RESEARCH ARTICLE OPEN ACCESS

Soft intuitionistic fuzzy sets with some new operations

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ABSTRACT: The soft set theory offers a general mathematical tool for dealing with uncertainty, fuzziness and vagueness. The concept of soft sets which can be seen as a new mathematical approach to vagueness is used in many applications including decision making and medical diagnosis problems. Later, it is generalized to soft fuzzy set. In this paper we present the definition and operations of soft intuitionistic fuzzy set. Furthermore, based on the analysis of several operations on soft intuitionistic fuzzy set in the study, we provide some new notions such as the restricted intersection, the restricted union and the restricted difference of two soft intuitionistic fuzzy sets. We also improve the notion of complement of a soft intuitionistic fuzzy set and prove certain De Morgan's laws hold in soft intuitionistic fuzzy set with respect to the new definitions.
Keywords: Soft set; Soft fuzzy set; Intuitionistic fuzzy set; Soft Intuitionistic fuzzy set.

I. INTRODUCTION

Fuzzy set that allow to manage imprecise and vague information was introduced by Lotfi A. Zadeh [4]. Such vagueness is reflected by the membership degree of the objects belonging to a concept [8]. Fuzzy sets theory has been widely and successfully applied in many different areas to handle such type of uncertainties. Nevertheless, it presents limitations to deal with imprecise and vague information when different sources of vagueness appear simultaneously. Due to this fact and in order to overcome such limitations, different extensions of fuzzy sets have been introduced in the literature such as intuitionistic fuzzy sets, by Atanassov [3],which allow to incorporate simultaneously the membership degree and the non-membership degree of each element. Soft set was later defined by Molodtsov [2] who initiated the concept of soft theory as a mathematical tool to deal with uncertainties. Maji et al [5] made a theoretical study on the Soft Set Theory in more detail and contributed towards the fuzzification of the notion of it and described the application of soft set theory to a decision making problem using rough sets. Recently Kong et al. [10, 11] applied the soft set theoretic approach in decision making problems. Soft fuzzy set was defined by Yao et al. [1] followed by intuitionistic fuzzy soft set defined by Xu Yong et al.[9].Alkhazaleh et al. [6] introduced the concept of fuzzy parameterized interval-valued fuzzy soft set and gave its application in decision making. Alkhazaleh et al. [7] introduced soft multisets as a generalization of Molodtsov's soft set and proposed the concept of possibility fuzzy soft set.

In the present paper we introduced some new concepts of soft intuitionistic fuzzy set. There has been incredible interest in the subject due to its diverse applications, ranging from engineering and computer science to social behaviour studies. Here we define a soft intuitionistic fuzzy set and its operations of equality, complement, union, intersection, OR and AND operators. We also provide some new notions such as the restricted intersection, the restricted union and the restricted difference of two soft intuitionistic fuzzy sets along with examples. We also improve the notion of complement of a soft intuitionistic fuzzy set and prove certain De Morgan's laws hold in soft intuitionistic fuzzy set with respect to the new definitions.

II. PRELIMINARIES

Definition 2.1. Let U be an initial set and E be a set of parameters. Let P(U) denotes the power set of U and A ⊂ E then a pair (F, A) is called a soft set over U if F is a mapping given by F : A →P(U).

Definition 2.2. Let U be an initial set and E be a set of parameters. Let F(U) denotes the fuzzy power set of U and A ⊂ E thena pair (F, A) is called a fuzzy soft setover U if F is a mapping given by F : A →F(U).

Definition 2.3. Let U be an initial set and E be a set of parameters. Let P(U) denotes the power set of U and A ⊂ E then a pair (F, A) is called a soft fuzzy set over U if F is a mapping given by F : A →P(U) and

$$F(x)=\{y\in U:\tilde{R}_{\alpha}(x,y)\geq\alpha, x\in A, y\in U, \alpha\in[0,1]\}\subset X\times Y$$

is defined as cut-set $\tilde{R}\in F\left(X\times Y\right)$.

