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Chaotic Dynamics of Fractional-order Volta System and Its Synchronization

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ABSTRACT. This paper addresses the chaotic behavior of fractional-order Volta system. Period-doubling routs to chaos have also been found. A simple, but effective linear coupling master-slave synchronization scheme is constructed. Finally, simulation results are presented to show the effectiveness of the proposed method.
Keywords: Fractional-order Volta system, Synchronization, Chaotic system.

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I. INTRODUCTION

Fractional-order chaotic systems, or systems that containing fractional order derivatives and integrals, have been addressed widely in the engineering research area. Some discussions and applications on this domain are presented by Podlubny [1], Luo et al. [2], and Hajiloo et al. [3] and the literatures therein. It should be pointed out that there is a growing number of real systems whose behavior can be opportunely described by using fractional system theory [4]. In [5], it is showed that most of the real systems are in fact with fractional order dynamics. It is also showed that the fractional order mathematical models are usually better than integer order ones to depict the plants with fractional order characteristics. For instance, the fractional capacitor and inductor can be used for the fractional order Chua's system [6], and numerical examples and measurements are carried out to confirm the fractional order characteristics of inductor.

Over the last two decades, since the pioneering work of Ott et al. [7], synchronization of chaotic systems has become more and more interesting to researchers in different domain. The problem of constructing a system, whose states mimics that of another chaotic system, is called completely synchronization. And the two chaotic systems are usually called drive (master) and response (slave) systems, respectively. Up to now, synchronization for chaotic fractional-order systems attracts more and more attention due to its potential applications in secure communication and control processing [8]. For instance, in [9], chaos synchronization of two fractional-order chaotic systems has been investigated. Also, synchronization of two chaotic fractional Chen systems and synchronization of two chaotic fractional Chua systems have been presented in Refs. [10], respectively.

In this paper, we numerically address the chaotic behaviors of fractional-order Volta chaotic system. Period-doubling routes to chaos in the fractional-order Volta system are also found. A synchronization method is also proposed. At last we study the synchronization performance through some simulations.

Chaotic Dynamics of fractional-order Volta System

Volta system can be described by the system of state differential equations [11]:

$$\begin{aligned}\dot{x}_1(t) &= -x_1(t) - ax_2(t) - x_3(t)x_2(t)\\ \dot{x}_2(t) &= -x_2(t) - bx_1(t) - x_1(t)x_3(t)\\ \dot{x}_3(t) &= cx_3(t) + x_1(t)x_2(t) + 1\end{aligned}\tag{1}$$

We can generalize system (1) to the fractional-order mode:

$$\begin{aligned}D^q x_1(t) &= -x_1(t) - ax_2(t) - x_3(t)x_2(t)\\ D^q x_2(t) &= -x_2(t) - bx_1(t) - x_1(t)x_3(t)\\ D^q x_3(t) &= cx_3(t) + x_1(t)x_2(t) + 1\end{aligned}\tag{2}$$

where $0 < q \le 1$ is the fractional derivative order.

In Fig.1 is shown the chaotic behavior toward fractional-order system (2), with system parameters $a = 19, b = 11, c = 0.73$ and fractional-order $q = 0.98$. The initial conditions are $x(0) = 8, y(0) = 2, \ z(0) = 1$.

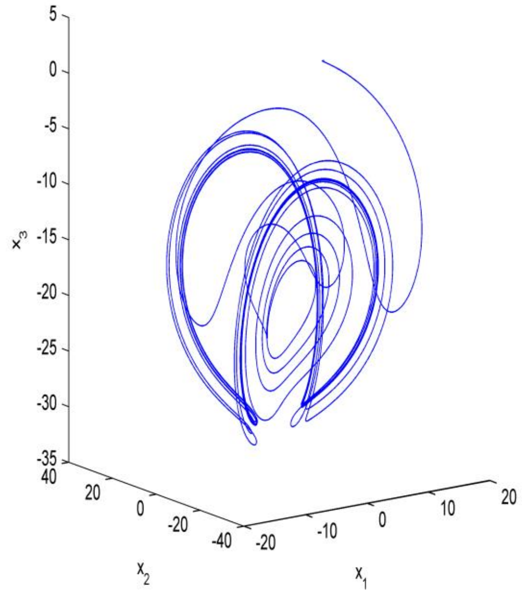

Fig.1 Chaotic attractor of fractional-order Volta system

Synchronization of fractional-order Volta chaotic system

Now, we write the master system as (2) and the coupled slave fractional-order system as:

$$D^q y_1(t) = -y_1(t) - ay_2(t) - y_3(t)y_2(t) + k_1(x_1(t) - y_1(t))$$
$$D^q y_2(t) = -y_2(t) - by_1(t) - y_1(t)y_3(t) + k_2(x_2(t) - y_2(t))$$
$$D^q y_3(t) = cy_3(t) + y_1(t)y_2(t) + 1 + k_3(x_3(t) - y_3(t))$$

(3)

where $k_i, i = 1,2,3$ are coupling strength constant parameters.

