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Effect of Heat Transfer on Peristaltic flow of Jeffrey fluid through Porous Medium in a Non-Uniform Channel

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ABSTRACT
In this paper, we have studied peristaltic flow of Jeffrey fluid and the effects of magnetic field and heat transfer on the fluid flow in a non-uniform porous channel. The equations have been fabricated by employing the wave frame and then reduced by assumption of long wavelength and low Reynolds number approximation. The solutions have been solved for the velocity, temperature and pressure gradient by using Adomian decomposition Method (ADM). The effects of physical parameters on the velocity, pressure gradient, pumping characteristics and temperature are discussed in detail with the aid of graphs.
***Keywords*:** Peristaltic flow, Jeffrey fluid, MHD, Darcy number, Porous medium, ADM

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I. INTRODUCTION

Peristalsis is a movement, caused by sequential muscle contraction that pushes the contents of the intestines or other tubular organs in one direction. It is an important and automatic process of the body which moves food through the digestive system, urine from the kidneys into the bladder. It is a normal function of the body and it can sometimes be felt in the belly (abdomen) as gas moves along. Some modern machinery imitates this design. The first systematic investigation of peristaltic movement was initiated by Latham in 1966 [1]. Non-Newtonian fluids generally exhibit a nonlinear relationship between the shear stress and the shear strain. Food stuffs (like banana juice, apple juice, chyme), blood, intra uterine fluid, etc. behave like non-Newtonian fluids. An attempt is made to study the non-Newtonian Jeffrey fluid model. It is one of the simplest fluid models compared to other fluids. Study of peristaltic flow of Jeffrey fluids useful in physiology and industry because of its large number of application and in mathematics due to its geometry and solutions of non-linear equations. For the conduction of heat transfer with peristaltic flow, there must be transport of fluid by contraction and expansion of kinetic energy. The several researches have been conducted to study the effects of heat transfer on peristaltic transport of Jeffrey fluids as cited [2-15]. This peristaltic process with heat transfer is helpful in the oxygenation. A porous medium consists of a number of tiny holes spread around the substance. For better analysis of numerous diseases like bladder and bacterial stones, cystitis and bacterial affection of kidneys, reflux conditions of peristaltic motion has been studied through porous medium. The influence of magnetic field on the peristaltic flow is considerably very large from the physiological point of view, such as the presence of haemoglobin molecule makes the blood a bio-magnetic fluid. Magnetic Resonance Imaging (MRI), magnetic devices and magnetic particles used as drug carriers are some of the applications of magnetic field in physiology as cited [16-19].

The concept of Adomian decomposition method (ADM) is a relatively new approach, which provides an analytic approximation to linear and non-linear problems. The method is quantitative rather than qualitative. It is analytic and requires neither linearization nor perturbation. It is also continuous with no resort to discretization. This analytic method provides the solution as an infinite series in which each term can be determined.

In view of the above, an attempt is made to study the effect of magnetic field and heat transfer on peristaltic flow of a Jeffrey fluid through a porous medium in a non-uniform channel under the assumptions of long wavelength and low Reynolds number. Expressions for the velocity and pressure gradient are obtained analytically. The effects of different physical imminent on the velocity, temperature and pumping characteristics are studied in detail.

II. MATHEMATICAL FORMULATION

Consider the peristaltic flow of an incompressible Jeffrey fluid with magnetic field and heat transfer. The Geometry of the Wall surface is maintained at non-uniform channel. The flow is produced by the sinusoidal waves propagating with constant speed c along the channel wall as given by the following.

$$\bar{H}(\bar{x},\bar{t}) = a(\bar{x}) + b\cos\left(\frac{2\pi}{\lambda}(\bar{X} - c\bar{t})\right), \tag{1}$$

where $a(\bar{x}) = a_0 + kx$ is the half width of the channel and b is the amplitude of the waves and t is the time, λ is the wavelength and $\bar{X}$ is the direction of wave propagation.