Definition 2.4. Consider U and E as a universe set and a set of parameters respectively. Let IFS(U) denotes the intuitionistic fuzzy power set of U and A ⊂E then a pair (F, A) is an intuitionistic fuzzy soft set over U if the mapping F is given by F : A →IFS(U) .

We recall these definitions in order to use them to introduce the concept of soft intuitionistic fuzzy set and to define some operations on soft intuitionistic fuzzy set namely equality, null, complement, union, intersection, AND and ORoperators.

III. SOFT INTUITIONISTIC FUZZY SET

3.1 Relation on Soft Intuitionistic Fuzzy Set

Let $\tilde{R}_\alpha = \left(\tilde{R}_{\mu_\alpha}, \tilde{R}_{\nu_\alpha}\right)$ be an intuitionistic fuzzy subset of $X \times Y$, and $\tilde{R}_\alpha$ is defined as intuitionistic fuzzy relationship from X to Y written as $X \xrightarrow{\tilde{R}} Y$. Let $\tilde{R}(X \times Y)$ denotes the degree of correspondence between X and Y based on the relationship $\tilde{R}$ and $F(X \times Y)$ denotes the family of an intuitionistic fuzzy relationship on X to Y.

The set $\tilde{R}_\alpha = \left\{(x,y) \in X \times Y : \tilde{R}_{\mu_\alpha}(x,y) \geq \alpha \text{ and } \tilde{R}_{\nu_\alpha}(x,y) \leq \alpha\right\} \subset X \times Y$ is defined as α- cut set if $\tilde{R} \in F(X \times Y)$ for $\alpha \in [0,1]$.

3.2 Soft Intuitionistic Fuzzy Set

Let U be an initial set and E be a set of parameters. Let P(U) denotes the power set of U and A ⊂ E then a pair (F,A) is called a soft intuitionistic fuzzy set over U if F is a mapping given by F:A →P(U) and $F(x) = \{y \in U : (x,y) \in \tilde{R}_\alpha, x \in A, y \in U, \alpha \in [0,1]\}$.

3.3 Subset of Soft Intuitionistic Fuzzy Sets

In this section we introduce the concept for subset of two soft intuitionistic fuzzy sets.

For two intuitionistic fuzzy sets $(\mathrm{F}, A)_{\tilde{R}}$ and $(\mathrm{G}, B)_{\tilde{R}}$ over common universe U, we say $(\mathrm{F}, A)_{\tilde{R}}$ is a soft intuitionistic fuzzy subset of $(\mathrm{G}, B)_{\tilde{R}}$ if:

(i) $A \subset B$ and

(ii) $\forall \varepsilon \in A, F(\varepsilon)$ is an intuitionistic fuzzy subset of $G(\varepsilon)$

i.e.

$A \subset B$ iff $\forall x \in \mathrm{E}, \mu_{\mathrm{A}}(x) \leq \mu_{\mathrm{B}}(x)$ and $\nu_{\mathrm{A}}(x) \leq \nu_{\mathrm{B}}(x)$

denoted by $(\mathrm{F}, A) \subset (\mathrm{G}, B)$.

3.4. Equality of Soft Intuitionistic Fuzzy Sets

Two soft intuitionistic fuzzy sets $(F, A)_{\tilde{R}}$ and $(G, B)_{\tilde{R}}$ over common universe U are said to be soft intuitionistic fuzzy sets equal if $(F, A)_{\tilde{R}}$ is a soft intuitionistic subset of $(G, B)_{\tilde{R}}$ and $(G, B)_{\tilde{R}}$ is a soft intuitionistic subset of $(F, A)_{\tilde{R}}$.

3.5. Complement of Soft Intuitionistic Fuzzy Set

The complement of soft intuitionistic fuzzy set $(F, A)_{\tilde{R}}$ is denoted by $(F, A)_{\tilde{R}}^{\,c}$ and defined as $(\mathrm{F}, A)_{\tilde{R}}^{\,c} = \left(\mathrm{F}^{\mathrm{c}}, \neg A\right)_{\tilde{R}}$, where $\mathrm{F}^{\mathrm{c}} : \neg A \to P(U)$ is a mapping, $\mathrm{F}^{\mathrm{c}}(A)$ = intuitionistic fuzzy complement of $F(\neg e), \forall e \in \neg A$.

