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The Optimal Tracking Control of Descriptor Time-Varying Continuous-Time Systems: An Approach Based on Preview Control

Fucheng Liao[1]*, Zhenqin Ren[2,3], Yanrong Lu[2]
[1]*School of Mathematics and Physics, University of Science and Technology Beijing, Beijing, China*
[2]*School of Automation and Electrical Engineering, University of Science and Technology Beijing, Beijing,China*
[3]*School of Information Technology, Luoyang Normal University, Luoyang, China*
** Corresponding author: Fucheng Liao*

ABSTRACT The optimal tracking problem of descriptor time-varying continuous-time systems is studied. First, under the assumption of impulse controllability, it is shown that the original system can be converted to a reduced-order normal linear time-varying system and an algebraic constraint by the pre-feedback technique and the restricted system equivalent (r.s.e) approach. Second, the state augmentation technique is applied to construct an augmented system, in which the state vector consists of tracking error, auxiliary input (appearing in pre-feedback), the state, and its derivative of the normal system. This procedure transforms the tracking control problem into an optimal regulation problem of the augmented system. Third, by utilizing the related results of preview control theory, a controller for the augmented system is presented through letting preview length tend to zero. Meanwhile, a controller to solve the original problem is also derived from the controller for the augmented system. Finally, two sorts of numerical simulation methods are proposed for the time-varying descriptor continuous-time systems, which have been demonstrated to be effective by various examples.
Keywords: time-varying descriptor systems; tracking problem; preview control theory

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I. INTRODUCTION

Descriptor systems have attracted extensive attention over the last four decades, and there have been fruitful results within the community of researchers on the topic, such as [1-3]. As study deepened, researchers started to investigate the related problems by combining descriptor systems and the preview control theory (related results about preview control can be found in [19,20]). For example, the authors of [4] designed an optimal controller with preview actionfor linear causal descriptor systems. In [5], the method of [4] was generalized to the case of discrete-time linear descriptor time-delay systems. Furthermore, based on the results of [4, 5], the authors of [6] handled the optimal preview tracking control for multi-rate discrete-time descriptor systems. Additionally, the results of [7] can be viewedas a continuous-time counterpart of [4]. Recently, [22] also considered the optimal preview control problem and proposed a numerical simulation algorithm for the original system that did not rely on the r.s.e. form.

The study of time-varying singular systems has also drawn increasing interest, and several results in this field have been published. Since impulsive responses were excluded in [8,9] by using admissible initial conditions, [10] applied the state feedback to eliminate the impulse behavior and derived a necessary and sufficient condition to guarantee the existence of the feedback control law. Further, in [11], the author transformed the original descriptor time-varying systems into a standard canonical form and established the related criteria of controllability and observability. Notice thatcontrollability and impulse controllability are two different concepts in descriptor systems, and so it is with observability and impulse observability. Hence, the authors of [12] investigated the conditions that can ensure the impulsive controllability and the impulse observability. [13] considered the same problem as in [12], The difference isthat a new impulse solution with respect tothe initial value problem of linear descriptor time-varying systemswas obtained,and meanwhile the impulse controllability as well as the impulse observability was redefined. Moreover, controllability and observability at infinity were studied in [14] under the scenario of analytical solvability. Recently, [15] proposed the finite-time stabilityfor linear descriptor time-varying impulse systems; this issue was further investigated in [16].Because the method in [15] relies on some restrictions on the system matrices, the authors of [17] developed new methods to address this problem. The most related literature to this paper could be [18], which discussed the linearquadratic optimal control problem of linear descriptor time-varying systems.

We are concerned in the present paper with the optimal tracking control problem of descriptor continuous-time time-varying systems with impulse controllability. Technically, the state augmentation technique, which is used in preview control theory, will be applied to convert the original problem into a stability problem of zero solution for a closed-loop augmented system. Hence, the results in [18] cannot be directly generalized to deal with this problem. Specifically, the reference signal is assumed to be previewable. Based on the results for optimal preview control in [23], an optimal controller is designed for the augmented system by setting the preview length to tend to zero.Then according to the relationship between the original input and the auxiliary input in pre-feedback, the controller for the original problem is also obtained. Moreover, two kinds of numerical simulation methods are given for time-varying descriptor continuous-time systems. Both methods are more general and can be applied to descriptor continuous-time systemsof any type.

Throughout this paper, a matrix $M(t)$ is referred to as invertible, if $M(t)$ is invertible for any $t\in[t_0,t_f]$. Furthermore, "iff" represents "if and only if".

II. RESTRICTED SYSTEM EQUIVALENT OF A DESCRIPTOR TIME-VARYING SYSTEM

Consider a descriptor time-varying system

$$\begin{cases} E(t)\dot{x}(t)=A(t)x(t)+B(t)u(t) \\ y(t)=C(t)x(t) \end{cases}, \quad t\in[t_0,t_f] \tag{1}$$

where $x(t)\in\mathbb{R}^n$, $u(t)\in\mathbb{R}^r$, $y(t)\in\mathbb{R}^m$ represent the state, control input, and output of system (1). $A(t)\in\mathbb{R}^{n\times n}$, $B(t)\in\mathbb{R}^{n\times r}$, $C(t)\in\mathbb{R}^{m\times n}$ are time-varying matrices with the appropriate dimensions. $E(t)\in\mathbb{R}^{n\times n}$ is singular matrix and has constant rank, i.e., $\mathrm{rank}E(t)=q<n$ holds for any $t\in[t_0,t_f]$.

Note that $\mathrm{rank}E(t)=q$, thus, according to the property in matrix theory, it follows that $E(t)$ is equivalent to $\begin{bmatrix} I_q & 0 \\ 0 & 0 \end{bmatrix}$. Namely, there exist two invertible matrices $M(t)$ and $N(t)$ such that

$$M(t)E(t)N(t)=\begin{bmatrix} I_q & 0 \\ 0 & 0 \end{bmatrix}$$

Let

$$x(t)=N(t)\tilde{x}(t) \tag{2}$$

then, pre-multiplying system (1) by $M(t)$ yields

$$\begin{cases} [M(t)E(t)N(t)]\dot{\tilde{x}}(t)=M(t)[A(t)N(t)-E(t)\dot{N}(t)]\tilde{x}(t)+[M(t)B(t)]u(t) \\ y(t)=[C(t)N(t)]\tilde{x}(t) \end{cases} \tag{3}$$

