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Determination of Electric and MagneticFieldsin Terms of Current by using the Lorentz Condition Approach

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ABSTRACT: There are so many approach used to determine the electric and magnetic fields. To determine an expression for the electric field and magnetic field in terms of current is used in this Lorentz condition approach. In this approach, the expression of electric and magnetic fields are determined.There are two expressions for the electric field and magnetic field in terms of current by using scalar and vector potentials.In the electric field, there are three components of static field, induction field, and radiation field but in magnetic field there are only two, induction and radiation fields.

Keywords: Scalar and Vector Potentials; Lorentz Condition approach; Electric field; Magnetic field.

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I. INTRODUCTION:

The lightning discharge is the phenomena in which electrostatic discharge occurs between two charged regions may be between the two clouds, or between a cloud and a ground [1] - [5]. If there are a sufficiently charges in these regions which produce the high electric potential between them in static case. If the amount of charges in the cloud increases over the surface of the earth sufficiently large, an equal electric charge of opposite polarity is induced on the earth surface [3] – [7]. The greater the accumulation of charges, higher will be the electric field produced. If the charges are in motion then there is a current, which produced the magnetic field [6] – [17]. There are so many approach used to determine the electric and magnetic fields. In the Lorentz Condition Approach, the expression of electric and magnetic fieldsshould be determined by using scalar and vector potential.In this paper, there are two expressions for the electric field and magnetic field in terms of current by usingscalar and vector potentials.

II. THEORY AND DISCUSSION:

The lightning return Stroke channel, as shown in the figure below, can be modeled as a straight line in vertical direction that is fixed at the point A. Another end extended with the speed v. Let i(z', t) be the current in the lightning channel, z' be the position along z-axis in a certain time t. When t = 0, the channel starting from A, from which the return stroke starting to propagate. Let P be the point of observer, which observes the return stroke at a distance r as shown in figure. The time taken by this light to travel this distance is r/c where c is the velocity of light. So the retarded current for the elemental section dz' is $i\left(z', t - \frac{R(z')}{c}\right)$.Let L'(t) be the total length of the return stroke channel at the time t, observed by the observer at the point P.

We can consider, the large number of dipoles in the lightning channel with the dz', length of dipole and due to this the electric and magnetic fields should be produced. The vector potential at the point P due to the return stroke channel starting from the point A where z' = 0 to the end z = L'(t) is given by[17].

$$A(r, \theta, \tau) = \frac{\mu_0}{4\pi}\int_0^{L'(t)} \frac{i\left(z', \tau - \frac{R(z')}{c}\right)}{R(z')}\hat{z}\,dz'$$

Whereτ is the time variable less than the total time t.

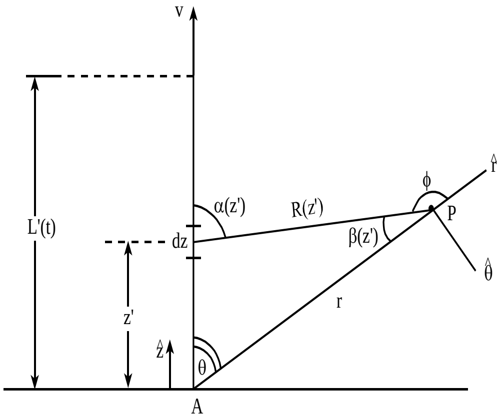

Figure:*Geometry of the problem to determine the Electric and Magnetic Fields by using the Lorentz Condition Approach*

From the Lorentz condition,

$$\nabla.\vec{A}+\frac{1}{c^2}.\frac{\partial\phi}{\partial t}=0$$

or, $$\frac{\partial\phi}{\partial t}=-c^2\nabla\vec{A}$$

$$\square \quad \square\square=-c^2\int_{r/c}^{t}\nabla A\, d\square$$

Now, the divergence of the scalar potential, [18] – [20]

$$\nabla\vec{A}=\frac{\mu_0}{4\pi}\int_0^{L'(\tau)}\nabla.\frac{i\left(z',\tau-\frac{R(z')}{c}\right)}{R(z')}\hat{z}\,dz'$$

$$=\frac{\mu_0}{4\pi}\int_0^{L'(\tau)}\left[\nabla\left(\frac{1}{R(z')}\right)i\left(z',\tau-\frac{R(z')}{c}\right)+\frac{1}{R(z')}\nabla i\left(z',\tau-\frac{R(z')}{c}\right)\right]\hat{z}\,dz'$$