3.6. Null Soft Intuitionistic Fuzzy Set

$(F, A)_{\tilde{R}}$, a soft intuitionistic fuzzy set over Uis said to be a null soft intuitionistic fuzzy set denoted by ϕ, if $\forall \varepsilon \in A, F(\varepsilon)$ = null intuitionistic fuzzy of U (null-set).

3.7. Union of Soft Intuitionistic Fuzzy Set

The union of two soft intuitionistic fuzzy sets $(F, A)_{\tilde{R}}$ and $(G, B)_{\tilde{R}}$ over a common universe U is the soft intuitionistic fuzzy set $(H, C)_{\tilde{R}}$, where $C = A \cup B$ and $\forall e \in C$

$$H(e) = \begin{cases} F(e) & \text{if } \quad e \in \text{A-B} \\ G(e) & \text{if } \quad e \in \text{B-A} \\ F(e) \cup G(e) & \text{if } \quad e \in \text{A} \cap \text{B} \end{cases}$$

we write $(F, A)_{\tilde{R}} \grave{E} (G, B)_{\tilde{R}} = (H, C)_{\tilde{R}}$

3.8. Intersection of Soft Intuitionistic Fuzzy Set

The intersection of two soft intuitionistic fuzzy sets $(F, A)_{\tilde{R}}$ and $(G, B)_{\tilde{R}}$ over a common universe U is the soft intuitionistic fuzzy set $(H, C)_{\tilde{R}}$, where $C = A \cup B$ and $\forall e \in C$ is defined by

$$H(e) = \begin{cases} F(e) & \text{if } \quad e \in \text{A-B} \\ G(e) & \text{if } \quad e \in \text{B-A} \\ F(e) \cap G(e) & \text{if } \quad e \in \text{A} \cap \text{B} \end{cases}$$

we write $(F, A)_{\tilde{R}} \cap (G, B)_{\tilde{R}} = (H, C)_{\tilde{R}}$

3.9. AND Operation on Soft Intuitionistic Fuzzy Set

If $(F,A)_{\tilde{R}}$ and $(G,B)_{\tilde{R}}$ be two soft intuitionistic fuzzy sets then, "$(\mathrm{F},A)_{\tilde{R}}\, AND(\mathrm{G},B)_{\tilde{R}}$" is an soft intuitionistic fuzzy set denoted by $(\mathrm{F},A)_{\tilde{R}}\,\overline{\wedge}(\mathrm{G},B)_{\tilde{R}}$ is defined by $(\mathrm{F},A)_{\tilde{R}}\,\overline{\wedge}(\mathrm{G},B)_{\tilde{R}}=(\mathrm{H},A\times B)_{\tilde{R}}$, where

$$H(\alpha,\beta)=F(\alpha)\cap G(\beta),\forall(\alpha,\beta)\in A\times B$$

3.10. OR Operation on Intuitionistic Fuzzy Soft Set

If $(\mathrm{F},A)_{\tilde{R}}$ and $(\mathrm{G},B)_{\tilde{R}}$ be two soft intuitionistic fuzzy sets then, "$(\mathrm{F},A)_{\tilde{R}}\, OR(\mathrm{G},B)_{\tilde{R}}$" is an soft intuitionistic fuzzy set denoted by $(\mathrm{F},A)_{\tilde{R}}\overset{o}{\vee}(\mathrm{G},B)_{\tilde{R}}$, is defined by $(\mathrm{F},A)_{\tilde{R}}\overset{o}{\vee}(\mathrm{G},B)_{\tilde{R}}=(\mathrm{O},A\times B)_{\tilde{R}}$, where $O(\alpha,\beta)=F(\alpha)\cup G(\beta),\forall(\alpha,\beta)\in A\times B$.