Due to the block structure of matrix $\begin{bmatrix} I_q & 0 \\ 0 & 0 \end{bmatrix}$, other matrices in system (3) can be correspondingly partitioned into the following forms:

$$M(t)[A(t)N(t)-E(t)\dot{N}(t)]=\begin{bmatrix} \tilde{A}_{11}(t) & \tilde{A}_{12}(t) \\ \tilde{A}_{21}(t) & \tilde{A}_{22}(t) \end{bmatrix},\ M(t)B(t)=\begin{bmatrix} \tilde{B}_1(t) \\ \tilde{B}_2(t) \end{bmatrix},$$

$$C(t)N(t)=[\tilde{C}_1(t)\quad \tilde{C}_2(t)]$$

Set $\tilde{x}(t)=\begin{bmatrix} \tilde{x}_1(t) \\ \tilde{x}_2(t) \end{bmatrix}$, then system (3) can be rewritten as

$$\begin{cases} \begin{bmatrix} I & \\ & 0 \end{bmatrix} \begin{bmatrix} \dot{\tilde{x}}_1(t) \\ \dot{\tilde{x}}_2(t) \end{bmatrix} = \begin{bmatrix} \tilde{A}_{11}(t) & \tilde{A}_{12}(t) \\ \tilde{A}_{21}(t) & \tilde{A}_{22}(t) \end{bmatrix} \begin{bmatrix} \tilde{x}_1(t) \\ \tilde{x}_2(t) \end{bmatrix} + \begin{bmatrix} \tilde{B}_1(t) \\ \tilde{B}_2(t) \end{bmatrix} u(t) \\ y(t) = [\tilde{C}_1(t) \quad \tilde{C}_2(t)] \begin{bmatrix} \tilde{x}_1(t) \\ \tilde{x}_2(t) \end{bmatrix} \end{cases} \tag{4}$$

or another form

$$\begin{cases} \dot{\tilde{x}}_1(t) = \tilde{A}_{11}(t)\tilde{x}_1(t) + \tilde{A}_{12}(t)\tilde{x}_2(t) + \tilde{B}_1(t)u(t) \\ 0 = \tilde{A}_{21}(t)\tilde{x}_1(t) + \tilde{A}_{22}(t)\tilde{x}_2(t) + \tilde{B}_2(t)u(t) \\ y(t) = \tilde{C}_1(t)\tilde{x}_1(t) + \tilde{C}_2(t)\tilde{x}_2(t) \end{cases} \tag{5}$$

As long as matrix $E(t)$ has constant rankany $t \in [t_0, t_f]$, the aforementioned transformation is always available. After this procedure, the original system (1) is decomposed into a reduced-order normal linear time-varying system and an algebraic equation.

Similar to the time-invariant descriptor system [7,24], if we obtain $\tilde{x}_2(t)$ from the following equation

$$0 = \tilde{A}_{21}(t)\tilde{x}_1(t) + \tilde{A}_{22}(t)\tilde{x}_2(t) + \tilde{B}_2(t)u(t) \tag{6}$$

and substitute it into the first equation, then system (5) will only be related to $\tilde{x}_1(t)$. In fact, the sufficient and necessary condition for solving $\tilde{x}_2(t)$ from (6) uniquely is that $\tilde{A}_{22}(t)$ is invertible. If the invertibility of $\tilde{A}_{22}(t)$ is always satisfied for any $t \in [t_0, t_f]$, then system (1) is termed as impulse-free.

Then, when system (1) is impulse-free, we obtain

$$\tilde{x}_2(t) = -\tilde{A}_{22}^{-1}(t)\tilde{A}_{21}(t)\tilde{x}_1(t) - \tilde{A}_{22}^{-1}(t)\tilde{B}_2(t)u(t)$$

Substituting it into system (5) yields

$$\begin{cases} \begin{bmatrix} I & \\ & 0 \end{bmatrix} \begin{bmatrix} \dot{\tilde{x}}_1(t) \\ \dot{\tilde{x}}_2(t) \end{bmatrix} = \begin{bmatrix} \tilde{A}_{11}(t) - \tilde{A}_{12}(t)\tilde{A}_{22}^{-1}(t)\tilde{A}_{21}(t) & 0 \\ \tilde{A}_{22}^{-1}(t)\tilde{A}_{21}(t) & I \end{bmatrix} \begin{bmatrix} \tilde{x}_1(t) \\ \tilde{x}_2(t) \end{bmatrix} + \begin{bmatrix} \tilde{B}_1(t) - \tilde{A}_{12}(t)\tilde{A}_{22}^{-1}(t)\tilde{B}_2(t) \\ \tilde{A}_{22}^{-1}(t)\tilde{B}_2(t) \end{bmatrix} u(t) \\ y(t) = [\tilde{C}_1(t) \quad \tilde{C}_2(t)] \begin{bmatrix} \tilde{x}_1(t) \\ \tilde{x}_2(t) \end{bmatrix} \end{cases} \tag{7}$$

This is equivalent to pre-multiplying equation (4) by the following invertible matrix

$$M_1(t) = \begin{bmatrix} I & -\tilde{A}_{12}(t)\tilde{A}_{22}^{-1}(t) \\ 0 & \tilde{A}_{22}^{-1}(t) \end{bmatrix}$$

In other words, if $M_1(t)M(t)$ is used in (3) instead of $M(t)$, then system (1) will be directly transformed into system (7). For convenience of simplifying the notations, hereafter $M_1(t)M(t)$ will be specified as $M(t)$, and system (7) will be denoted as the following form:

$$\begin{cases} \begin{bmatrix} I & \\ & 0 \end{bmatrix} \begin{bmatrix} \dot{\tilde{x}}_1(t) \\ \dot{\tilde{x}}_2(t) \end{bmatrix} = \begin{bmatrix} \tilde{A}_{11}(t) & 0 \\ \tilde{A}_{21}(t) & I \end{bmatrix} \begin{bmatrix} \tilde{x}_1(t) \\ \tilde{x}_2(t) \end{bmatrix} + \begin{bmatrix} \tilde{B}_1(t) \\ \tilde{B}_2(t) \end{bmatrix} u(t) \\ y(t) = [\tilde{C}_1(t) \quad \tilde{C}_2(t)] \begin{bmatrix} \tilde{x}_1(t) \\ \tilde{x}_2(t) \end{bmatrix} \end{cases} \tag{8}$$

Here, system (8) is termed as the r.s.e. form of system (1).

Note that $\tilde{A}_{22}(t)$ is not always invertible in (6), thus $\tilde{x}_2(t)$ will not be solved uniquely. To this end, we introduce the state pre-feedback:

$$u(t) = K_2(t)\tilde{x}_2(t) + v(t) \tag{9}$$

to (6); then we have

$$0=\tilde{A}_{21}(t)\tilde{x}_1(t)+[\tilde{A}_{22}(t)+\tilde{B}_2(t)K_2(t)]\tilde{x}_2(t)+\tilde{B}_2(t)v(t) \qquad (10)$$

We now proceed to analyze pre-feedback (9); it is evident that this control can be expressed as the following form

$$u(t)=[0 \quad K_2(t)]\begin{bmatrix}\tilde{x}_1(t)\\ \tilde{x}_2(t)\end{bmatrix}+v(t)$$

According to (2), we have $\tilde{x}(t)=N^{-1}(t)x(t)$, and thereby the above controller can be rewritten as

$$u(t)=[0 \quad K_2(t)]N^{-1}(t)x(t)+v(t)=K(t)x(t)+v(t) \qquad (11)$$

By combining system (1) with the above pre-feedback control, we obtain

$$\begin{cases}E(t)\dot{x}(t)=[A(t)+B(t)K(t)]x(t)+B(t)v(t)\\ y(t)=C(t)x(t)\end{cases} \qquad (12)$$

Note that if $K_2(t)$ can be suitably selected such that $[\tilde{A}_{22}(t)+\tilde{B}_2(t)K_2(t)]$ is invertible, or if matrix $K(t)$ is designed such that system (12) is impulse-free, system (1) is impulse controllable.

