $z_i^L \in \mathbb{R}^n$ is the state, $u_i^L \in \mathbb{R}^m$ is the control input, and $\{ w_i^L : 1 \leq i \leq N_L \}$ denotes a set of independent ρ - dimensional standard Wiener processes. The matrices A_i, B_i and C_i have compatible dimensions.

Let = $[A_i, B_i, C_i]$ be defined as the dynamical parameters associated with leader i,
$1 \leq i \leq N_L$ where we assume that, for all $1 \leq i \leq N$, are in the compact set $\ominus_L$. The initial states $\{(0) : 1 \leq i \leq N_L\}$ are assumed to be independent of $\{ w_i^L : 1 \leq i \leq N_L\}$. In addition we assume that $sup_{1\leq i\leq N}\ E\|Z_i^L\|^2 < \infty$.

The admissible control set for leader i, $1 \leq i \leq N_L$, is given by
$\mathcal{U}_i^L = \{ : {}_i^L(t)$ is adapted to sigma-field $\sigma(z_i^L(s), s \leq t, 1 \leq i \leq N_L)$

and

$$\|z_i^L(T)\| = O(\sqrt{T}), \quad \int_0^T \|z_i^L(T)\|^2 dt = O(T) \text{ a.s } \}. \quad 4$$

Again the cost function of the leaders is based on a trade-off between moving towards a common reference trajectory and keeping cohesion of the flock of leaders by tracking their Centroid.

Let $(z^{L,NL})(.) = \lambda h(.) + (1 - \lambda)\ (z^{L,})(.) \quad 5$

Where λ is a scalar in (0, 1), h belongs to $C_n{}^b$ is a reference trajectory known to all the leaders and

$$(z^{L,N_L})(.) = \frac{1}{N_L} \sum_{i=1}^{N_L} z_i^L \quad (.) \quad 6$$

is the centroid of the leaders. The objective of each individual leader i, $1 \leq i \leq N_L$ is to minimize its long time average (LTA) (i.e., ergodic) cost function given by

$$J_i^{L,N_L}(u_i^L, u_{-i}^L) = \lim_{T\to\infty} sup\ \frac{1}{T}\int_0^T (\|Z_i^L - \Psi^L(z^{L,N_L}) + \|_Q^2 + \|u_i^L\|_R^2)\, dt \quad 7$$

where the matrices Q and R are symmetric positive semi- definite and symmetric positive definite respectively, with compatible dimensions and

$$u_{-i}^L = (u_1^L, \ldots, u_{i-1}^L, u_{i+1}^L, u_{N_L}^L), \quad u_i^L = (u_1^L, \ldots\ldots\ldots, u_{N_L}^L)$$

To indicate the dependent J_i^L on $(u_i^L(.), u_{-i}^L(.))$ and the leaders population size N_L, we write it as $J_i^{L,N_L}(u_i^L, u_{-i}^L)$. Note that the leaders mean field cost coupling (5) is the same as mean field coupling in the basic models considered in [5], but with time varying offset term h(.). If $\lambda = 1$ then the leaders become independent such that each leader is interested in optimality tracking h(.) .

1.2 Follower Stochastic LQG Dynamic Game Model

Similarly the dynamics for the N_F followers are given by

$$dz_i^F = (A_i z_i^F + B_i u_i^F)dt + C_i\, dw_i^F, \quad t \geq 0, \quad 1 \leq i \leq NF \quad 8$$

where $z^F{}_i \in \mathbb{R}^n$ is the state $u_i^F \in \mathbb{R}^m$ is the control input, and $\{ w_i^F : 1 \leq i \leq N_F \}$ denotes a set of independent ρ - dimensional standard Wiener processes independent both $\{w_i^L : 1 \leq i \leq N_L\}$ and $\{(0) : 1 \leq i \leq N_L\}$. The matrices A_i, B_i and C_i have compatible dimensions.

Let $\theta_i = [A_i, B_i, C_i]$ be defined as the dynamical parameter associated with follower i,

$1\leq i \leq N_F$, where we assume that, for all $1\leq i \leq N_F$ are in the compact set $\ominus_F$. The initial states

$\{z_i^F(0) : 1\leq i \leq N_F\}$ are assumed to be independent and also independent of

$\{w_i^F: 1 \leq i \leq N_F\}$, $\{w_i^L : 1 \leq i \leq N_L\}$ and $\{z_i^F(0) : 1\leq i \leq N_F\}$.