IV. SOME NEW OPERATIONS IN SOFTINTUITIONISTIC FUZZY SET THEORY

4.1. Restricted union

Let $(\mathrm{F},A)_{\tilde{R}}$ and $(\mathrm{G},B)_{\tilde{R}}$ be two soft intuitionistic fuzzy sets over a common universe U such that $A\cap B\neq\phi$. The restricted union of $(\mathrm{F},A)_{\tilde{R}}$ and $(\mathrm{G},B)_{\tilde{R}}$ is denoted by $(\mathrm{F},A)_{\tilde{R}}\cup_{\Re}(\mathrm{G},B)_{\tilde{R}}$ and is defined as $(\mathrm{F},A)_{\tilde{R}}\cup_{\Re}(\mathrm{G},B)_{\tilde{R}}=(\mathrm{H},C)_{\tilde{R}}$ where $C=A\cap B$ and $c\in C,H(c)=F(c)\cup G(c)$.

Example 1.

Assume that $U=\{h_1,h_2,h_3\}$ is a set of houses under consideration and $E=\{e_1,e_2,e_3,e_4\}$ is the set of parameters where each parameter is an intuitionistic fuzzy word or a sentence involving intuitionistic fuzzy words $A,B\subset E$ suchthat $A=\{\text{beautiful}(e_1),\text{comfortable}(e_2),\text{cheap}(e_4)\}$ and $B=\{\text{beautiful}(e_1),\text{big}(e_3),\text{cheap}(e_4)\}$.

The soft intuitionistic fuzzy sets $(F,A)_{\tilde{R}}$ and $(G,B)_{\tilde{R}}$ describe the “attractiveness of the houses” which Mr. X and Mr. Y are going to buy, respectively.

Let intuitionistic fuzzy relationship for Mr. X is

$$\tilde{R}=\left\{\frac{(0.3,0.4)}{(h_1,e_1)}+\frac{(0.5,0.3)}{(h_1,e_2)}+\frac{(0.1,0.4)}{(h_1,e_4)}+\frac{(0.8,0.0)}{(h_2,e_1)}+\frac{(0.1,0.5)}{(h_2,e_2)}+\frac{(0.3,0.6)}{(h_2,e_4)}+\frac{(0.0,0.4)}{(h_3,e_1)}+\frac{(0.5,0.5)}{(h_3,e_2)}+\frac{(0.3,0.4)}{(h_3,e_4)}\right\}$$

Thus, we can view the soft intuitionistic fuzzy set $(F,A)_{\tilde{R}}$ as a collection of intuitionistic fuzzy approximations (which are under the fuzzy relationship $\tilde{R}$) as below:

$$(F,A)_{\tilde{R}}=\left\{\text{beautiful house}=\left\{\frac{(0.3,0.4)}{h_1},\frac{(0.8,0.0)}{h_2},\frac{(0.0,0.4)}{h_3}\right\},\text{ comfortable house}=\left\{\frac{(0.5,0.3)}{h_1},\frac{(0.1,0.5)}{h_2},\frac{(0.5,0.5)}{h_3}\right\},\right.$$

$$\left.\text{cheap house}=\left\{\frac{(0.1,0.4)}{h_1},\frac{(0.3,0.6)}{h_2},\frac{(0.3,0.4)}{h_3}\right\}\right\}$$

and suppose intuitionistic fuzzy relationship for Mr. Y is

$$\tilde{R}=\left\{\frac{(0.1,0.5)}{(h_1,e_1)}+\frac{(0.4,0.5)}{(h_1,e_3)}+\frac{(0.1,0.6)}{(h_1,e_4)}+\frac{(0.4,0.3)}{(h_2,e_1)}+\frac{(0.1,0.7)}{(h_2,e_3)}+\frac{(0.3,0.6)}{(h_2,e_4)}+\frac{(0.4,0.3)}{(h_3,e_1)}+\frac{(0.2,0.5)}{(h_3,e_3)}+\frac{(0.3,0.4)}{(h_3,e_4)}\right\}$$