We know, from the figure

$$R(z')=\sqrt{r^2+z'^2-2rz'\cos\theta}$$

$$\frac{dR(z')}{dr}=\frac{r-z'\cos\theta}{R(z')}$$

$$\frac{dR(z')}{d\theta}=\frac{rz'\sin\theta}{R(z')}$$

$$\frac{\partial i}{\partial R}=-\frac{1}{c}\frac{\partial i}{\partial t},\quad \text{where } i\equiv i\left(z',t-\frac{R(z')}{c}\right)$$

$$\frac{dR(z')}{dz'}=\frac{2z'-2r\cos\theta}{2R(z)}=\frac{z'-r\cos\theta}{R(z')}$$

$$\nabla\left(\frac{1}{R}\right)=-\frac{1}{R^2}.\frac{\partial R}{\partial L}=-\frac{1}{R^2}.\frac{z'-r\cos\theta}{R(z')}$$

$$=\frac{-z'-r\cos\theta}{R^3(z')}$$

$$\nabla i\equiv\frac{\partial i}{\partial z}=\frac{\partial i}{\partial R}.\frac{\partial R}{\partial z}=-\frac{1}{c}.\frac{\partial i}{\partial t}\times\frac{z'-r\cos\theta}{R(z')}$$

.

On substituting in the above equation

$$\nabla.A=\frac{\mu_0}{4\pi}\int_0^{L'(\tau)}\left\{-\left[\frac{z'-r\cos\theta}{R^3(z')}\right]i\left(z',\tau-\frac{R(z')}{c}\right)+\frac{1}{R(z')}\frac{z'-r\cos\theta}{R(z')}\right.$$
$$\left.\times\left(-\frac{1}{c}\frac{\partial i(z',\tau-R(z')/c)}{\partial\tau}\right)\right\}dz'+\frac{\mu_0}{4\pi}\frac{L'(\tau)-r\cos\theta}{cR^2(L')}\times i\left(L',\tau-\frac{R(L')}{c}\frac{dL'(\tau)}{d\tau}\right)$$

$$=\frac{1}{4\pi\epsilon_0c^2}\int_0^{L'(\tau)}\left[\frac{z'-r\cos\theta}{R^3(z')}i\left(z',\tau-\frac{R(z')}{c}\right)+\frac{z'-r\cos\theta}{cR^2(z')}\frac{\partial i(z',\tau-R(z')/c)}{\partial\tau}\right]dz'\quad+$$
$$\frac{1}{4\pi\epsilon_0c^2}\frac{L'(\tau)-r\cos\theta}{cR^2(L')}i\left(L',\tau-\frac{R(L')}{c}\right)\frac{dL'(\tau)}{d\tau}$$

So, on substituting,

$$\square\square\square \quad \square\square c^{\square}\int_{r/c}^{t}(\nabla.A)\square\square\square$$

$$=\int_{r/c}^{t}\left[\frac{1}{4\pi\epsilon_0}\int_0^{L'(\tau)}\left\{\frac{z'-r\cos\theta}{R^3(z')}i\left(z',\tau-\frac{R(z')}{c}\right)+\frac{z'-r\cos\theta}{cR^2(z')}\frac{\partial i}{\partial t}\left(z',\tau-\frac{R(z')}{c}\right)\right\}dz'\quad+\right.$$
$$\left.\frac{1}{4\pi\epsilon_0}\frac{L'(\tau)-r\cos\theta}{cR^2(L')}i\left(L',\tau-\frac{R(L')}{c}\right)\frac{dL'(\tau)}{d\tau}\right]d\square$$

On interchanging the order of integration, and as the time changing from r/c to t, the channel length L'() also changing from, 0 to L'(t).