In the following, system (1) will be assumed to be impulse controllable. For this assumption and system (12), there exists the following result.

Lemma 1 [18]. System (1) is impulse controllable over $[t_0,t_f]$ iff there exist gain matrix $K(t)$ and invertible matrices $M(t)$ and $N(t)$ such that

$$M(t)\begin{bmatrix}E(t) & A(t)+B(t)K(t)-E(t)\dot{N}(t)N^{-1}(t) & B(t)\end{bmatrix}\begin{bmatrix}N(t) & 0 & 0\\ 0 & N(t) & 0\\ 0 & 0 & I_r\end{bmatrix}$$

$$=\begin{bmatrix}I_q & 0 & \bar{A}_{11}(t) & 0 & \bar{B}_1(t)\\ 0 & 0 & \bar{A}_{21}(t) & I_{n-q} & \bar{B}_2(t)\end{bmatrix} \qquad (13)$$

According to the discussion of the scenario of impulse-freeness and Lemma 1, if system (1) is impulse controllable, then we can choose pre-feedback (11) such that system (12) is impulse-free.

III. PROBLEM FORMULATION

We continue to consider the descriptor time-varying system (1). Definethe tracking error as

$$e(t)=y(t)-y_d(t) \qquad (14)$$

where $y_d(t)$ is reference signal.The object is to design a controller with preview action such that the output $y(t)$ can track $y_d(t)$ accurately. To this end, for system (12), the following performance index function is introduced

$$J=e^T(t_f)Fe(t_f)+\int_{t_0}^{t_f}\left[e^T(t)Q_e e(t)+\dot{v}^T(t)R\dot{v}(t)\right]dt \qquad (15)$$

where the weighted matrices satisfy $F\ge 0$, $Q_e>0$, $R>0$. As pointed out in [7], by applying the derivativeof the control input $v(t)$ in J, the integral of the tracking error will be contained in an optimal controller, which can assist to eliminate static errors.

Before proceeding further, let us list some basic assumptions.

A1. System (1) is impulse controllable;

A2. Each element of $A(t)$ 、 $B(t)$ and $C(t)$ is continuously differentiable over the defined interval $[t_0,t_f]$.

Under A1, according to Lemma 1, system (12), formed by system (1) and pre-feedback (11), is impulse-free. Therefore, there are two invertible matrices $M(t)$ and $N(t)$ such that system (12) is r.s.e. to

$$\begin{cases}\begin{bmatrix}I_q & 0\\ 0 & 0\end{bmatrix}\dot{\bar{x}}(t)=\begin{bmatrix}\bar{A}_{11}(t) & 0\\ \bar{A}_{21}(t) & I_{n-q}\end{bmatrix}\bar{x}(t)+\begin{bmatrix}\bar{B}_1(t)\\ \bar{B}_2(t)\end{bmatrix}v(t)\\ y(t)=\begin{bmatrix}\bar{C}_1(t) & \bar{C}_2(t)\end{bmatrix}\bar{x}(t)\end{cases} \qquad (16)$$

Furthermore, denoting $\bar{x}(t)=\begin{bmatrix}\bar{x}_1(t)\\ \bar{x}_2(t)\end{bmatrix}$ gives

$$\begin{cases}\dot{\bar{x}}_1(t)=\bar{A}_{11}(t)\bar{x}_1(t)+\bar{B}_1(t)v(t)\\ 0=\bar{A}_{21}(t)\bar{x}_1(t)+\bar{x}_2(t)+\bar{B}_2(t)v(t)\\ y(t)=\bar{C}_1(t)\bar{x}_1(t)+\bar{C}_2(t)\bar{x}_2(t)\end{cases} \tag{17}$$

where $\bar{A}_{11}(t)\in R^{q\times q}$, $\bar{A}_{21}(t)\in R^{(n-q)\times q}$, $\bar{B}_1(t)\in R^{q\times r}$, $\bar{B}_2(t)\in R^{(n-q)\times r}$, $\bar{C}_1(t)\in R^{m\times q}$, $\bar{C}_2(t)\in R^{m\times(n-q)}$. Note that $\bar{x}_1(t)=\begin{bmatrix}I & 0\end{bmatrix}N^{-1}(t)x(t)$, which will be used later.

To design the controller for system (1), a hypothesis, which is about the reference signal $y_d(t)$, is needed.

A3. $y_d(t)$ is piecewise differentiable over $[t_0,t_f]$.

Remark 1. We need to differentiate $A(t)$, $B(t)$ and $C(t)$ with respect to t in the process of constructing an augmented system. Therefore, these matrices are required to be differentiable in A2. Moreover, according to A3, it follows that reference signal $y_d(t)$ is not differentiable at some isolated points. At this juncture, we take the one-sided derivative of $y_d(t)$.

IV. CONSTRUCTION OF AN AUGMENTED SYSTEM AND DESIGN OF THE CONTROLLER

In the following, the approach of [21,23] will be applied to solvethe key problem of this paper. First of all, an augmented system, which is based on the reduced-order normal time-varying system, will be constructed below.Specifically, by solving $\bar{x}_2(t)$ from the second equation of (17) and substituting it into the third one, we can obtain the following reduced-order normal system:

$$\begin{cases}\dot{\bar{x}}_1(t)=\bar{A}_{11}(t)\bar{x}_1(t)+\bar{B}_1(t)v(t)\\ y(t)=\hat{C}_1(t)\bar{x}_1(t)+\hat{C}_2(t)v(t)\end{cases} \tag{18}$$

with $\hat{C}_1(t)=\bar{C}_1(t)-\bar{C}_2(t)\bar{A}_{21}(t)$, $\hat{C}_2(t)=-\bar{C}_2(t)\bar{B}_2(t)$. Then the original problem is finally converted to the tracking problem of the reduced-order normal system (18), where the reference signal is still $y_d(t)$.

We proceed to the construction of the augmented system. Differentiating both sides of system (18) and tracking error (14) with respect to t gives

$$\begin{cases}\ddot{\bar{x}}_1(t)=\dot{\bar{A}}_{11}(t)\bar{x}_1(t)+\bar{A}_{11}(t)\dot{\bar{x}}_1(t)+\dot{\bar{B}}_1(t)v(t)+\bar{B}_1(t)\dot{v}(t)\\ \dot{y}(t)=\dot{\hat{C}}_1(t)\bar{x}_1(t)+\hat{C}_1(t)\dot{\bar{x}}_1(t)+\dot{\hat{C}}_2(t)v(t)+\hat{C}_2(t)\dot{v}(t)\end{cases} \tag{19}$$

and

$$\begin{aligned}\dot{e}(t)&=\dot{y}(t)-\dot{y}_d(t)\\ &=\dot{\hat{C}}_1(t)\bar{x}_1(t)+\hat{C}_1(t)\dot{\bar{x}}_1(t)+\dot{\hat{C}}_2(t)v(t)+\hat{C}_2(t)\dot{v}(t)-\dot{y}_d(t)\end{aligned} \tag{20}$$

Combining the first equation of (19) with (20) and using the following two identities

$\dot{\bar{x}}_1(t)=\dot{\bar{x}}_1(t)$, $\dot{v}(t)=\dot{v}(t)$

we have

$$\begin{cases}\dot{z}(t)=\bar{A}(t)z(t)+\bar{B}(t)\dot{v}(t)+G\dot{y}_d(t)\\ e(t)=\bar{C}z(t)\end{cases} \tag{21}$$

System (21) is referred to as the augmented system, where $e(t)=\bar{C}z(t)$ is the observation equation.