Thus, we can view the soft intuitionistic fuzzy set $(G,B)_{\tilde{R}}$ as a collection of fuzzy approximations (which are under the fuzzy relationship $\tilde{R}$) as below:

$$(G,B)_{\tilde{R}}=\left\{\text{beautiful house}=\left\{\frac{(0.3,0.4)}{h_1},\frac{(0.8,0.0)}{h_2},\frac{(0.4,0.3)}{h_3}\right\},\text{big house}=\left\{\frac{(0.4,0.5)}{h_1},\frac{(0.1,0.7)}{h_2},\frac{(0.2,0.5)}{h_3}\right\},\right.$$

cheap house$=\left\{\frac{(0.1,0.6)}{h_1},\frac{(0.5,0.3)}{h_2},\frac{(0.3,0.4)}{h_3}\right\}\Bigg\}$

To find the $(\mathrm{F},A)_{\tilde{R}} \bigcup_{\Re} (\mathrm{G},B)_{\tilde{R}} = (\mathrm{H},C)_{\tilde{R}}$

We have $C=\{\text{beautiful}(e_1),\text{cheap}(e_4)\}$, then

$$H(e_1)=F(e_1)\cup G(e_1)$$
$$=\left\{\frac{\max(0.2,0.1),\min(0.8,0.6)}{h_1},\frac{\max(0.4,0.5),\min(0.5,0.3)}{h_2},\frac{\max(0.6,0.3),\min(0.2,0.4)}{h_3}\right\}$$
$$=\left\{\frac{(0.3,0.4)}{h_1},\frac{(0.8,0.0)}{h_2},\frac{(0.4,0.3)}{h_3}\right\}, \text{ and}$$
$$H(e_4)=F(e_4)\cup G(e_4)$$
$$=\left\{\frac{\max(0.2,0.1),\min(0.8,0.6)}{h_1},\frac{\max(0.4,0.5),\min(0.5,0.3)}{h_2},\frac{\max(0.6,0.3),\min(0.2,0.4)}{h_3}\right\}$$
$$=\left\{\frac{(0.2,0.6)}{h_1},\frac{(0.5,0.3)}{h_2},\frac{(0.6,0.2)}{h_3}\right\}$$

Thus, we can view the soft intuitionistic fuzzy set as a collection of fuzzy approximations (which are under the fuzzy relationship $\tilde{R}$) as below:

$$(H,C)_{\tilde{R}}=\left\{\text{beautiful house}=\left\{\frac{(0.3,0.4)}{h_1},\frac{(0.8,0.0)}{h_2},\frac{(0.4,0.3)}{h_3}\right\},\text{cheap house}=\left\{\frac{(0.2,0.6)}{h_1},\frac{(0.5,0.3)}{h_2},\frac{(0.6,0.2)}{h_3}\right\}\right\}$$

4.2. Restricted intersection

Let $(\mathrm{F},A)_{\tilde{R}}$ and $(\mathrm{G},B)_{\tilde{R}}$ be two soft intuitionistic fuzzy sets over a common universe U such that $A\cap B\neq\phi$. The restricted intersection of $(\mathrm{F},A)_{\tilde{R}}$ and $(\mathrm{G},B)_{\tilde{R}}$ is denoted by $(\mathrm{F},A)_{\tilde{R}} \bigcap_{\Re} (\mathrm{G},B)_{\tilde{R}}$ and is defined as $(\mathrm{F},A)_{\tilde{R}} \bigcap_{\Re} (\mathrm{G},B)_{\tilde{R}} = (\mathrm{H},C)_{\tilde{R}}$ where $C=A\cap B$ and $c\in C, H(c)=F(c)\cap G(c)$.

Example 2.

Consider $(\mathrm{F},A)_{\tilde{R}}$ and $(\mathrm{G},B)_{\tilde{R}}$ be two soft intuitionistic fuzzy sets over a common universe U as defined in example 1.