In addition, we assume that $sup_{1\leq i\leq N} E\|Z_i^F(0)\|^2 < \infty$.

The admissible control set for follower i, $1 \leq i \leq N_F$ is given by

$\mathcal{U}_i^F = \{u_i^F : u_i(t)$ which is adapted to sigma -field $\sigma(z(s), z_k^L(s), s \leq t, 1 \leq i \leq N_F, 1\leq j \leq N_L)$ and

$\|z_i^F(T)\| = O(\sqrt{T}), \int_0^T \|z_i^F(T)\|^2 dt = O(T)$ a.s } 9

The followers react by tracking a convex combination of their own centroid and the centroid the leaders. We let

$\Psi^F(z^{L,NL}, z^{F,NF})(.) = \mu z^{L,NL}(.) + (1-\mu)(z^{F,})(.)$ 10

where μ is a scalar in (0,1), $(z^{F,N_F})(.) = \frac{1}{N_F}\sum_{i=1}^{N_F} z_i^F(.)$ 11

is the centroid of the followers and $z^{L,N}L$ is the centroid of the leaders defined in (5). The objective of each individual follower i, $1\leq i \leq N_F$ is minimized its long time average (LTA) (i.e., ergodic) cost function, given by

$J_i^{F,N}(u_i^F, u_{-i}^F, u^L) = \lim_{T\to\infty} sup \frac{1}{T}\int_0^T (\|Z_i^F - \Psi^F(z$ dt 12

where the matrices Q and R are symmetric positive semi- definite and symmetric positive definite respectively, with compatible dimensions and

$u_{-i}^F = (u_1^F, \ldots u_{i-1}^F, u_{i+1}^F, u_{N_F}^F)$, $u^L = (u_1^L, \ldots$, $N = N_L+N_F$.

To indicate the dependency of J_i^F on ($u_i^F(.), u_{-i}^F(.), u^L(.)$ and N is the population size of the system , we write it as $J_i^{L,N_L}(u_i^L, u_{-i}^L, u^L)$.

We note that in this model (i) the leaders are coupled to each other through their cost functions and respond to each of other their reference, and (ii) the followers attempt to track the convex combination of both their own centroid and the centroid of the leader.

1.3 Application

The leader-follower modelling of this chapter is motivated by many practical problems in which some agents in a group have more information than the others.

Application of the model is leader-follower dynamic version of Keynes' beauty contest games in economics. Keynes proposed beauty contest games where a newspaper print would print some photographs and people would vote for the prettiest faces. Everyone who picked the most popular face automatically entered a lottery to win a prize. Keynes remarked that the stock market is similar to beauty contest games where each investor would like to guess the other investors guesses. (see example 1 in [1]). A similar approach to MF stochastic control is considered in [1] to study large population static aggressive games such as Keynes' beauty contest games. Now we formulate a leader-follower LQG dynamic version of Keynes' beauty contest games. Here we consider a large population of players divided into two groups: (i) the leaders, as large well informed players (e.g., institutional investors in the stock market), and (ii) the followers (e.g., retail investors in the stock market). The state of each player is its publicly announced prediction of the prettiest face where $^L(.)$ denotes the state of the i[th] leader $(1 \leq i \leq N_L)$ and $z_i^F(.)$ denotes the state of the i[th] follower $(1 \leq i \leq N_F)$. The leaders and followers have linear stochastic dynamics given in 3 and 12 with different classes of parameters $\ominus_L$ and $\ominus_F$. The average prediction of the leaders and followers are given by their centroids

$$z^{L,N_L}(.) = \left(\frac{1}{N_L}\right)\sum_{i=1}^{N_L} z_i^L \text{ and } z^{F,N_F}(.) = \left(\frac{1}{N_F}\right)\sum_{i=1}^{N_F} z_i^F$$

respectively.

Based on the quadratic payoff functions considered in [2], we formulate cost functions of the agents as follows. The leaders would like to minimize their cost functions (7) based on a trade-off between making guesses, close to the exogenous private informative h(which is unknown to the followers) and guessing close to their own average prediction $z^{L,N}L$ (.). On the other hand, the follower would like to guess, close to some convex combination of their own average prediction $z^{F,}$ (.) and the average prediction of the leader $z^{L,}$ (.).

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