The constitutive equation for stress tensor $\bar{S}$ in Jeffrey fluid is given by

$$\bar{S} = \frac{\mu}{1+\lambda_1}(\dot{\tau} + \lambda_2 \ddot{\tau}), \tag{2}$$

where λ_1 is the rate of relaxation time to retardation time, λ_2 is the retardation time, $\dot{\tau}$ is the shear rate and dot over the quantities indicate differentiation with respect to time.

Introducing a wave frame $(\bar{x},\bar{y})$ moving with velocity c away from the fixed frame $(\bar{X},\bar{Y})$ by the transformation.

$$\bar{x} = \bar{X} - c\bar{t}, \bar{y} = \bar{Y}, \bar{u} = \bar{U} - c, \bar{v} = \bar{V}$$
$$\bar{p}(\bar{x}) = \bar{P}(\bar{X},\bar{t}), \tag{3}$$

where $(\bar{u},\bar{v})$ and $(\bar{U},\bar{V})$ are the velocity components, $\bar{p}$ *and* $\bar{P}$ are the pressures in wave and fixed frames of references, respectively.

Equations governing the flow field in a wave frame are

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{u}}{\partial \bar{y}} = 0, \tag{4}$$

$$\rho\left(\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}\right) = -\frac{\partial \bar{p}}{\partial \bar{x}} + \frac{\partial \bar{\tau}_{xx}}{\partial \bar{x}} + \frac{\partial \bar{\tau}_{xy}}{\partial \bar{x}} - \frac{\mu}{k_0}(\bar{u}+c) - \sigma B_0^2(\bar{u}+c), \tag{5}$$

$$\rho\left(\bar{u}\frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{v}}{\partial \bar{y}}\right) = -\frac{\partial \bar{p}}{\partial \bar{y}} + \frac{\partial \bar{\tau}_{yx}}{\partial \bar{x}} + \frac{\partial \bar{\tau}_{yy}}{\partial \bar{x}} - \frac{\mu}{k}\bar{v}, \tag{6}$$

$$\varsigma\left(\bar{u}\frac{\partial \bar{T}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{T}}{\partial \bar{y}}\right) = \frac{k}{\rho}\nabla^2\bar{T} + 2\bar{v}\left[\left(\frac{\partial \bar{u}}{\partial \bar{x}}\right)^2 + \left(\frac{\partial \bar{v}}{\partial \bar{y}}\right)^2\right] + \bar{v}\left[\left(\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{x}}\right)^2\right]. \tag{7}$$

where ρ is the density, k_0 is the permeability of the porous medium, ς is the specific heat at constant volume, $\bar{v}$ is the kinematic viscosity of the fluid, k is the temperature of the fluid,

$$\bar{\tau}_{xx} = \frac{2\delta}{1+\lambda_1}\left[\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\lambda_2 \delta c}{a}\left(\bar{u}\frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \bar{v}\frac{\partial^2 \bar{u}}{\partial \bar{x}\partial \bar{y}}\right)\right], \tag{8}$$

$$\bar{\tau}_{xy} = \frac{1}{1+\lambda_1}\left[\frac{\partial \bar{u}}{\partial \bar{y}} + \frac{\lambda_2 \delta c}{a}\left(\bar{u}\frac{\partial^2 \bar{u}}{\partial \bar{x}\partial \bar{y}} + \delta^2 \bar{u}\frac{\partial^2 \bar{v}}{\partial \bar{x}^2}\right)\right], \tag{9}$$

(9)

$$\bar{\tau}_{yy} = \frac{2\delta}{1+\lambda_1}\left[\frac{\partial \bar{v}}{\partial \bar{y}} + \frac{\lambda_2 \delta c}{a}\left(\bar{v}\frac{\partial^2 \bar{v}}{\partial \bar{y}^2} + \bar{u}\frac{\partial^2 \bar{v}}{\partial \bar{x}\partial \bar{y}}\right)\right], \tag{10}$$

$$\bar{\tau}_{yx} = \frac{1}{1+\lambda_1}\left[\delta^2\frac{\partial \bar{v}}{\partial \bar{x}} + \frac{\lambda_2 \delta c}{a}\left(\delta^2 \bar{v}\frac{\partial^2 \bar{v}}{\partial \bar{x}\partial \bar{y}} + \bar{v}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right)\right]. \tag{11}$$