Note that $y(t)$ is the measured output of system (1), and $y_d(t)$ is known at the current time t, hence $e(t)$ is also measurable according to equation (14). This means that it is reasonable to choose $e(t)$ as the output of system (21).

We now turn to the process of obtaining optimal control of system (21). Since the state vector of this system is $z(t)$, the relevant variables of performance index function (15) is correspondingly needed to be replaced by $z(t)$. Specifically, we can substitute $\bar{C}z(t)$ for $e(t)$ and $\bar{C}z(t_f)$ for $e(t_f)$; the result is

$$J = z^T(t_f)\bar{F}z(t_f) + \int_{t_0}^{t_f} [z^T(t)\bar{Q}z(t) + \dot{v}^T(t)R\dot{v}(t)]dt \qquad (22)$$

where $\bar{F} = \bar{C}^T F\bar{C} = diag(F,0,0,0)$, $\bar{Q} = \bar{C}^T Q_e\bar{C} = diag(Q_e,0,0,0)$.

Eventually, the tracking problem of system (1) is changed into the finite-time optimal state regulation problem of system (21). In fact, if the optimal controller exists, the $e(t)$ will asymptotically converge to an appropriately small neighborhood of the zero vector, which is exactly what we need.

To obtain the optimal controller, the reference signal $y_d(t)$ is first assumed to be previewable and the preview length is set as l_r. Then main result follows immediately according to the literature [23].

Theorem 1. The optimal controller for linear descriptor time-varying system (21) with respect to the performance index function (22) is given by

$$\dot{v}(t) = -R^{-1}\bar{B}^T(t)\bar{P}(t)z(t) - R^{-1}\bar{B}^T(t)g(t) \qquad (23)$$

where $\bar{P}(t)$ is an $(m+2q+r)\times(m+2q+r)$ matrix. Moreover, $\bar{P}(t)$ and $g(t)$ are, respectively, the solutions of the following two boundary problems of differential equation

$$\begin{cases} -\dot{\bar{P}}(t) = \bar{A}^T(t)\bar{P}(t) + \bar{P}(t)\bar{A}(t) - \bar{P}(t)\bar{B}(t)R^{-1}\bar{B}^T(t)\bar{P}(t) + \bar{Q} \\ \bar{P}(t_f) = \bar{F} \end{cases} \qquad (24)$$

and

$$\begin{cases} \dot{g}(t) = \bar{A}_C(t)g(t) - \bar{P}(t)G\dot{y}_d(t) \\ g(t_f) = 0 \end{cases} \qquad (25)$$

Here, $t \in [t_0, t_f]$ and $\bar{A}_C(t) = \bar{P}(t)\bar{B}(t)R^{-1}\bar{B}^T(t) - \bar{A}^T(t)$.

Notice that what we want is to design a control input $u(t)$ for the system (1). This result will be discussed in the following section.

V. DESIGN OF CONTROLLER FOR SYSTEM (1)

Since $u(t) = K(t)x(t) + v(t)$, in order to obtain $u(t)$, we need to solve $v(t)$. To this end, it follows from (23) that we have to solve $g(t)$ from differential equation (25). In light of the basic theories of differential equations, we set $\Phi(t)$ is the fundamental solution matrix of $\dot{g}(t) = \bar{A}_C(t)g(t)$. As a result, the solution of (25) has the form of

$$g(t) = -\int_{t_f}^{t} \Phi(t)\Phi^{-1}(s)\bar{P}(s)G\dot{y}_d(s)ds \qquad (26)$$

Substituting (26) into (23) and partitioning $R^{-1}\bar{B}^T(t)\bar{P}(t)$ into the following form in terms of the structure of $z(t)$

$$R^{-1}\bar{B}^T(t)\bar{P}(t) = \begin{bmatrix} K_e(t) & K_{x1}(t) & K_{\dot{x}1}(t) & K_v(t) \end{bmatrix}$$

we get

$$\dot{v}(t) = -K_v(t)v(t) - K_e(t)e(t) - K_{x1}(t)\bar{x}_1(t) - K_{\dot{x}1}(t)\dot{\bar{x}}_1(t)\ldots$$
$$-R^{-1}\bar{B}^T(t)\int_t^{t_f} \Phi(t)\Phi^{-1}(s)\bar{P}(s)G\dot{y}_d(s)ds \quad (27)$$

Here, the integral term $-R^{-1}\bar{B}^T(t)\int_t^{t_f} \Phi(t)\Phi^{-1}(s)\bar{P}(s)G\dot{y}_d(s)ds$ utilizes the derivative value of the reference signal after the current time. According to the assumption on $y_d(t)$, this term can be further written as

$$-R^{-1}\bar{B}^T(t)\int_t^{(t+l_r)\Lambda t_f}\Phi(t)\Phi^{-1}(s)\bar{P}(s)G\dot{y}_d(s)ds$$

where $(t+l_r)\Lambda t_f=\min(t+l_r,t_f)$. Because $y_d(t)$ is not previewable, neither is $\dot{y}_d(t)$, by letting $l_r\to 0$, we have

$$\dot{v}(t)=-K_v(t)v(t)-K_e(t)e(t)-K_{x1}(t)\bar{x}_1(t)-K_{\dot{x}1}(t)\dot{\bar{x}}_1(t) \qquad (28)$$

Correspondingly, a result can be immediately derived from Theorem 1.

Theorem 2. The control input $v(t)$ for the time-varying system (21), which minimizes the performance index function (22), can be determined by equation (28), where $\begin{bmatrix}K_e(t) & K_{x1}(t) & K_{\dot{x}1}(t) & K_v(t)\end{bmatrix}=R^{-1}\bar{B}^T(t)\bar{P}(t)$, $\bar{A}_C(t)=\bar{P}(t)\bar{B}(t)R^{-1}\bar{B}^T(t)-\bar{A}^T(t)$, and $\bar{P}(t)$ is a solution matrix of the Riccati differential equation (24).

