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Using Variational Iteration Method To Solve Burger's Equations

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ABSTRACT
The variational iteration method (VIM) is a method that is being increasingly used for solving non-linear equations. Recent applications have included using (VIM) for solving Burger's equations. Looking at methods for resolving these equations remains important because of the potential applications to real-world scenarios, and this can be demonstrated through the examples provided in this paper. Historically, issues with numerical methods for resolving measurements of turbulence and flow through viscous fluids have been problematic due to rounding issues, and variational iteration is being used by many mathematicians as a solution. This paper outlines this viewpoint and provides examples of its successful application to Burger's equations.
Keywords: Burger's equations,Burger's couple equations, non-linear equations, variational iteration method, numerical methods.

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I. INTRODUCTION

Real-world phenomena are most often described using non-linearequations that essentially describe the fundamental aspects of physical behaviors from complex biological systems to patterns of energy generation. The often-ubiquitous nature of these forms of equation and the large classes of nonlinear equations, which do not have a precise analytic solution, have ensured that establishing numerical methods to provide exact solutions for non-linear equations remains an important aspect of modern mathematic research (Abdou and Soliman, 2005; Alquran, 2012). Amongst these equations are Burger's and Burger's couple equations, which are of great significance when handling real-life analysis and the range of diverse physical problems that are related to engineering and applied sciences (Mohyud-Din et al., 2010; Biazar and Aminikhah, 2007). As a result, methods to resolve these equations continue to be of importance, and this focus is reflected the range of methods that have been established and tested, including the Backlund and Darboux transformations (Mao and Liu, 2018), the inverse scattering method (Zakharov, 1980), the sine-cosine method (Alquran, 2012), and the homogenous balance method (Wang, 1996). This research will examine in particular the application of the variational iteration method as a mechanism for resolving non-linear equations, with particular focus on how this can be applied to Burger's equations.

Burger's equations specifically have been applied to a variety of scenarios, which consider approximation theory of flow through viscous fluids and in relation to models of turbulence, and this is evident in a range of research that has utilized this approach in equation resolution. Methods for resolving these equations using in practical examples have proved problematic and overcomplicated historically, and alternative methods have been explored by various mathematicians over time. As a result, many mathematicians have worked on developing alternative methods for resolving these equations in a manner that can be applied to various nonlinear and linear equations, particularly focusing on how this can be achieved in a more efficient and simplified manner than previous methods. This paper will provide an overview of the main method for utilizing the variation iteration method and how this can be applied to the Burger equations, before providing some examples of these applications. The paper will conclude by looking at how successful these applications have been and the potential use of this approach.

II. ANALYSIS OF THE METHOD

Burger's equations are most commonly used to describe mathematical models of turbulence and the approximation theories relating to the flow of shock waves travelling through a viscous fluid (Biazar and Aminikah, 2007; Cole, 1951). Currently, many numerical methods that look at finite differences and characteristics require a large amount of computation to resolve, and this affects accuracy in models of turbulence and measurements of shock wave flow because of the issues associated with round-off errors in particular (Biazar and Aminikah, 2007). The variational iteration method for solving nonlinear equations such as Burger's equations is fundamentally based on the Langrange multiplier and, as a result, has been credited as an approach that offers greater simplicity and easy execution (Mistry and Pradhan, 2012). This is supported by the work of Ghorbani and Saberri-Nadjafi (2007) and He (2007), who suggest that He's polynomials are more compatible with Adomian's polynomials and easier to calculate. This suggests that the variational iteration method is more user-friendly and could potentially be applied more frequently within many fields, particularly in research related to physics.

III. APPLICATIONS

The variational iteration method has been introduced by researchers as a mechanism for resolving these equations without the difficulties, which are associated with calculations associated with Adomian polynomials (as discussed in Mistry and Pradhan, 2012). In the main, these polynomials are incompatible with the physical nature of problems (Mohyud-Din et al) and, as a result, alternative methods have been discussed and explored by many mathematicians over time. In addition, for many nonlinear equations, often the Lagrange multiplier can provide difficult to identify, and this can be overcome with the variational iteration method by applying restricted variations to many of the nonlinear terms (as demonstrated in the examples of He, 2007). The variation iteration method was first fully developed by He as a method for solving both linear and non-linear differential equations (He, 1999; 2007). Through a range of testing and subsequent application of this method, it has been found to be both efficient and reliable against a wide variety of scientific and engineering applications (Sadighi and Ganji, 2007; He, 2007; Taghizadeh et al., 2011).

The variational iteration method has been applied to Burger's equations in both one- and two-dimensional forms. Abdou and Soliman (2005) illustrate the fundamental concepts of (VIM) through the following example of a differential equation modified from the general Lanrgange multiplier method (Burger, 1948), using L as a linear operator, N as a nonlinear operator, and g(x) as an inhomogeneous term or analytical function. This is the most common form in contemporary mathematical research of the variational iteration method when used in conjunction with Burger's equations. For example, this is a similar framework suggested by Biazar and Aminkhah (2009) when outlining the basics of this particular mathematical approach. A summary of the main modified multiplier method is provided below:

$$Lu + Nu = g(x)$$

According to (VIM), using in Abdou and Soliman's (2005) work and the methods that have been applied within their research a correct functional can be constructed from this approach in the format detailed below:

$$u_{n+1}(x) = u_n(x) + \int_0^x \lambda\big(Lu_n(\tau) + N\widetilde{u_n}(\tau) - g(\tau)\big)d\tau$$

Where λ is a general Lagrangian multiplier, which 'can be identified optimally via the variational theory' (Abdou and Soliman, 2005, p. 247). Within this functional n denotes the h-order approximation, and$\widetilde{u_n}$ is the restricted variation – where$\delta\widetilde{u_n} = 0$ (Abdou and Soliman, 2005, p. 247). Utilising this approach helps to overcome some of the limitations of alternative methods for solving Burger's equations and can be employed to provide more practical solutions and more accurate data in a wide variety of problems in physics in particular (Biazar and Amiknkhah, 2009). This is principally because the variational iteration method can provide mathematicians with a powerful tool for solving a large number of problems in a manner that provides the exact solution or at the very least a closed approximation solution to a given problem (Biaxar and Aminkhah, 2009).