Let us define the synchronization errors as

$$e_i(t) = x_i(t) - y_i(t), i = 1,2,3. \quad (4)$$

Then form (2) and (3) we can obtain

$$D^q e_1(t) = -(1+k_1)e_1(t) - (a + x_3(t))e_2(t) - y_2(t)e_3(t)$$
$$D^q e_2(t) = (b + y_3(t))e_1(t) - (1+k_2)e_2(t) - x_1(t)e_3(t)$$
$$D^q y_3(t) = x_1(t)e_2(t) + y_2(t)e_1(t) + (c - k_3)e_3(t)$$

(5)

From Fig.1 we have that $|x_i(t)| \le 40, i = 1,2,3$. Then if we can choose k_i opportunely, then dynamical system (5) will be asymptotic stable.

Now we will give the simulation results. In the simulation, the coupling strength constant parameters are chosen as $k_1 = 2, k_2 = 2, k_3 = 2.$ The initial condition of the slave system are selected as $y_1(0) = -2, y_2(t) = -3, y_3(t) = -5.$ The simulation results can be seen in Fig.2 to Fig.5. From the simulation results, we can see that the synchronization errors have a fast convergence and good synchronization performance has been achieved.

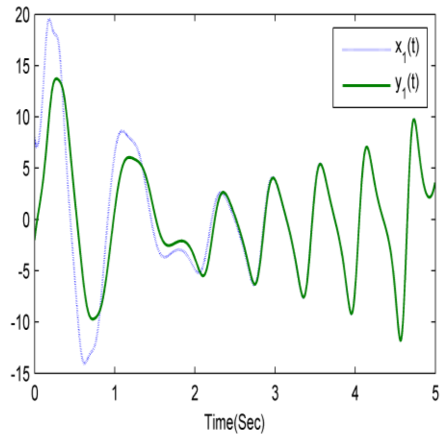

Fig.2 Synchronization of $x_1(t)$ and $y_1(t)$.

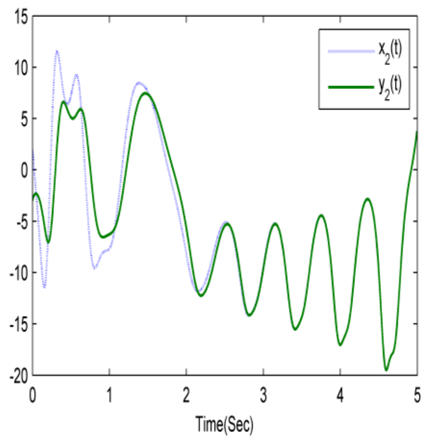

Fig.3 Synchronization of $x_2(t)$ and $y_2(t)$.

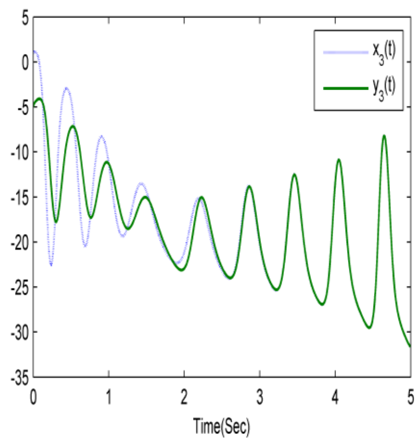

Fig.4 Synchronization of $x_3(t)$ and $y_3(t)$.

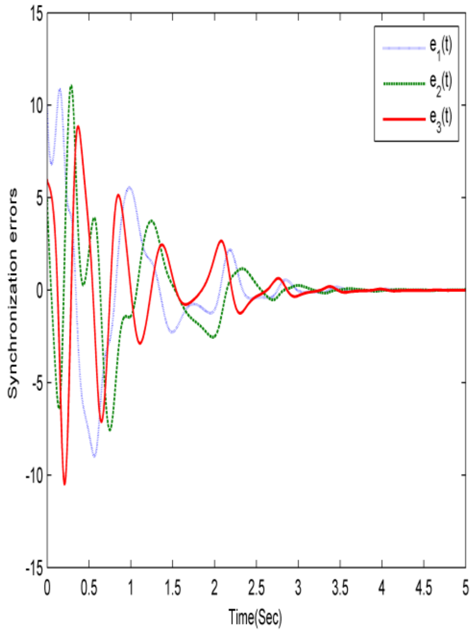

Fig.4 Time response of synchronization errors.

II. CONCLUSIONS

In this paper, we have investigated the chaotic behavior of the fractional Volta system. We found that chaos exists in this system with order 0.98. A simple linear coupling master-slave synchronization approach has also been proposed to synchronize the fractional-order Volta system. In the simulation results we know that good synchronization performance has been achieved.

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