The boundary conditions are

$$\bar{u}(\bar{y}) = 0,\ \bar{T}(\bar{y}) = \bar{T}_0 \quad at\ \bar{y} = 0, \tag{12}$$

$$\bar{u}(\bar{y}) = -c,\ \bar{T}(\bar{y}) = \bar{T}_1 \quad at\ \bar{y} = \bar{h}. \tag{13}$$

Introducing the non-dimensional quantities

$$x = \frac{\bar{x}}{\lambda},\ y = \frac{\bar{y}}{a},\ u = \frac{\bar{u}}{c},\ v = \frac{\bar{v}}{c\delta},\ t = \frac{c\bar{t}}{\lambda},\ \mathrm{Re} = \frac{\rho c a}{\mu},$$
$$Fr = \frac{c^2}{ga}, \alpha = \frac{b}{a_0},\ \theta = \frac{\bar{T} - \bar{T}_0}{\bar{T}_1 - \bar{T}_0},\ \delta = \frac{a}{\lambda},$$
$$Da = \frac{k_0}{a^2},\ \mathrm{Pr} = \frac{\rho v \varsigma}{k},\ Ec = \frac{c^2}{\varsigma(\bar{T}_1 - \bar{T}_0)},\ p = \frac{a^2 \bar{p}}{\mu_0 c \lambda},$$
$$h = \frac{\bar{h}}{a_0} = 1 + \frac{\lambda k x}{a_0} + \alpha\cos\left(\frac{2\pi x}{\lambda}\right), M^2 = \frac{a^2 \sigma B_0^2}{\mu}. \tag{14}$$

where $\mathrm{Re} = \frac{\rho c a}{\mu}$ is the Reynolds number , $M^2 = \frac{a^2 \sigma B_0^2}{\mu}$ is the Hartmann number.

In view of (14), the equations (4)-(7), reduces to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{15}$$

$$\mathrm{Re}\,\delta\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \delta\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} - \left(\frac{1}{Da}(u+1) + M^2(u+1)\right) \tag{16}$$

$$\mathrm{Re}\,\delta^3\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \delta^2\frac{\partial \tau_{yx}}{\partial x} + \delta\frac{\partial \tau_{yy}}{\partial y} - \frac{\delta^2}{Da}v, \tag{17}$$

$$\mathrm{Re}\,\delta\left(u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y}\right) = \frac{1}{\mathrm{Pr}}\left(\delta^2\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right) + Er4\delta^2\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + Er\delta^4\left(\frac{\partial v}{\partial x}\right)^2 + Er2\delta^2\frac{\partial u}{\partial y}\frac{\partial v}{\partial x} \tag{18}$$

Under the assumptions of long wavelength $(\delta \ll 1)$ and low Reynolds number the equations (16)-(18) becomes

$$\frac{\partial p}{\partial x}=\frac{1}{1+\lambda_1}\frac{\partial^2 u}{\partial y^2}-\left[\frac{1}{Da}+M^2\right](u+1)\,, \qquad (19)$$

$$\frac{\partial p}{\partial y}=0\,, \qquad (20)$$

$$\frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2}+Ec\frac{\partial^2 u}{\partial y^2}=0\,. \qquad (21)$$

The non-dimensional boundary conditions are

$$u(y)=0,\ \theta(y)=0\ \ at\ y=0 \qquad (22)$$

$$u(y)=-1,\ \theta(y)=1\ \ at\ y=h \qquad (23)$$

III. SOLUTION

Solving Equation (19) using Adomian decomposition method along with boundary conditions (22)-(23), we get

$$u=c_1\cosh Ny+c_2\sinh Ny+\left(\frac{1+\lambda_1}{N^2}\frac{dp}{dx}+1\right)\left(\cosh Ny-1\right) \qquad (24)$$

$$N^2=\frac{1+\lambda_1}{Da},c_1=0\ and$$

where $c_2=\frac{N}{\sinh Nh}\left(1-\left(\frac{1+\lambda_1}{N^2}\frac{dp}{dx}+1\right)\left(\cosh Ny-1\right)\right)$.