We know, the time t $=\frac{L'(\tau)}{v}+\frac{R[L'(\tau)]}{c}=\frac{z'}{v}+\frac{R(z')}{c}$

Substituting in the above equation and keeping the limit of integration then we get an expression for scalar potential as,

$$\square=-\frac{1}{4\pi\epsilon_0}\int_0^{L'(t)}\left[\frac{z'-r\cos\theta}{R^3(z')}\int_{\frac{z'}{v}+\frac{R(z')}{c}}^{t}i\left(z',t-\frac{R(z')}{c}\right)d\tau+\frac{z'-r\cos\theta}{cR^2(z')}i\left(z',t-\frac{R(z')}{c}\right)\right]dz'$$

Now, the gradient of scalar potential, $\nabla\square$, and the time derivative of the vector potential, $\frac{\partial A}{\partial t}$

Substituting on the equation,

$E=-\nabla\square-\frac{\partial A}{\partial t}$ then we get the equation as follows.

$$E(r,\square,t)=-\frac{\hat{r}}{4\pi\epsilon_0}\int_0^{L'(t)}\frac{\cos\theta-3\left\{-\frac{(z'-r\cos\theta)}{R(z')}.\frac{r-z'\cos\theta}{R(z')}\right\}}{R^3(z')}\int_{t_b}^{t}i\left(z',\tau-\frac{R(z')}{c}\right)d\square\square dz'$$

$$-\frac{1}{4\pi\epsilon_0}\hat{r}\int_0^{L'(t)}\frac{\cos\theta-3\left\{-\frac{z'-r\cos\theta}{R(z')}.\frac{r-z'\cos\theta}{R(z')}\right\}}{CR^2(z')}i\left(z',t-\frac{R(z')}{c}\right)dz'$$

$$-\frac{1}{4\pi\epsilon_0}\hat{r}\int_0^{L'(t)}\frac{\cos\theta+\frac{(z'-r\cos\theta)(r-z'\cos\theta)}{R^2(z')}}{CR^2(z')}\frac{\partial i\left(z',t-\frac{R(z')}{c}\right)}{\partial t}dz'$$

$$+\frac{1}{4\pi\epsilon_0}\hat{\theta}\int_0^{L'(t)}\frac{\sin\theta+3\frac{(r\cos\theta-z')(z'\sin\theta)}{R^2(z')}}{R^3(z')}\int_{t_b}^{t}i\left(z',\tau-\frac{R(z')}{c}\right)d\square\square dz'$$

$$+\frac{1}{4\pi\epsilon_0}\hat{\theta}\int_0^{L'(t)}\frac{\sin\theta+3\frac{(r\cos\theta-z')(z'\sin\theta)}{R^2(z')}}{CR^2(z')}i\left(z',t-\frac{R(z')}{c}\right)dz'$$

$$+\frac{1}{4\pi\epsilon_0}\hat{\theta}\int_0^{L'(t)}\frac{\sin\theta+\frac{(r\cos\theta-z')(z'\sin\theta)}{R^2(z')}}{CR^2(z')}\frac{\partial i\left(z',t-\frac{R(z')}{c}\right)}{\partial t}dz'$$

$$-\frac{1}{4\pi\epsilon_0}\hat{r}\left\{\frac{\cos\theta-\frac{(r\cos\theta-L')}{R(L')}\cdot\frac{(r-L'\cos\theta)}{R(L')}}{C^2R(L')}\right\}i\left(L',t-\frac{R(L')}{c}\right)\frac{dL'}{dt}$$

$$+\frac{1}{4\pi\epsilon_0}\hat{\theta}\left\{\frac{\sin\theta+\frac{(r\cos\theta-L')}{R(L')}\cdot\frac{L'\sin\theta)}{R(L')}}{C^2R(L')}\right\}i\left(L',t-\frac{R(L')}{c}\right)\frac{dL'}{dt}$$

Here, $\frac{dL'}{dt}$ is the speed of the current wave-front observed from the point P, v be the velocity of the return stroke wave-front. In the above equation, the term containing the factor $\frac{1}{R^3(z')}$ is related to static component, $\frac{1}{CR^2}$ is related to the induction component and $\frac{1}{C^2R}$ is related to the radiation component.When □ = 90□ , then $\hat{\theta} = -\hat{z}$, $\hat{r}$ is along the horizontal ground.

