

Birkás prime numbers

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ABSTRACT

After defining, Birkás prime numbers will be presented from 227 to 7777272777. How many Birkás prime numbers are there in the interval $(10^{p-1}, 10^p)$ (where p is a prime number)? On the one hand, it has been counted by computer among the prime numbers with up to 23-digits. On the other hand, the function (1) gives the approximate number of Birkás prime numbers in the interval $(10^{p-1}, 10^p)$. Near-proof reasoning has emerged from the conformity of Mills' prime numbers with Birkás prime numbers. The set of Birkás prime numbers is probably infinite.

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I. INTRODUCTION

The sets of special prime numbers within the set of prime numbers are well-known. For instance, the Erdős-primes (the sum of the digits is prime) [8], Fibonacci-primes ($F_0=0, F_1=1, F_n=F_{n-1}+F_{n-2}$), Gauss-primes (in the form $4n+3$), Leyland-primes (in the form x^y+y^x , where $1 \leq x \leq y$), Pell-primes ($P_0=0, P_1=1, P_n=2P_{n-1}+P_{n-2}$), Bölcsföldi-Birkás-Ferenczi primes (all digits are prime and the number of digits is prime), etc. Question: Which further sets of special prime numbers are there within the set of prime numbers? We have found a further set of special prime numbers within the set of prime numbers. It is the set of Birkás prime numbers.

II. BIRKÁS PRIME NUMBERS [3], [9], [10], [11].

Definition: a positive integer number is a Birkás prime number, if

a/ the positive integer number is prime, b/ all digits are 2 or 7, c/ the number of digits is prime, d/ the sum of digits is prime.

The set of prime numbers meeting the conditions a/ and c/ is also well-known: it is the set of prime-long prime numbers [3], [9]. Positive integer numbers meeting all the four conditions (a/, b/, c/, d/) at the same time are Birkás prime numbers (Fig.1, Fig.2).

Birkás prime number p has the following sum form:

$k(p)$
 $k(p)$

$$p = \sum_{j=0} e_j(p) \cdot 10^j \quad \text{where } e_j(p) \in \{2, 7\} \text{ and } k(p)+1 \text{ is prime and } e_0(p) \in \{7\} \text{ and } \sum e_j(p) \text{ is prime.}$$

$j=0$

$j=0$

The Birkás prime numbers are as follows (the last digit can only be 7):

227,
 2227727,
 22222272727, 22222777277, 22227222727, 22227227777, 22227277727, 22227722227, 22227772727,
 22272272227, 22272277277, 22272727727, 22277227727, 22277772227, 22722222277, 2272222727,
 22722277727, 22727222227, 22727277227, 22772227727, 22772722277, 27222222277, 27222227777,
 27222722227, 27222727277, 27222777227, 27227222777, 27272222227, 27272227727, 27272722727,
 27277777777, 27722227277, 27727272227, 27727722227, 27772227227, 27772777777, 27777777277,
 72222722227, 72222722777, 72222772277, 72227222777, 72227722727, 72227772227, 72272222227,
 72272227277, 72272277227, 72277222727, 72722222777, 72722727227, 72727222727, 72772222227,
 72777777727, 77222227727, 77272272227, 77277777777, 77772727777, etc.

H(p) is the factual frequency of Birkás prime numbers in the interval $(10^{p-1}, 10^p)$.
 H(2)=0, H(3)=1, H(5)=0, H(7)=4, H(11)=54, H(13)=207, H(17)=2485, H(19)=7950, H(23)=59043, etc.
 S(p) function gives the number of Birkás prime numbers in the interval $(10^{p-1}, 10^p)$. We think that

$$S(p) = 1,192 \times 1,6^p, \quad \text{where } p \text{ is prime.} \quad (1)$$

The factual number of Birkás primes and the number of Birkás primes calculated according to function (1) are as follows:

Number of digits p	The factual number of Birkás primes in the interval $(10^{p-1}, 10^p)$ H(p)	The number of Birkás primes according to function $S(p) = 1,192 \times 1,6^p$	$H(p)/S(p)$
2	0	3,0515	0
3	1	4,8824	0,21
5	0	12,4990	0
7	4	31,9975	0,13
11	54	209,6986	0,26
13	207	536,8291	0,26
17	2485	3518,1630	0,71
19	7950	9006,4974	0,88
23	59043	59024,9811	1,00