Biazar and Aminkhah (2009) consider how this approach can be used in relation to one- and two-dimensional Burger's equations, outlining both the initial condition and boundary conditions to provide a correctional function and ultimately deriving an exact solution. Although the results of their application are almost exact when compared to the Adomina decomposition methods (Gorguis, 2006; Kaya and Yokus, 2002), the main advantage of adopting (VIM) in their approach is that the technique can help to solve the main problem without having to carry out any discretization of any of the variables (Biazar and Amikhah, 2009). Misty and Pradhan (2012) have also successfully applied (VIM) to the findings of Burger's equations in (1+1), (1+2), and (1+3) dimensional. In their work, the findings suggested that the solutions that can be obtained by (VIM) have an 'infinite power series for appropriate initial conditions which, in turn, can be expressed in a closed form, the exact solution' (Mistry and Pradhan, 2012, p. 246).

Example 1

Consider that the one-dimensional Burger's equation

$$u_x + uu_x - u_{xx} = 0 \qquad (1)$$

with initial condition

$$u(x,0) = x \qquad (2)$$

To solve Eq. (1) by means of the variational iteration method, we construct a correction functional
which reads

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t \lambda(\frac{\partial u_n}{\partial t} + u_n \frac{\partial u_n}{\partial x} - \frac{\partial^2 u_n}{\partial x^2})d\tau \quad (3)$$

where

$$\lambda'(\tau) = 0 \quad (4)$$

$$1 + \lambda(\tau)_{\tau=t} (5)$$

The Lagrange multiplier can therefore be simply identified as $\lambda = -1$, and substituting into Eq. (3) formula can be obtained:

$$u_{n+1}(x,t) = u_n(x,t) - \int_0^t (\frac{\partial u_n}{\partial t} + u_n \frac{\partial u_n}{\partial x} - \frac{\partial^2 u_n}{\partial x^2})d\tau \quad (6)$$

And from Eq.(2) and Eq.(6) we obtain the following successive approximations:

$$u_1 = x - tx$$

$$u_2 = x - (tx - t^2x + \frac{1}{3}t^3x)$$

$$u_3 = x + tx - t^2x + \frac{1}{3}t^3x - (tx - t^2x + t^3x - \frac{2}{3}t^4x + \frac{1}{3}t^5x - \frac{1}{9}t^6x + \frac{1}{63}t^7x$$

$$\vdots$$

$$u(x,t) = u_1 + u_2 + u_3 + \cdots \quad (7)$$

The solution of $u(x,t)$ in a closed from is

$$u(x,t) = \frac{x}{1+t} \quad (8)$$

IV. CONCLUSIONS

Research continues to review the application of (VIM) to non-linear equations. However, with reference to Burger's equations specifically, there is evidence that has supported the efficient solutions, which are obtained when this approach is applied. It is considered that in many situations, (VIM) offers benefits over other approaches in terms of its efficiency and its ability to overcome the complexities, which are associated with many of the calculations associated with Adomian polynomials. In addition to the successful resolution of Burger's equations demonstrated and discussed in this paper, there is also further support for using this approach to solve other non-linear equations. Applications of this method have been successfully resolved across a number of different nonlinear equations. However, with Burger's equations in particular, there has been practical success in obtaining exact results. This has shown how the variational iteration method can solve more effectively, more easily, and ultimately more accurately a large class of nonlinear problems.

REFERENCES

[1]. Abdou, M.A. and Soliman, A.A. (2005) Variation iteration method for solving Burger's and coupled Burger's equations, Journal of Computational and Applied Mathematics, Vol.181, pp. 245–251.
[2]. Alquran, M.T. (2012) Solitons and Periodic Solutions to Nonlinear Partial Differential Equations by the Sine-Cosine Method,Applied Mathematics and Information Sciences,Vol.6(1), pp. 85–88.
[3]. Biazar, J. and Aminikhah, H. (2008) Exact and numerical solutions for non-linear Burger's equation by VIM,Mathematical and Computer Modelling, Vol.49, pp. 1394–1400.
[4]. Cole, J.D. (1951) On a quasilinear parabolic equations occurring in aerodynamics,Quarterly Applied Mathematics,Vol.9, pp. 225–236.
[5]. Ghorbani, A. and Saberi-Nadjfi, S. (2007) He's homotopy perturbation method for calculating adomian polynomials,International Journal of Nonlinear Sciences and Numerical Simulation,Vol.8 (2), pp. 229–232.
[6]. Gorguis, A. (2006) A comparison between Cole–Hopf transformation and the decomposition method for solving Burgers' equations,Applied Mathematics and Computing,Vol.173, pp. 126–136.
[7]. He, J.-H. (2007) Variational iteration method - Some recent results and new interpretations,Journal of Computational and Applied Mathematics,Vol.207, pp. 3–17.

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