To find the $(\mathrm{F},A)_{\tilde{R}} \bigcap_{\Re} (\mathrm{G},B)_{\tilde{R}} = (\mathrm{H},C)_{\tilde{R}}$

We have $C=\{\text{beautiful}(e_1),\text{cheap}(e_4)\}$, then

$$H(e_1)=F(e_1)\cap G(e_1)$$
$$=\left\{\frac{\min(0.3,0.1),\max(0.4,0.5)}{h_1},\frac{\min(0.8,0.4),\max(0.0,0.3)}{h_2},\frac{\min(0.0,0.4),\max(0.4,0.3)}{h_3}\right\}$$
$$=\left\{\frac{(0.1,0.5)}{h_1},\frac{(0.4,0.3)}{h_2},\frac{(0.0,0.4)}{h_3}\right\}, \text{ and}$$
$$H(e_4)=F(e_4)\cap G(e_4)$$
$$=\left\{\frac{\min(0.2,0.1),\max(0.8,0.6)}{h_1},\frac{\min(0.4,0.5),\max(0.5,0.3)}{h_2},\frac{\min(0.6,0.3),\max(0.2,0.4)}{h_3}\right\}$$

$$=\left\{\frac{(0.1,0.8)}{h_1},\frac{(0.4,0.5)}{h_2},\frac{(0.3,0.4)}{h_3}\right\}$$

Thus, we can view the soft intuitionistic fuzzy set as a collection of fuzzy approximations (which are under the fuzzy relationship $\tilde{R}$) as below:

$$(H,C)_{\tilde{R}}=\left\{\text{beautiful house}=\left\{\frac{(0.1,0.5)}{h_1},\frac{(0.4,0.3)}{h_2},\frac{(0.0,0.4)}{h_3}\right\},\text{cheap house}=\left\{\frac{(0.1,0.8)}{h_1},\frac{(0.4,0.5)}{h_2},\frac{(0.3,0.4)}{h_3}\right\}\right\}$$

4.3. Restricted difference

Let $(\mathrm{F},A)_{\tilde{R}}$ and $(\mathrm{G},B)_{\tilde{R}}$ be two soft intuitionistic fuzzy sets over the same universe U such that $A\cap B\neq\phi$. The restricted difference of $(\mathrm{F},A)_{\tilde{R}}$ and $(\mathrm{G},B)_{\tilde{R}}$ denoted by $(\mathrm{F},A)_{\tilde{R}}\square_{\Re}(\mathrm{G},B)$, and is defined as $(\mathrm{F},A)_{\tilde{R}}\square_{\Re}(\mathrm{G},B)_{\tilde{R}}=(\mathrm{H},C)_{\tilde{R}}$, where $C=A\cap B$ and for all $c\in C, H(c)=F(c)-G(c)$, the difference of the sets $F(c)$ and $G(c)$.

4.4. Relative null soft intuitionistic fuzzy set

Let U be an initial universe set, E be the universe set of parameters, and $A\subset E$ then $(\mathrm{F},A)_{\tilde{R}}$ is called a relative null soft intuitionistic fuzzy set (with respect to the parameter set A), denoted by ϕ_A, if $F(e)=\phi$ for all $e\in A$.

4.5. Relative whole soft intuitionistic fuzzy set

Let U be an initial universe set, E be the universe set of parameters, and $A\subset E$ then $(\mathrm{G},A)_{\tilde{R}}$ is called a relative whole soft intuitionistic fuzzy set (with respect to the parameter set A), denoted by $\cup_A$, if $F(e)=U$ for all $e\in A$.

The relative whole soft intuitionistic set $\cup_E$ with respect to the universe set of parameters E is called the absolute soft intuitionistic fuzzy set over U.

4.6. Relative complement

The relative complement of a soft intuitionistic fuzzy set $(\mathrm{F},A)_{\tilde{R}}$ is denoted by $(\mathrm{F},A)_{\tilde{R}}^{\ r}$ and is defined by $(\mathrm{F},A)_{\tilde{R}}^{\ r}=(\mathrm{F}^{\mathrm{r}},A)_{\tilde{R}}$ where $F^r:A\to P(U)$ is a mapping defined by $F^r(\alpha)=U-F(\alpha)$ for all $\alpha\in A$.