Solving Equation (21) using Adomian decomposition method along with boundary conditions (22)-(23), we get

$$\theta=c_3+c_4-\frac{\Pr Ec}{8}\left(\frac{1+\lambda_1}{N^2}\frac{dp}{dx}+1\right)^2\frac{\left(\cosh Nh-1\right)^2}{\left(\sinh Nh\right)^2}\left(2N^2y^2+\cosh 2Ny\right)+\cosh 2Ny-2N^2y^2-2\sinh 2Ny\left(\frac{\cosh Nh-1}{\sinh Nh}\right) \qquad (25)$$

where $c_3=\frac{\Pr Ec}{8}\left(\frac{1+\lambda_1}{N^2}\frac{dp}{dx}+1\right)^2\left(1+\frac{\left(\cosh Nh-1\right)^2}{\left(\sinh Nh\right)^2}\right)$,

The rate of volume flow rate through each section in a wave frame, is calculated as

$$q=\int_0^h u\ dy \qquad (26)$$

The volume flow rate q in the wave frame of reference is given by

$$q=\frac{-Nh-\left(\frac{1+\lambda_1}{N^2}\frac{dp}{dx}+1\right)\left(\left(\cosh Ny-1\right)^2+\left(Nh-\sinh Nh\right)\sinh Nh\right)}{N\sinh Nh} \qquad (27)$$

The flux at any axial station in the laboratory frame is

$$Q(x,t)=\int_0^h (u+1)dy=q-h \qquad (28)$$

The average volume flow rate over one period $\left(T=\lambda/c\right)$ of the peristaltic wave is defined as

$$\bar{Q}=\frac{1}{T}\int_0^T Q\ dt=q+2 \qquad (29)$$

The pressure gradient can be written as

$$\frac{dp}{dx}=\frac{N^2}{1+\lambda_1}\left(\frac{-Nh-Nq\sinh Nh}{\left(\cosh Nh-1\right)^2+\left(Nh-\sinh Nh\right)\sinh Nh}-1\right) \qquad (30)$$

The dimensionless pressure rise per one wavelength in the wave-frame are defined as

$$\Delta p=\int_0^1\frac{dp}{dx}dx \qquad (31)$$

IV. RESULTS DISCUSSION

In this paper the analytical results of peristaltic flow of Jeffrey fluid in a non-uniform porous channel under the influence of MHD and heat transfer are obtained. To study the behaviour and physical imminent of the solutions, numerical calculations for several values of Jeffrey fluid parameter λ_1, Hartmann number M, Darcy number Da, Porous medium φ have been calculated using MATHEMATICA software.

Figure (1)-(3) shows the variations of the axial pressure gradient dp/dx with respect the axial x in which it has oscillatory behaviour in the whole range of the x-axis for different values of $M, Da, \lambda_1, \varphi$. Figure (1) shows that magnitude of pressure gradient with the increase in M. It isalso seen that pressure gradient attains maximum value when $x=0.5$, and the pressure gradient near the channel wall is small. This enables us the fact that flow can easily pass in the middle of the channel. Figure (2) shows that variation of pressure gradient with the parameter λ_1 is very much similar to that of the parameters M as shown in figure (1). Figure (3) shows that the magnitude of pressure gradient dp/dx with the increasing Da. It is noted here that pressure gradient decreases with increasing Da.