Suppose that $\Psi(t)$ is the fundamental solution matrix of $\dot{v}(t)=-K_v(t)v(t)$, then system (28) has the following state response:

$$v(t)=\Psi(t)v(t_0)-\int_{t_0}^t\left\{\Psi(t)\Psi^{-1}(\tau)\left[K_e(\tau)e(\tau)+K_{x1}(\tau)\bar{x}_1(\tau)+K_{\dot{x}1}(\tau)\dot{\bar{x}}_1(\tau)\right\}\right.d\tau \quad (29)$$

In addition, setting $v(t_0)=0$ yields

$$v(t)=-\int_{t_0}^t\left\{\Psi(t)\Psi^{-1}(\tau)\left[K_e(\tau)e(\tau)+K_{x1}(\tau)\bar{x}_1(\tau)+K_{\dot{x}1}(\tau)\dot{\bar{x}}_1(\tau)\right\}\right.d\tau \qquad (30)$$

which is exactly the control input $v(t)$ of system (21).

Noting $\bar{x}_1(t)=\begin{bmatrix}I & 0\end{bmatrix}N^{-1}(\mathrm{t})x(\mathrm{t})$, $\dot{\bar{x}}_1=\begin{bmatrix}I & 0\end{bmatrix}\dot{N}^{-1}(\mathrm{t})x(\mathrm{t})+\begin{bmatrix}I & 0\end{bmatrix}N^{-1}(\mathrm{t})\dot{x}(\mathrm{t})$ and using them in (30), we can finally get the control input for system (1) based on $u(t)=K(t)x(t)+v(t)$, that is

$$u(t)=K(t)x(t)...$$
$$-\int_{t_0}^t\left\{\Psi(t)\Psi^{-1}(\tau)\left[K_e(\tau)e(\tau)+K_{x1}(\tau)\begin{bmatrix}I & 0\end{bmatrix}N^{-1}(\tau)x(\tau)+K_{\dot{x}1}(\tau)\begin{bmatrix}I & 0\end{bmatrix}\dot{N}^{-1}(\tau)x(\tau)+K_{\dot{x}1}(\tau)\begin{bmatrix}I & 0\end{bmatrix}N^{-1}(\tau)\dot{x}(\tau)\right]\right.d\tau$$
(31)

By decomposing the integral of the sum function into the sum of the function integrals, we conclude the following result:

Theorem 3. Assume that $E(t)$ is a singular matrix with constant rank, and assumptions A1-A3 hold, under the performance index function (15), the controller for the time-varying descriptor system (1) is given as

$$u(t)=K(t)x(t)-\int_{t_0}^t\Psi(t)\Psi^{-1}(\tau)K_e(\tau)e(\tau)\,\mathrm{d}\tau...$$
$$-\int_{t_0}^t\Psi(t)\Psi^{-1}(\tau)\left[K_{x1}(\tau)\begin{bmatrix}I & 0\end{bmatrix}N^{-1}(\tau)+K_{\dot{x}1}(\tau)\begin{bmatrix}I & 0\end{bmatrix}\dot{N}^{-1}(\tau)\right]x(\tau)d\tau...$$
$$-\int_{t_0}^t\Psi(t)\Psi^{-1}(\tau)K_{\dot{x}1}(\tau)\begin{bmatrix}I & 0\end{bmatrix}N^{-1}(\tau)\dot{x}(\tau)\,\mathrm{d}\tau \qquad (32)$$

where $\bar{A}_C(t)=\bar{P}(t)\bar{B}(t)R^{-1}\bar{B}^T(t)-\bar{A}^T(t)$; $K_e(t)$, $K_{x1}(t)$, $K_{\dot{x}1}(t)$, $K_v(t)$, as well as $\bar{P}(t)$, are defined in Theorem 2; $K(t)$, $N(t)$ are determined by (13). Moreover, $\Psi(t)$ is the solution to the initial value problem of $\dot{v}(t)=-K_v(t)v(t),v(t_0)=I$.

In (32), $K(t)x(t)$ is employed to eliminate the impulse in system (1); $-\int_{t_0}^t\Psi(t)\Psi^{-1}(\tau)K_e(\tau)e(\tau)\,\mathrm{d}\tau$ is an integral term about the tracking error, which has the ability to eliminate the static error; and

$$-\int_{t_0}^t\Psi(t)\Psi^{-1}(\tau)\left[K_{x1}(\tau)\begin{bmatrix}I & 0\end{bmatrix}N^{-1}(\tau)+K_{\dot{x}1}(\tau)\begin{bmatrix}I & 0\end{bmatrix}\dot{N}^{-1}(\tau)\right]x(\tau)d\tau$$

is the state feedback.

In Theorem 3, the theoretical analysis can be facilitated by writing control input $u(t)$ in the form of (32). However, (32) is inconvenient for practical problems, as it needs the solution $\Psi(t)$ of the differential equation $\dot{v}(t)=-K_v(t)v(t),v(t_0)=I$. To this end, we intend to propose a numerical method for finding the controller based on equation (28). Meanwhile, we will also provide a simulation approach to achieve the closed-loop response in the following two sections, respectively.

VI. THE FIRST REALIZATION METHOD

The sampling time is assumed to be T ; based on Euler's method, equation (28) can be discretized as:

$$v((i+1)T) = v(iT) - TK_v(iT)v(iT) - TK_e(iT)e(iT)...$$
$$-TK_{x1}(iT)\overline{x}_1(iT) - TK_{\dot{x}1}(iT)\dot{\overline{x}}_1(iT) \qquad (33)$$

Substituting $\dfrac{x_1(iT) - x_1((i-1)T)}{T}$ for $\dot{\overline{x}}_1(iT)$ and applying the formula $\overline{x}_1(t) = \begin{bmatrix} I & 0 \end{bmatrix} N^{-1}(t)x(t)$, we can obtain

$$v((i+1)T) = v(iT) - TK_v(iT)v(iT) - TK_e(iT)e(iT)...$$
$$-TK_{x1}(iT)\overline{x}_1(iT) - TK_{\dot{x}1}(iT)[x_1(iT) - x_1((i-1)T)] - TR^{-1}\overline{B}^T(iT)g(iT)$$
$$= [I - TK_v(iT)]v(iT) - TK_e(iT)e(iT) - TK_{x1}(iT)\begin{bmatrix} I & 0 \end{bmatrix} N^{-1}(iT)x(iT)...$$
$$-TK_{\dot{x}1}(iT)[\begin{bmatrix} I & 0 \end{bmatrix} N^{-1}(iT)x(iT) - \begin{bmatrix} I & 0 \end{bmatrix} N^{-1}((i-1)T)x((i-1)T)] \qquad (34)$$

In view of $u(t) = K(t)x(t) + v(t)$, $u(t)$ of the discretization form can be written as:

$$u((i+1)T) = K((i+1)T)x((i+1)T) + v((i+1)T)$$
$$v((i+1)T) = [I - TK_v(iT)]v(iT) - TK_e(iT)e(iT) - TK_{x1}(iT)\begin{bmatrix} I & 0 \end{bmatrix} N^{-1}(iT)x(iT)...$$
$$-TK_{\dot{x}1}(iT)[\begin{bmatrix} I & 0 \end{bmatrix} N^{-1}(iT)x(iT) - \begin{bmatrix} I & 0 \end{bmatrix} N^{-1}((i-1)T)x((i-1)T)] \qquad (35)$$