Fig.1

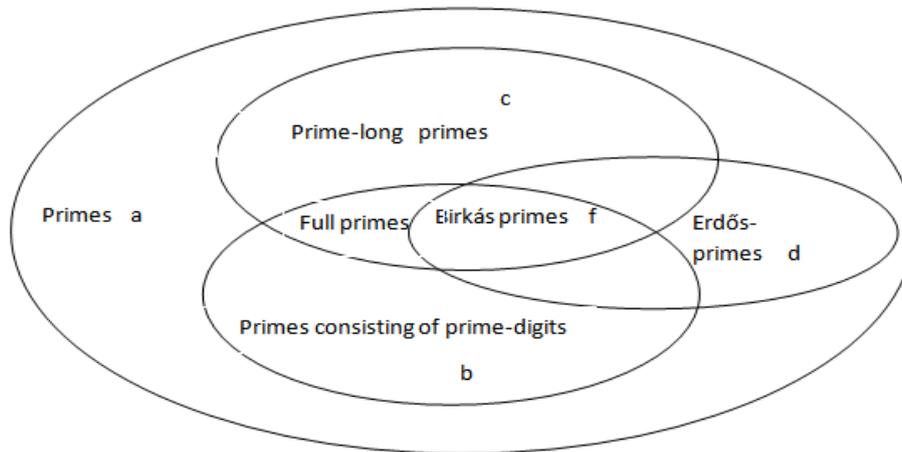
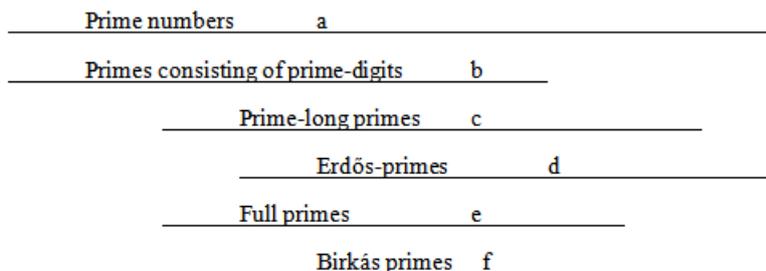


Fig.2



III. NUMBER OF THE ELEMENTS OF THE SET OF BIRKÁS PRIME NUMBERS [3], [9],[10], [11].

Let's take the set of Mills' prime numbers!

Definition: The number $m = [M \text{ ad } 3^n]$ is a prime number, where $M = 1,306377883863080690468614492602$ is the Mills' constant, and $n = 1, 2, 3, \dots$ is an arbitrary positive integer number. It is already known that the number of the elements of the set of Mills' prime numbers is infinite. The Mills' prime numbers are the following: $m = 2, 11, 1361, 2521008887, \dots$

The connection $n \rightarrow m$ is the following: $1 \rightarrow 2, 2 \rightarrow 11, 3 \rightarrow 1361, 4 \rightarrow 2521008887, \dots$. The Mills' prime number $m = [M \text{ ad } 3^n]$ corresponds with the interval $(10^{m-1}, 10^m)$ and vice versa. For instance: $2 \rightarrow (10, 10^2)$, $11 \rightarrow (10^{10}, 10^{11})$, $1361 \rightarrow (10^{1360}, 10^{1361})$, etc. and vice versa. The number of the elements of the set of Mills' prime numbers is infinite. As a consequence, the number of the intervals $(10^{m-1}, 10^m)$ that contain at least one Mills' prime number is infinite. The number of Birkás primes in the interval $(10^{m-1}, 10^m)$ is $S(m) = 1,192 \times 1,6^m$. The number of Birkás prime numbers is probably infinite: $\lim_{p \rightarrow \infty} H(p) = \infty$ is probably where p is prime.

IV. CONCLUSION

Countless different sets of special prime numbers have been known. We have found the following set of special prime numbers within the set of prime numbers. There may be further sets of special prime numbers that we do not know yet. Finding them will be task of researchers of the future.

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