The pressure rise Δp versus flow rate $\bar{Q}$ for different values of Jeffrey fluid parameter λ_1, porous medium φ and magnetic field M, are plotted in figures (4)- (6). Figure (4) represents the variation of Δp versus flow rate $\bar{Q}$ for different values of λ_1. We observe that the rate of pumping decreases with increasing λ_1. Figure (5) represents the variation Δp ofversus flow rate $\bar{Q}$ for different values of φ. We observe that pressure rise increases with increasing φ. figure (6) represents the variation of Δp versus flow rate $\bar{Q}$ for different values of M. We observe that pressure rise increases with increase in M.

The temperature profiles for different values of Jeffrey fluid parameter λ_1, Darcy number Da, magnetic field M are plotted in figures (7)-(10). Figure (7) represents the variation of temperature for different values of λ_1. It is observed that Temperature profile decreases with increase of λ_1. Figure (8) represents the variation of temperature for different values of Da. It is observed that temperature profile decreases with increasing Da .figure (9) represents the variation of temperature for different values of M. It is observed that temperature profile increases with increasing M. figure (10) represents the variation of temperature for different values of φ. It is observed that temperature profile increases with increasing φ.

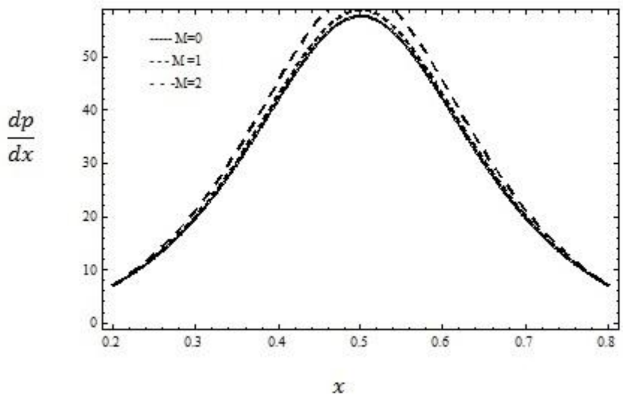

Fig (1) Influence of M on dp/dx when Da =0.1, $x = 0.3, \varphi = 0.5, \lambda_1 = 0.3$

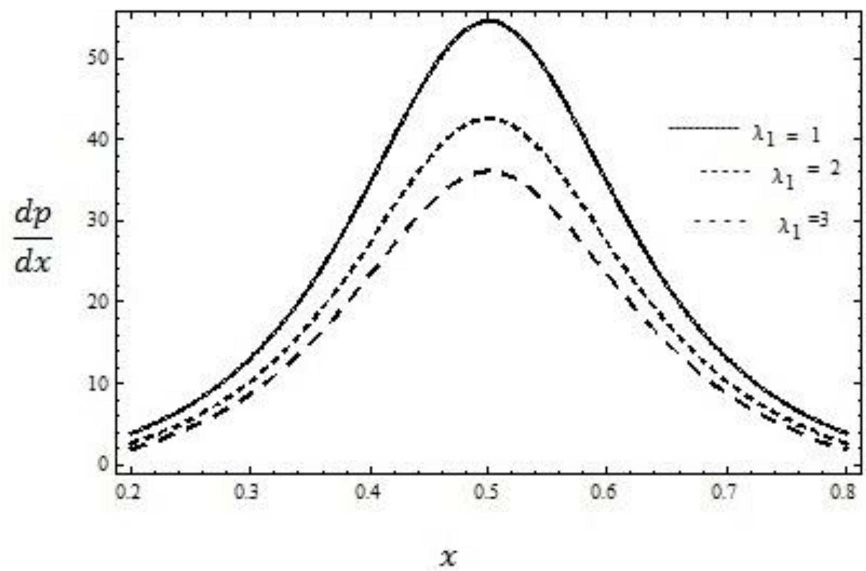

Fig (2) Influence of λ_1 on dp/dx when $M = 0.5, x = 0.3, \varphi = 0.5, Da = 0.$

