

## A Variable p-CMA Algorithm for Blind Channel Equalization

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### ABSTRACT

In this work, we propose a novel variant of the conventional p- Constant Modulus Algorithm (p-CMA) algorithm. The rationale behind our work is that the exponent p in the standard CMA can play significant role in the convergence speed and steady-state performance of the algorithm. It was observed that the larger value of p can provide faster convergence at the cost of higher steady-state excess mean-square-error (EMSE). On the other hand, smaller value of p can give lower EMSE at the cost of the slower speed of convergence. Thus, in this work, we develop a time variant p parameter by utilizing the energy of the estimation error such that the value of p takes larger values in the beginning of adaptation and reduces to smaller values near steady-state. Thus, it can achieve both faster convergence and smaller steady-state EMSE. Simulation results are provided to support our theoretical claims.

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### I. INTRODUCTION

The basic idea behind an adaptive blind equalizer is to minimize or maximize some admissible blind objective or cost function through the choice of filter coefficients based on the equalizer output [1–4]. In the context of adaptive blind equalization, the widely adopted algorithm is Constant Modulus Algorithm (CMA2-2) [2,5,6,7]. For quadrature amplitude modulation (QAM) signaling, however, a tailored version of CMA2-2, commonly known as Multi-modulus Algorithm (MMA2-2), is considered to be more suitable. The MMA2-2 is capable of jointly achieving blind equalization and carrier phase recovery [8–13], whereas the CMA2-2 requires a separate phase-lock loop for carrier phase recovery.

#### Blind Equalization and conventional CMA Algorithm

The Figure 1 explains the CMA blind equalization and its mechanism. The system of CMA depends on the CM array and the adaptive equalizer to cancel the negative effect of multipath fading.  $E(k)$  is the input signal of CMA array and  $y(k)$  is the output of it. It is also the input of the adaptive equalizer. The adaptive filter deals with the symbol which comes from multiple sources individually.

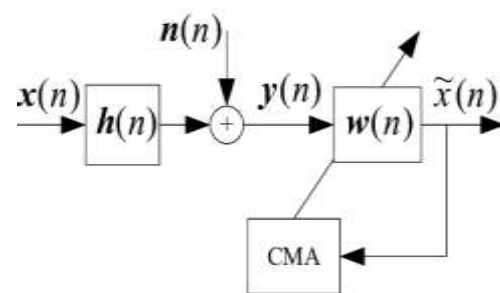


Figure 1 : block diagram of CMA blind equalizer

CMA has a cost function and this cost function is shown in Equation (1).

$$J = E [ (|Z(k)|^p - R_2)^2 ] \quad (1)$$

Where  $z(k)$  is the output of the equalizer and it is given by

$$z(k) = y^T(k)W(k) \quad (2)$$

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where  $w(k)$  is the vector of equalizer coefficients. The update rule for the  $w(k)$  can be obtained by employing stochastic gradient approach. This requires the evaluation of gradient of the cost function w.r.t. the weight vector  $w(k)$  which results in the following recursive formulation

$$W(k+1) = W(k) - 4\mu_{CMA} e(k) y^*(k) \quad (3)$$

where  $\mu_{CMA}$  is the mean value of step size  
 $(0 < \mu_{CMA} < 1/\lambda_{max})$  (4)

and  $e(k)$  is the estimation error given by

$$\varepsilon_2(k) = z(k)(|z(k)|^2 - R_2)$$

(5)

With  $R_2$  is called dispersion constant which is evaluated using

$$R_2 = \frac{E[|S(k)|^4]}{E[|S(k)|^2]^2}$$

(6)

### Proposed Variable p-CMA

The cost function of the standard p-CMA is given by

$$J = E[(|Z(k)|^p - R_2)^2]$$

(7)

Therefore, the weight update of P-CMA will be

(8)

$$W_{P-CMA}(k+1) = W_{P-CMA}(k) - \mu_{P-CMA} e_{P-CMA}(k) y^*(k)$$

where

$$e_{P-CMA}(k) = y(k)(|y(k)|^p - R_2) |y(k)|^{p-2}$$

(9)

In the proposed variable p-CMA, we introduce a time varying p parameter such that the cost function will take the following form

$$J = E[(|Z(k)|^{p(k)} - R_2)^2]$$

(10)

where

(9)

$$e_{P-CMA}(k) = y(k)(|y(k)|^{p(k)} - R_2) |y(k)|^{p(k)-2}$$

We proposed to adapt the time varying p(k) according to the energy of estimation error such that it attain larger value if the estimation error is large and reduces to smaller values if the estimation error is small. This is achieved by employing an intermediate variable  $\tilde{p}(k)$  and adapted using the following mechanism

(11)

$$\tilde{P}(k) = \beta \tilde{P}(k+1) + \gamma |e_{P-CMA}(k)|^2, \quad (0 < \beta < 1, \gamma > 0)$$

erehw

(12)

$$P(k) = \begin{cases} P_{upper} & \text{if } \tilde{P}(k) \geq P_{upper} \\ P_{lower} & \text{if } \tilde{P}(k) \leq P_{lower} \\ \tilde{P}(k) & \text{otherwise} \end{cases}$$

The above condition make sure that the variable p(k) should not acquire the values that diverges the algorithm. For this purpose, we will first estimate Pupper and Plower by Monte Carlo simulation then we initialize our P(k) with these values to achieve faster convergence in the beginning. Later the update rule automatically reduce the P(k) to attain lower steady state error.