We now turn to the investigation of the simulation algorithm. Owing to the singularity of $E(t)$ over $[t_0, t_f]$, the state response $x(t)$ cannot be directly derived from system (1). In terms of the implicit Euler's method, applying the approximation $\dot{x}((i+1)T) \approx \dfrac{x((i+1)T) - x(iT)}{T}$ to system (1) gives

$$E((i+1)T)\left[x((i+1)T) - x(iT)\right] = T\left[A((i+1)T)x((i+1)T) + B((i+1)T)u((i+1)T)\right]$$

A routine computation gives rise to the following formula

$$[E((i+1)T) - TA((i+1)T)]x((i+1)T) = E((i+1)T)x(iT) + TB((i+1)T)u((i+1)T)$$

Inserting the first equation of (35) into the last equation, we obtain

$$[E((i+1)T) - TA((i+1)T) - TB((i+1)T)K((i+1)T)]x((i+1)T)$$
$$= E((i+1)T)x(iT) + TB((i+1)T)v((i+1)T)$$

Therefore, if sampling time T is suitably chosen such that

$$[E((i+1)T) - TA((i+1)T) - TB((i+1)T)K((i+1)T)]$$

is invertible for all i, then the simulation algorithm can be obtained as follows:

$$\begin{cases} x((i+1)T) = [E((i+1)T) - TA((i+1)T) - TB((i+1)T)K((i+1)T)]^{-1}[E((i+1)T)x(iT)... \\ \qquad\qquad + TB((i+1)T)v((i+1)T)] \\ v((i+1)T) = [I - TK_v(iT)]v(iT) - TK_e(iT)e(iT) - TK_{x1}(iT)\begin{bmatrix} I & 0 \end{bmatrix} N^{-1}(iT)x(iT)... \\ \qquad\qquad - TK_{\dot{x}1}(iT)[\begin{bmatrix} I & 0 \end{bmatrix} N^{-1}(iT)x(iT) - \begin{bmatrix} I & 0 \end{bmatrix} N^{-1}((i-1)T)x((i-1)T)] \end{cases}$$

(36)

where $\overline{P}(t)$ is the solution of equation (24).

Notably, it is necessary to ensure that $t_f - t_0$ is an integer multiple of T in the discretization process.

VII. THE SECOND REALIZATION METHOD

The basic idea of the second method includes two points: (1) discretization; (2) utilizing the method described in the literature [23] to build an iterative scheme. Specifically, according to Euler's method, system (1) is first discretized into the following form

$$E((i+1)T)x((i+1)T)=[E((i+1)T)+TA(iT)]x(iT)+TB(iT)u(iT) \quad (37)$$

Choosing a matrix M and adding the identity

$$Mx((i+1)T)=Mx((i+1)T) \quad (38)$$

into both sides of the equation (37), we have

$$[M+E((i+1)T)]x((i+1)T)=Mx((i+1)T)+[E((i+1)T)+TA(iT)]x(iT)+TB(iT)u(iT)$$
(39)

If the closed-loop output of system (1) can track the reference signal, then we take $x((i+1)T)\approx x(iT)$ for large enough i. Meanwhile, M is selected such that $[M+E((i+1)T)]$ is invertible for all i. Then it follows from (39) that

$$x((i+1)T)=[M+E((i+1)T)]^{-1}\{[M+E((i+1)T)+TA(iT)]x(iT)+TB(iT)u(iT)\}$$
(40)

Furthermore, on the basis of equation (35), $u(iT)$ has the following form:

$$\begin{aligned} u(iT)=K(iT)x(iT)&+[I-TK_v((i-1)T)]v((i-1)T)-TK_e((i-1)T)e((i-1)T)\ldots \\ &-TK_{x1}((i-1)T)\begin{bmatrix} I & 0\end{bmatrix}N^{-1}((i-1)T)x((i-1)T)\ldots \\ &-TK_{\dot{x}1}((i-1)T)[\begin{bmatrix} I & 0\end{bmatrix}N^{-1}((i-1)T)x((i-1)T)\ldots \\ &-\begin{bmatrix} I & 0\end{bmatrix}N^{-1}((i-2)T)x((i-2)T)] \end{aligned} \quad (41)$$

If we plug (41) back into (40), we will obtain a closed-loop system, which will not be included here.

In particular, by selecting the parameters M and T, the eigenvalues of the matrix

$$[M+E((i+1)T)]^{-1}[M+E((i+1)T)+TA(iT)+TB(iT)K(iT)]$$

can be adjusted effectively, which will have a positive effect on the tracking speed and the overshoot during the tracking process. From this perspective, the introduction of matrix M is a correct decision.

VIII. NUMERICAL SIMULATION

Consider the system (1) with

$$E(t)=\begin{bmatrix} 0 & \frac{1}{t^2+1} \\ 0 & 1\end{bmatrix},\ A(t)=\begin{bmatrix} 10 & -\frac{1}{t^2+1} \\ 0 & -1\end{bmatrix},\ B(t)=\begin{bmatrix} -1 \\ 1\end{bmatrix},\ C(t)=\begin{bmatrix} 0 & 1\end{bmatrix},$$

$t\in[t_0,t_f]=[0,200]$

the reference signal $y_d(t)$ is selected as:

$$y_d(t)=\begin{cases} 0, & 0\le t\le 40 \\ 1+\sin\left(\frac{\pi}{10}(t-45)\right). & 45<t\le 50 \\ 2, & t>50 \end{cases}$$

and the controller will be designed according to Theorem 3.

It is obviously that the coefficient matrices of the above system and the reference signal satisfy assumptions A2 and A3. Alternatively, since $\text{rank}E(t)=1$ holds for any $t\in[0,200]$, then based on the discussions in section 2, there exist the following two invertible matrices:

$$M(t)=\begin{bmatrix} 0 & 1 \\ \frac{1}{10} & -\frac{1}{10(t^2+1)}\end{bmatrix},\ N(t)=\begin{bmatrix} 0 & 1 \\ 1 & 0\end{bmatrix}$$

such that

$$M(t)E(t)N(t)=\begin{bmatrix} 1 & 0 \\ 0 & 0\end{bmatrix}, \quad M(t)[A(t)N(t)-E(t)\dot{N}(t)]=M(t)A(t)N(t)=\begin{bmatrix} -1 & 0 \\ 0 & 1\end{bmatrix},$$

$$M(t)B(t)=\begin{bmatrix}1\\-\dfrac{t^2+2}{10(t^2+1)}\end{bmatrix},\ C(t)N(t)=\begin{bmatrix}1 & 0\end{bmatrix}$$

then system (1) is r.s.e. to

$$\begin{cases}\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}\dot{\bar{x}}(t)=\begin{bmatrix}-1 & 0\\0 & 1\end{bmatrix}\bar{x}(t)+\begin{bmatrix}1\\-\dfrac{t^2+2}{10(t^2+1)}\end{bmatrix}v(t)\\ y(t)=\begin{bmatrix}1 & 0\end{bmatrix}\bar{x}(t)\end{cases}\tag{42}$$

Note that (42) has the form of system (8), which implies that the system in this example is impulse-free according to Lemma 1.