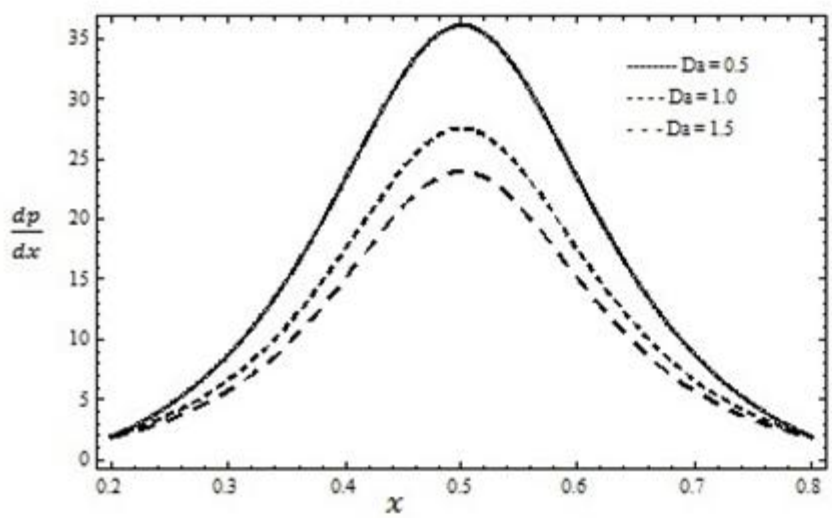

Fig (3) Influence of Da on dp/dx when $M = 0.5, x = 0.3, \varphi = 0.5,\ \lambda_1 = 3$