Clearly, $(\mathrm{F},A)_{\tilde{R}}^{\ r}=\cup_E\square_{\Re}(\mathrm{F},A)_{\tilde{R}}$ and $\left((\mathrm{F},A)_{\tilde{R}}^{\ r}\right)^r=(\mathrm{F},A)_{\tilde{R}}$. It is worth noting that in the above definition of complement, the parameter set of the complement $(\mathrm{F},A)_{\tilde{R}}^{\ r}$ is still the original parameter set A, instead of $\neg A$ as given in Definition 3.5. To emphasize this difference, the complement given in the Definition 3.5 will be called neg-complement (or pseudo-complement).

V. DE MORGAN'S LAWS IN SOFT INTUITIONISTIC FUZZY SET THEORY

In this section, we first show that the following De Morgan's type of results hold in soft set theory for the newly definedrelative complement, restricted union and restricted intersection.

Theorem: Let $(\mathrm{F},A)_{\tilde{R}}$ and $(\mathrm{G},B)_{\tilde{R}}$ be two soft intuitionistic fuzzy sets over the same universe U such that $A\cap B\neq\phi$. Then

(i) $\left((\mathrm{F},A)_{\tilde{R}}\cup_{\Re}(\mathrm{G},B)_{\tilde{R}}\right)^r=(\mathrm{F},A)_{\tilde{R}}^{\ r}\cap_{\Re}(\mathrm{G},B)_{\tilde{R}}^{\ r}$

(ii) $\left((\mathrm{F},A)_{\tilde{R}}\cap_{\Re}(\mathrm{G},B)_{\tilde{R}}\right)^r=(\mathrm{F},A)_{\tilde{R}}^{\ r}\cup_{\Re}(\mathrm{G},B)_{\tilde{R}}^{\ r}$

Proof.

(i) Let $(\mathrm{F},A)_{\tilde{R}}\cup_{\Re}(\mathrm{G},B)_{\tilde{R}}=(\mathrm{H},C)_{\tilde{R}}$

where $H(c)=F(c)\cup G(c)$ for all $c\in C=A\cap B\neq\phi$

Since $\left((\mathrm{F},A)_{\tilde{R}}\cup_{\Re}(\mathrm{G},B)_{\tilde{R}}\right)^r=(\mathrm{H},C)_{\tilde{R}}^{\ r}$, by definition

$H^r(c)=U-\left[F(c)\cup G(c)\right]=\left[U-F(c)\right]\cap\left[U-G(c)\right]$ for all $c\in C$.

Now $(\mathrm{F},A)_{\tilde{R}}^{\ r} \cap_{\Re} (\mathrm{G},B)_{\tilde{R}}^{\ r} = (\mathrm{F^r},A)_{\tilde{R}} \cap_{\Re} (\mathrm{G^r},B)_{\tilde{R}} = (\mathrm{K},C)_{\tilde{R}}$ where $C = A \cap B$.

So by definition, we have

$$\begin{aligned} K(c) &= F^r(c) \cap G^r(c) \\ &= (U - F(c)) \cap (U - G(c)) \\ &= H^r(c) \text{ for all } c \in C. \end{aligned}$$

Hence $\left((\mathrm{F},A)_{\tilde{R}} \cup_{\Re} (\mathrm{G},B)_{\tilde{R}}\right)^r = (\mathrm{F},A)_{\tilde{R}}^{\ r} \cap_{\Re} (\mathrm{G},B)_{\tilde{R}}^{\ r}$.