We next construct the augmented system in the form of (21). In light of the reduced-order normal system extracting from system (42) and the structure of system (21), the coefficient matrices of the augmented system are

$$\bar{A}(t)=\begin{bmatrix}0&0&1&0\\0&0&1&0\\0&0&-1&0\\0&0&0&0\end{bmatrix},\ \bar{B}(t)=\begin{bmatrix}0\\0\\1\\1\end{bmatrix},\ G=\begin{bmatrix}-1\\0\\0\\0\end{bmatrix},\ \bar{C}=\begin{bmatrix}1&0&0&0\end{bmatrix}$$

Letting $\bar{Q}=diag(Q_e,0,0,0)$, $\bar{F}=diag(F,0,0,0)$ and $R=2$ with $Q_e=20$ and $F=1$, and selecting the sampling period T as 0.1, we will perform the simulation according to the algorithms provided in sections 7 and 8, respectively.

A. Simulation based on the first approach

Since $K(t)=0$, the iterative strategy (36) can be rewritten as

$$\begin{cases}x((i+1)T)=[E((i+1)T)-TA((i+1)T)]^{-1}[E((i+1)T)x(iT)+TB((i+1)T)v((i+1)T)]\\ v((i+1)T)=[I-TK_v(iT)]v(iT)-TK_e(iT)e(iT)-TK_{x1}(iT)\begin{bmatrix}I & 0\end{bmatrix}N^{-1}(iT)x(iT)\\ -K_{\dot{x}1}(iT)[\begin{bmatrix}I & 0\end{bmatrix}N^{-1}(iT)x(iT)-\begin{bmatrix}I & 0\end{bmatrix}N^{-1}((i-1)T)x((i-1)T)]\end{cases}\tag{43}$$

After solving the Riccati equation (24) by Euler's method (the iterative step is taken as T), algorithm (43) can be employed to accomplish the simulation experiment. Figure 1 illustrates the output response.

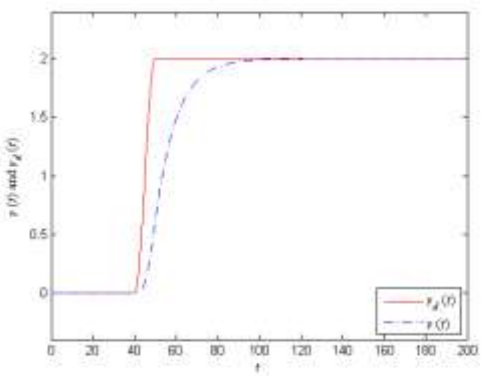

Figure 1. Closed-loop output curve by utilizing the first approach

It can be observed from Figure 1 that the closed-loop output of system (1) tracks $y_d(t)$ accurately, which confirms the effectiveness of the designed controller and the simulation algorithm in section 6.

B. Simulation based on the second approach

Combining (40) with (41) and utilizing $K(t)=0$, we have

$$\begin{cases} x((i+1)T)=[M+E((i+1)T)]^{-1}\{[M+E((i+1)T)+TA(iT)]x(iT)+TB(iT)u(iT)\} \\ u(iT)=[I-TK_v((i-1)T)]u((i-1)T)-TK_e((i-1)T)e((i-1)T)... \\ \qquad -TK_{x1}((i-1)T)\begin{bmatrix} I & 0 \end{bmatrix} N^{-1}((i-1)T)x((i-1)T)... \\ \qquad -TK_{\dot{x}1}((i-1)T)\{\begin{bmatrix} I & 0 \end{bmatrix} N^{-1}((i-1)T)x((i-1)T)... \\ \qquad -\begin{bmatrix} I & 0 \end{bmatrix} N^{-1}((i-2)T)x((i-2)T)\} \end{cases}$$

Choosing $M=\begin{bmatrix} 100 & 10 \\ 0 & 10 \end{bmatrix}$, Figure 2 plots the output response according to the simulation algorithm provided in section 7, from which we can see that the algorithm is very effective.

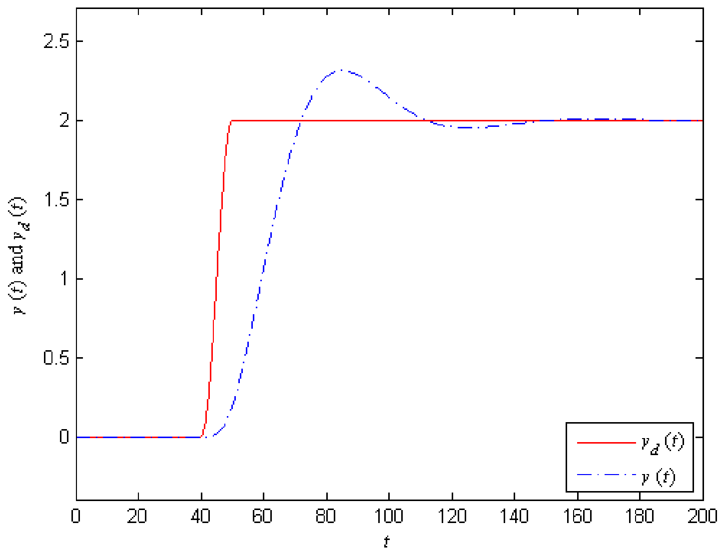

Figure 2. Closed-loop output curve by utilizing the second approach

Comparing Figure 2 with Figure 1, it can be clearly seen that there exists a large overshoot in Figure 2. Meanwhile, the adjustment time is longer than that in Figure 1, as well. In fact, as we mentioned in section 7, these deficiencies can be improved by properly selecting matrix M. For example, taking $M=\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}$, then the simulation result based on the second approach is depicted in Figure 3.

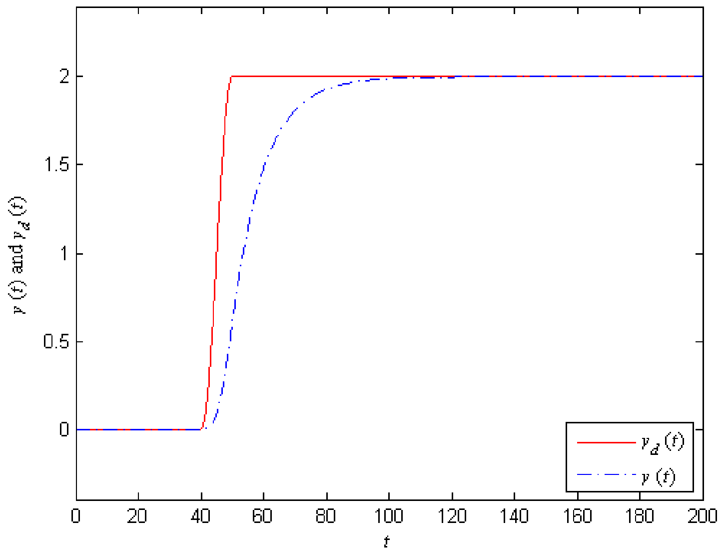

Figure 3. Closed-loop output curve 2 by utilizing the second approach

Through plotting Figure 1 and Figure 3 in the same figure (Figure 4), it can be clearly seen that Figure 3 is almost the same as Figure 1.