Let us assumed that the ground to be infinitely conducting then the effect of the ground plane as a image channel carrying an image current. In the above equation, the two terms (of the above the ground and the image) are in opposite vector of $\hat{r}$ and so cancelled each other. (Due to the equal and opposite cancelled them).Only the field expression containing the vector $\hat{\theta}$ added up and we get the total field expression as:

$$E(r,□,t)=\frac{1}{2\pi\epsilon_0}\int_0^{L'(t)}\frac{2-3\sin^2\alpha(z')}{R^3(z')}\int_{t_b}^{t}i\left(z',\tau-\frac{R(z')}{c}\right)d□dz'+\frac{1}{2\pi\epsilon_0}\int_0^{L'(t)}\frac{2-3\sin^2\alpha(z')}{cR^2(z')}i\left(z',t-\frac{R(z')}{c}\right)dz' -$$

$$\frac{1}{2\pi\epsilon_0}\int_0^{L'(t)}\frac{\sin^2\alpha(z')}{cR^2(z')}\frac{\partial i\left(z',t-\frac{R(z')}{c}\right)}{\partial t}dz'-\frac{1}{2\pi\epsilon_0}\frac{\sin^2\alpha(L')}{c^2R(L')}i\left(L',t-\frac{R(L')}{c}\right)\frac{dL'}{dt}$$

Where, $\sin □=\frac{r\sin\theta}{R(z')}$ i.e. $\sin□(z')=\frac{r\sin\theta}{R(z')}$; $\sin□(L')=\frac{r\sin\theta}{R(L')}$

If there is no current discontinuity at the propagating wave-front, i.e. if $i\left(L',t-\frac{R(L')}{c}\right)=0$, then the above equation becomes,

$$E(r,□,t)=\frac{1}{2\pi\epsilon_0}\int_0^{L'(t)}\frac{2-3\sin^2\alpha}{R^3(z')}\int_{t_b}^{t}i\left(z',\tau-\frac{R(z')}{c}\right)d□dz' + \frac{1}{2\pi\epsilon_0}\int_0^{L'(t)}\frac{2-3\sin^2\alpha}{cR^2(z')}i\left(z',t-\frac{R(z')}{c}\right)dz' -$$

$$\frac{1}{2\pi\epsilon_0}\int_0^{L'(t)}\frac{\sin^2\alpha}{c^2R(z')}\frac{\partial i\left(z',t-\frac{R(z')}{c}\right)}{\partial t}dz'$$

The magnetic field is given by $B=\nabla\times A$. For a vertical channel, there is only a horizontal component of the magnetic field. On taking the curl of the vector potential, the magnetic field at the ground level is

$$B = \frac{1}{2\pi\epsilon_0c^2}\int_0^{L'(t)}\left[\frac{\sin^2\alpha(z')}{R^2(z')}i\left(z',t-\frac{R(z')}{c}\right)+\frac{\sin\alpha(z')}{cR(z')}\frac{\partial i\left(z',t-\frac{R(z')}{c}\right)}{\partial t}\right]dz' +$$

$$\frac{1}{2\pi\epsilon_0c^2}\frac{\sin\alpha(L')}{cR(L')}i\left(L',t-\frac{R(L')}{c}\right)\frac{dL'}{dt}$$

If there is no current discontinuity at the propagating wavefront i.e. if $i\left(L',t-\frac{R(L')}{c}\right)$ = 0, then the last term of the above equation vanishes. Hence the total magnetic field at the point on the ground level is

$$B=\frac{1}{2\pi\epsilon_0c^2}\int_0^{L'(t)}\left[\frac{\sin\alpha}{R^2(z')}i\left(z',t-\frac{R(z')}{c}\right)+\frac{\sin\alpha}{cR(z')}\frac{\partial i\left(z',t-\frac{R(z')}{c}\right)}{\partial t}\right]dz'$$

The first term containing $1/R^2(z)$ indicate the induction component and the term containing 1/CR(Z) indicates the radiation components.

III. CONCLUSION:

There are two equations which represents electric and magnetic fields. In the equation of the electric field, there are three terms in which the first term represent the electrostatic field which have R^{-3} term, the second term contain $c^{-1}R^{-2}$ which represents the induction field, and the last term represents the radiation field containing terms $1/c^2R(z')$. But in the equation of the magnetic field, there are two terms in which the first term represent the induction field which contain $R^{-2}(z)$ and the last term represents the radiation field containing terms 1/cR(z'). Hence in the electric field, there are three components of static field, induction field, and radiation field but in magnetic field there are only induction and radiation fields.

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