(ii) Let $(\mathrm{F},A)_{\tilde{R}} \cap_{\Re} (\mathrm{G},B)_{\tilde{R}} = (\mathrm{H},C)_{\tilde{R}}$

where $H(c) = F(c) \cap G(c)$ for all $c \in C = A \cap B \neq \phi$

Since $\left((\mathrm{F},A)_{\tilde{R}} \cap_{\Re} (\mathrm{G},B)_{\tilde{R}}\right)^r = (\mathrm{H},C)_{\tilde{R}}^{\ r}$, by definition

$$H^r(c) = U - [F(c) \cap G(c)] = [U - F(c)] \cup [U - G(c)] \text{ for all } c \in C.$$

Now

$$(\mathrm{F},A)_{\tilde{R}}^{\ r} \cup_{\Re} (\mathrm{G},B)_{\tilde{R}}^{\ r} = (\mathrm{F^r},A)_{\tilde{R}} \cup_{\Re} (\mathrm{G^r},B)_{\tilde{R}} = (\mathrm{K},C)_{\tilde{R}} \text{ where } C = A \cap B.$$

So by definition, we have

$$\begin{aligned} K(c) &= F^r(c) \cup G^r(c) \\ &= (U - F(c)) \cup (U - G(c)) \\ &= H^r(c) \text{ for all } c \in C \end{aligned}$$

Hence $\left((\mathrm{F},A)_{\tilde{R}} \cap_{\Re} (\mathrm{G},B)_{\tilde{R}}\right)^r = (\mathrm{F},A)_{\tilde{R}}^{\ r} \cup_{\Re} (\mathrm{G},B)_{\tilde{R}}^{\ r}$.

VI. CONCLUSION

In this paper we introduced new concepts of soft intuitionistic fuzzy set. There has been incredible interest in the subject due to its diverse applications, ranging from engineering and computer science to social behaviour studies. In this study we defined a soft intuitionistic fuzzy set and its operations of equality, complement, union, intersection, OR and AND operators. We have given some new notions such as the restricted intersection, the restricted union and the restricted difference of two soft intuitionistic fuzzy sets along with examples. We also improved the notion of complement of a soft intuitionistic fuzzy set and proved certain De Morgan's laws hold in soft intuitionistic fuzzy set with respect to the new definitions.

REFERENCES:

[1]. B. Yao, L. Jin-liang and Y. Rui-xia, Fuzzy soft set and soft fuzzy set. Fourth International Conference on Natural Computation, 4 (2008), 252-255.

[2]. D. Molodtsov, Soft set theory first result. An International Journal of Computers and mathematics with Applications, 37(1999), 19-31.

[3]. K.T. Atanassov, Intuitionistic fuzzy set. Fuzzy Sets and Systems, 20 (1986), 87-96.

[4]. L. A. Zadeh, Fuzzy sets.Information and Control, 8 (1965), 338-353.

[5]. P.K. Maji, R. Biswas, A.R. Roy, Soft set theory.Comput. Math. Appl., 45 (2003), 555-562.

[6]. S. Alkhazaleh, A. R. Salleh and N. Hassan, Fuzzy parameterized interval-valued fuzzy soft set. Applied Mathematical Sciences, 67 (2011), 3335-3346.

[7]. S. Alkhazaleh. A. R. Salleh and N. Hassan, Soft multisets theory. Applied Mathematical Sciences, 72 (2011), 3561-3573.

[8]. Y. Xu, J. Liu, L. Mart´ınez, and D. Ruan, Some views on information fusion and logic based approaches in decision making under uncertainty. Journal of Universal Computer Science, 16(2010),3-19,.

[9]. Y. Xu, Y. Sun and D. Li, Intuitionistic fuzzy soft set. Science & ResearchDepartment Dalian Naval Academy Dalian China, 2010.

[10]. Z. Kong, L. Gao, L. Wang and S. Li, The normal parameter reduction of soft sets and its algorithm.Comput. Math. Appl., 56 (2008), 3029-3037.

[11]. Z. Kong, L. Gao and L. Wang, Comment on A fuzzy soft set theoretic approach to decision making problems. J. Comput. Appl. Math. 223 (2009), 540-542.

Deepak Kumar" Soft intuitionistic fuzzy sets with some new operations" International Journal of Engineering Research and Applications (IJERA), vol. 9, no. 10, 2019, pp 16-22