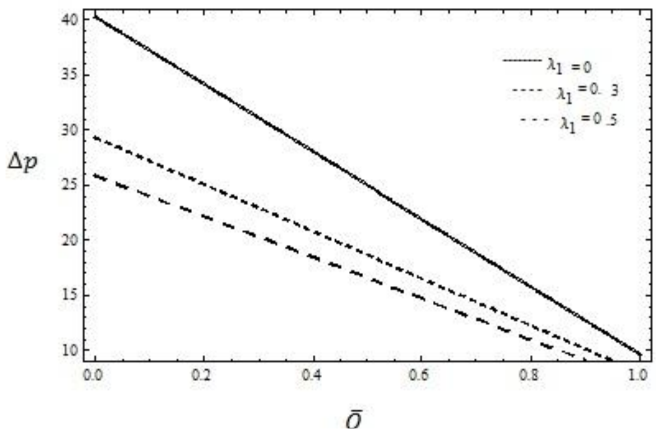

Fig (4) Influence of λ_1 on Δp when $M = 1, x = 0.3, \varphi = 0.2, Da = 0.1$

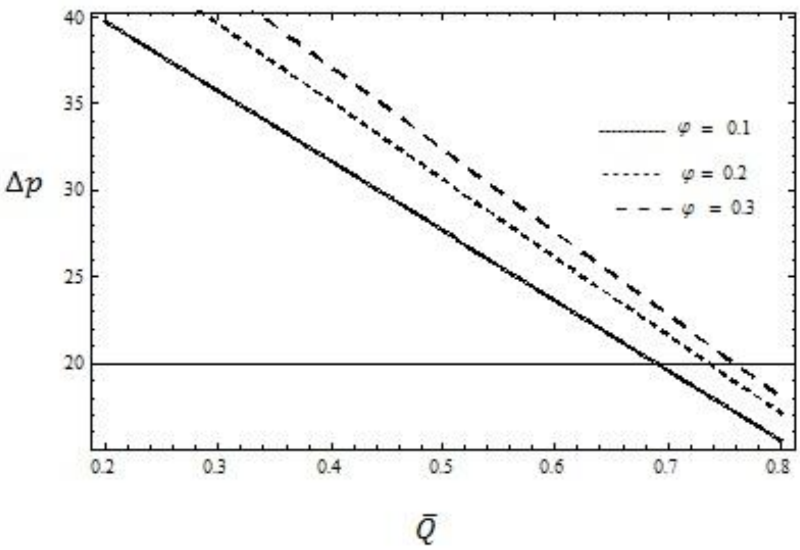

Fig (5) Influence of φ on Δp when $M = 2, x = 0.3,, Da = 0.05,\ \lambda_1 = 0$

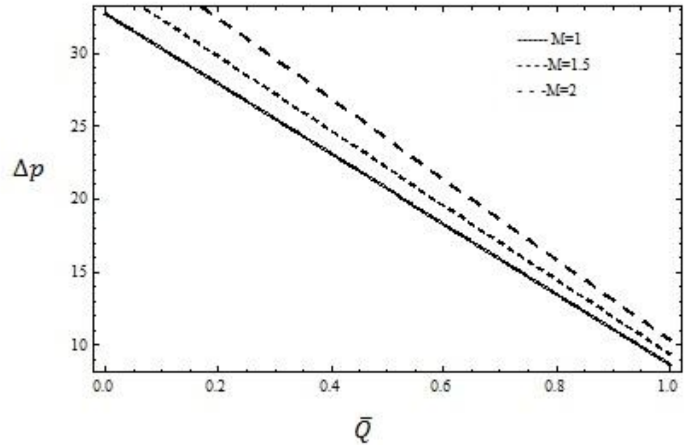

Fig (6) Influence of M on Δp when $\lambda_1 = 0.5, x = 0.3, \varphi = 0.2, Da = 0.1$