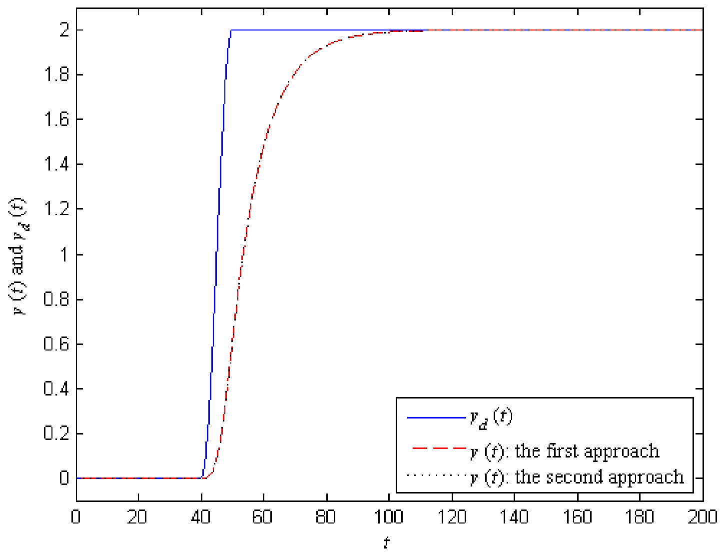

Figure 4. Closed-loop output curves by utilizing both approaches

In the following, we denote the output responses of Figure 1 and Figure 3 as $y_1(t)$ and $y_2(t)$, respectively. By calculating $y_1(t) - y_2(t)$ and illustrating the result in Figure 5, we find the biggest deviations between $y_1(t)$ and $y_2(t)$ are just ± 0.0015 magnitude, which further confirms the result observed in Figure 4.

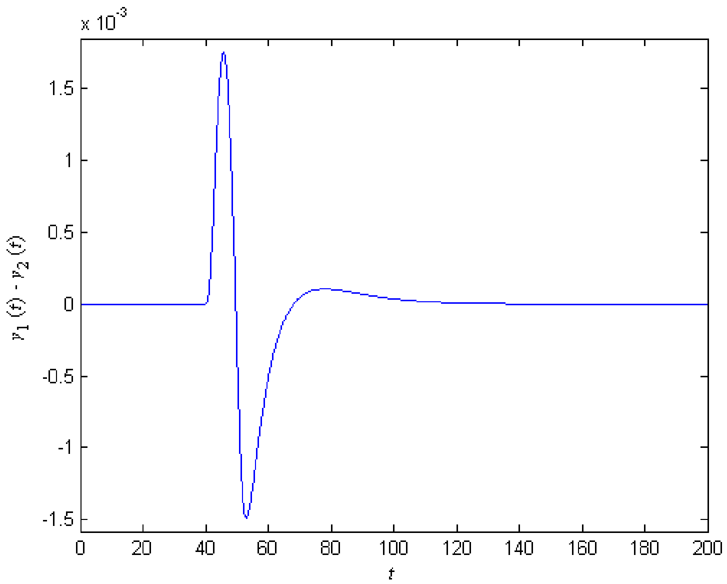

Figure 5. Deviation between $y_1(t)$ and $y_2(t)$

Furthermore, setting $M = \begin{bmatrix} -1000 & -1 \\ 1 & 1 \end{bmatrix}$, Figure 6 demonstrates the output responses of the closed-loop system under the above two methods. Figure 7 exhibits the partial enlargement of Figure 6 during $0 \le t \le 110$. It can be seen from both figures that the output response under the second approach is slightly slower than that under the first approach during the period $40 < t < 60$, and the scenario is opposite after $t > 60$.

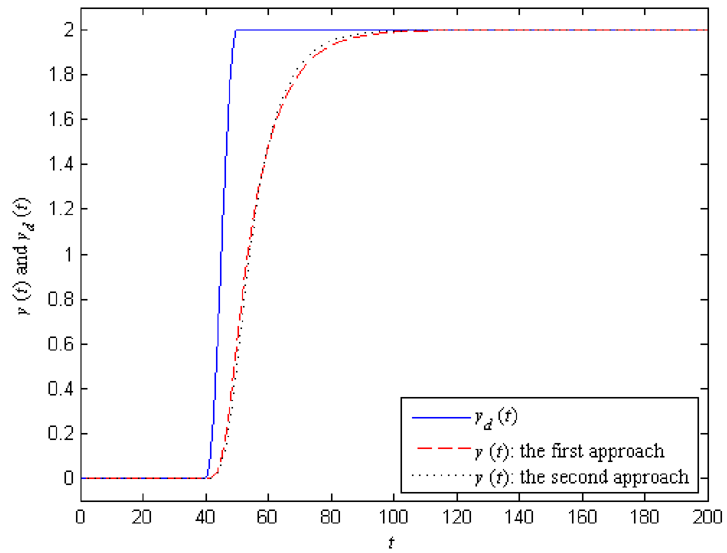

Figure 6. Closed-loop output curves by utilizing both approaches

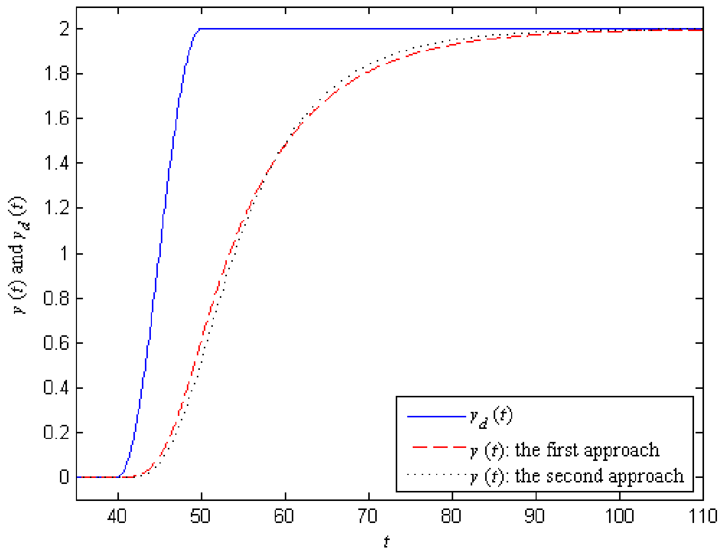

Figure 7. The partial enlargement of Figure 6

IX. CONCLUSION

The optimal tracking problem for time-varying descriptor systems was investigated. First, the reference signal was assumed to be previewable. If necessary, the pre-feedback could be used to eliminate the impulse existingin the original system. With the aid of the coordinate transformation and the state augmentation techniques, the tracking problem of the time-varying descriptor system was transformed into a stability problem with a zero solution for the augmented system. With the help of preview control theory, a controller for ensuring the stability of the zero solution was obtained by letting the preview length tend to zero. Meanwhile, a control input for the original problem was also obtained by virtue of the pre-feedback. This paper also presented two kinds of numerical simulation methods for the descriptor time-varying system. Simulation results verified the effectiveness of the given theorems and proposed simulation algorithms.

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