## II. SIMULATION RESULT

In this section, we present various simulation results to show the performance of the proposed variable p-CMA algorithm. The performance of the proposed algorithm with that of the standard CMA algorithm is compared. In experiments, the value of signal-to-noise ratio (SNR) is set to 20 dB. Residual inter-symbol interference (ISI) is used as a performance measure in the reported results. Three different examples are considered.

### Example 1: Laplacian Environment

In the first example, the experiment is performed in Laplacian environment. The results are shown in Fig. 2. It can be seen from the results that the proposed algorithm converges approximately 2500 iteration earlier than its counterparts to reach the same floor of steady-state ISI of -43 dB. This shows the supremacy of the proposed algorithm.

### Example 2: Uniform Environment

In the second example, the experiment is performed in Uniform environment. The results are shown in Fig. 3. It can be seen from the results that the proposed algorithm converges approximately 2300 iteration earlier than its counterparts to reach the same floor of steady-state ISI of -44dB. Thus, the proposed algorithm has superior performance in Uniform environment too.

### Example 3: Gaussian Environment

In the third example, the experiment is performed in Gaussian environment. The results are shown in Fig. 4. It can be seen from the results that the proposed algorithm converges approximately 2000 iteration earlier than its counterparts to reach the same floor of steady-

state ISI of -41dB. Here again the supremacy of the proposed algorithm is observed.

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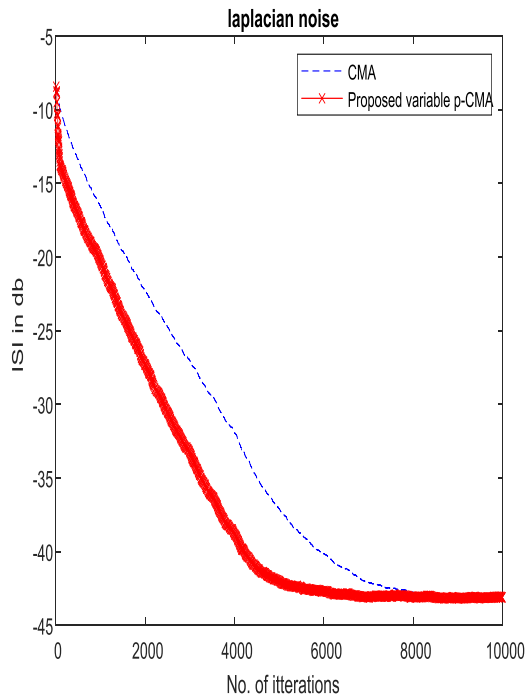


Figure 2: ISI comparison in Laplacian noise

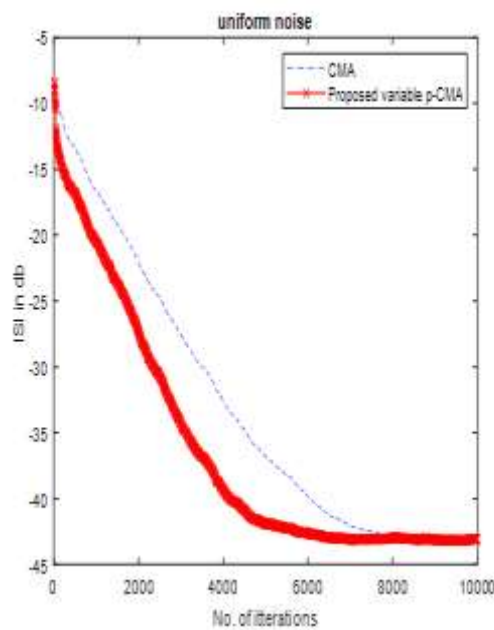


Figure 3: ISI comparison in Uniform noise

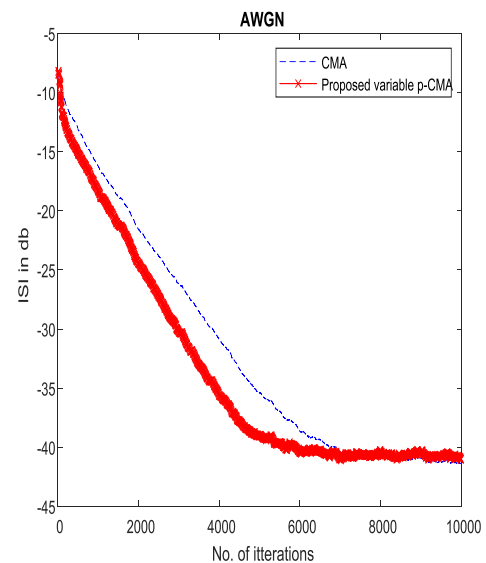


Figure 4: ISI comparison in Gaussian noise

### III. CONCLUSION

A variable  $p$ - Constant Modulus Algorithm ( $p$ -CMA) algorithm is developed in this work. The exponent  $p$  is used to enhance the convergence speed and steady-state performance of the standard CMA algorithm. The exponent  $p$  is made time varying in accordance to the energy of the estimation error which enforces larger values of  $p$  in the initial adaptation while smaller values near steady-state. Simulation results are presented for blind equalization in three different environments: Laplacian, Uniform, and Gaussian environments. These simulation results show that the proposed algorithm converges faster than the conventional CMA algorithm in all the three environments validating the theoretical claims.

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