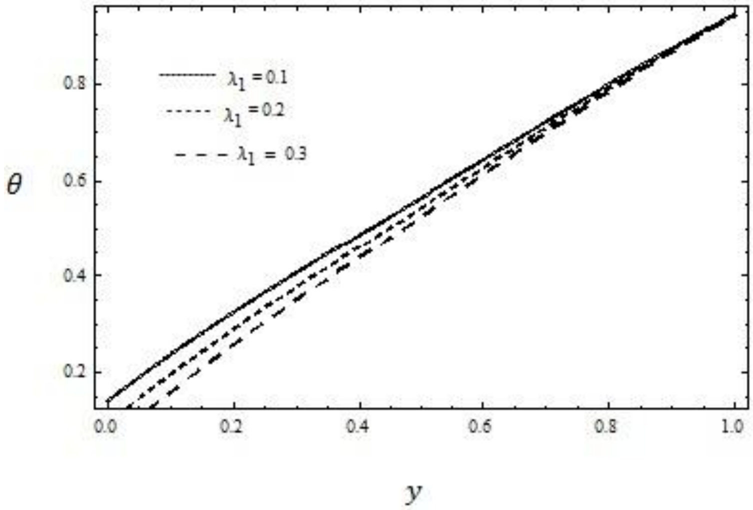

Fig (7) Influence of λ_1 on θ when $M = 0.5, x = 0.2\ , Da = 1, \varphi = 0.3.$

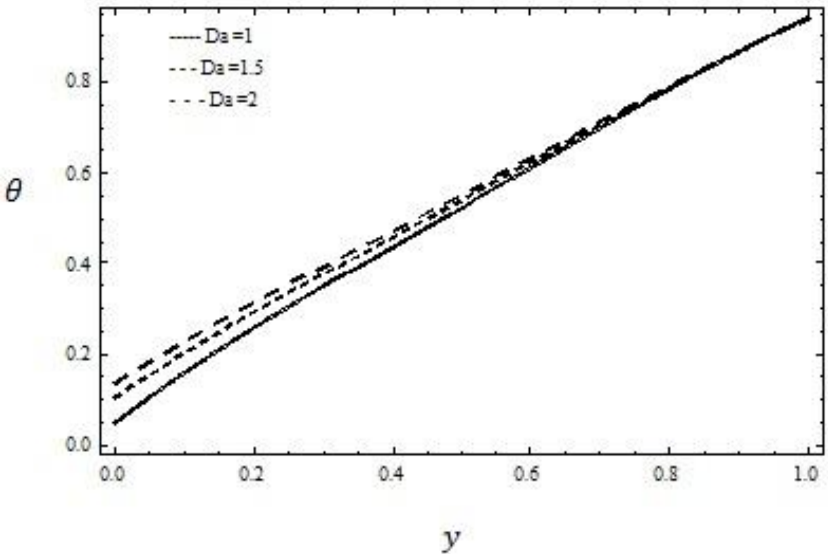

Fig (8) Influence of Da on θ when $\lambda_1 = 0.3, x = 0.2\ , \varphi = 0.3, M = 0.5.$

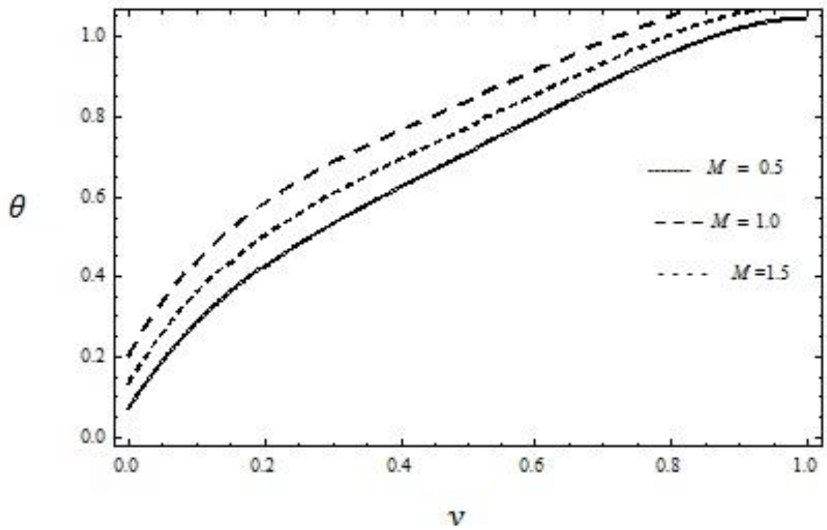

Fig (9) Influence of M on θ when $\lambda_1 = 0.3, x = 0.2\,, Da = 1, \varphi = 0.3.$

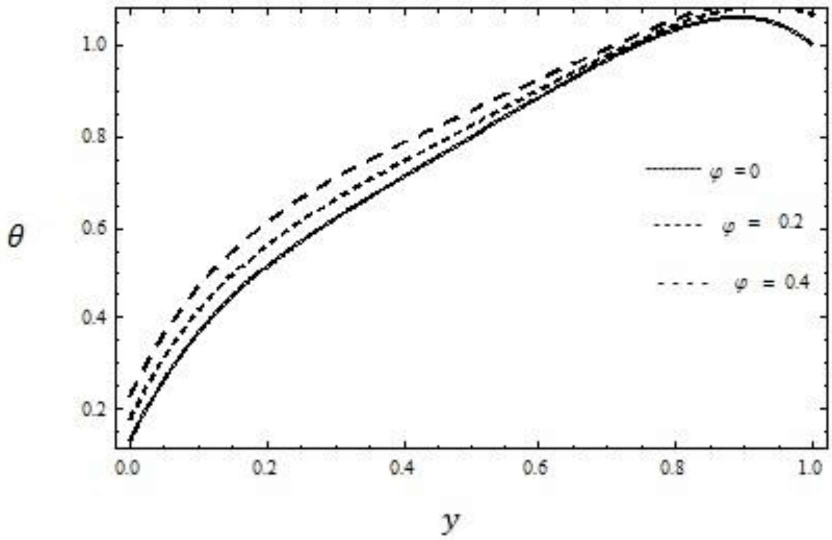

Fig (10) Influence of φ on θ when $\lambda_1 = 0.3, x = 0.2\,, Da = 1, M = 2.5.$

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