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# An Interval-Valued Method For Solving Fuzzy Multi-Level Multi-Objective Integer Fractional Programming Problem

Omar M. Saad*, Mervat M. Elshafei**, Marwa M. Sleem***
**(Department of Mathematics, Faculty of Science, Helwan University, P.O. Box11795, Cairo, Egypt,*
***(Department of Mathematics, Faculty of Science, Helwan University,P.O. Box11795, Cairo, Egypt,*
****(Department of Basic Sciences, Thebes Higher Institute of Engineering, Maadi, P.O.Box11434, Cairo, Egypt,*
*Corresponding author ; Omar M. Saad*

**ABSTRACT**
This paper presents an interval-valued method to solve fuzzy multi-level multi-objective integer fractional programming problem (**F-MMIFP**). The problem under consideration involves fuzzy parameters in the objective functions, in the left-hand side and in the right-hand side of the constraints. The suggested method depends on the concept of the α-level set of the fuzzy numbers. The **α-MMIFP** problem can be converted to interval-valued multi-level multi-objective integer fractional programming problem (**IV-MMIFP**), which can be transformed to interval-valued multilevel single objective integer fractional programming problem (**IV-MSIFP**) by using the nonnegative weighting sum approach. Then, this resulting problem can be rewritten in the form of real-valued multi-level single objective integer fractional programming problem (**RV-MSIFP**) using the interval-valued optimization technique together with the convex linear combination of the first and least point of the intervals in the constraints. Finally, a non-dominated solution of the problem of concern (**F-MMIFP**) is obtained. In addition, an algorithm is described in finite steps to solve problem (**F-MMIFP**). An illustrative numerical example is included to demonstrate the proposed solution algorithm.
**Keywords**: Fractional Programming, Fuzzy Programming, Integer Programming, Interval Programming, Multi-level Programming, Multi-objective Programming, Non-linear Programming.

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## I. INTRODUCTION

Interval-valued fuzzy programming is the modelling aspects of optimization problems in which model parameters are defined in the form of bounded intervals [8,9]. Charnes et al suggested a primal algorithm for interval linear-programming problems. (See [7]).

Multi-level multi-objective programming problems have more than one decision maker and more than goal, where the hierarchical system be composed of a n-level and n-objective decision maker [12, 28, 29, 30, 31].

Non-linear fractional integer (**NFI**) programming is one of the most popular models used in decision-making and in optimization problems. The **NFI** programming problem aims at minimize (maximize) non-linear fractional objective function subject to a set of linear constraints [1, 9, 14, 15, 19, 26, 27, 34].

M.S. Osman et al in [22, 23], presented multi-level multi-objective quadratic fractional programming problems with fuzzy parameters: A **FGP** approach for multi-level multi-objective quadratic fractional programming problem with fuzzy parameters was proposed.

An algorithm to solve a bi-level multi-objective fractional integer programming problem involving fuzzy numbers in the right-hand side of the constraints was presented in E. A. Youness et al [35].The suggested algorithm combine the method of Taylor series together with the Kuhn Tucker conditions to solve -level multi-objective fractional integer programming problem(**FBLMOFIPP**)then Gomory's cutsareadded till the integer solution is obtained.

M.S. Osman et al. in [24] introduced an interactive approach for solving multi-level multi-objective fractional programming problem (**ML-MOFP**) with fuzzy parameters is introduced. The proposed interactive approach makes an extended work of Shi and Xia (1997). In the first phase, the numerical crisp model of the problem **(ML-MOFP)** has been developed at a confidence level without changing the fuzzy gist of the problem. Then, the linear model for the problem (**ML-MOFP)** is formulated. In the second phase, the interactive approach simplifies the linear multi-level multi-objective model by converting it into separate multi-objective programming problems. Also, each separate multi-objective programming problem of the linear model is solved by the ε-constraint method and the concept of satisfactoriness.

In this paper, an interval-valued method is proposed to solve the multi-level multi-objective integer fractional programming problem with fuzzy parameters in the objective functions, in both the right hand side and in the left hand side of constraints of problem (**F-MMIFP**). This paper is organized as follows: we start in Section II by stating the problem formulation of the mathematical model with the solution concept. In Section III, some interval analysis is provided. Then, interval-valued multi-level multi-objective integer fractional programming problem is given in Section IV.A solution algorithm for solving problem (**F-MMIFP**) is suggested in Section V. An illustrative numerical example clarifies the theory and the solution algorithm is suggested in Section VI. Finally, the paper is concluded in Section IV where some points of further research in future are reported.

## II. PROBLEM FORMULATION AND THE SOLUTION CONCEPT

Consider the fuzzy multi-level multi-objective integer fractional programming problem (**F-MMIFP**) of the following form:

(**F-MMIFP):**
**[1st-level]**
$\underset{x_1}{Max}\, F_1(x,\tilde{\theta}) = \underset{x_1}{Max}\big(f_{11}(x,\tilde{\theta}), f_{12}(x,\tilde{\theta}), \dots, f_{1k_1}(x,\tilde{\theta})\big),$
*where* $x_2, x_3, \dots, x_n$ *solves*
**[2nd-level]**
$\underset{x_2}{Max}\, F_2(x,\tilde{\theta}) = \underset{x_2}{Max}\big(f_{21}(x,\tilde{\theta}), f_{22}(x,\tilde{\theta}), \dots, f_{2k_2}(x,\tilde{\theta})\big),$
.
. (1-a)
.
*where* $x_n$ *solves*
**[tth-level]**
$\underset{x_n}{Max}\, F_t(x,\tilde{\theta}) = \underset{x_n}{Max}\big(f_{t1}(x,\tilde{\theta}), f_{t2}(x,\tilde{\theta}), \dots, f_{tk_t}(x,\tilde{\theta})\big),$
**Subject to**
$x \in X(\tilde{a}_{ij}, \tilde{b}_i) = \{x \in \mathcal{R}^n \,|\, \sum_{j=1}^{n} \tilde{a}_{ij} x_j \le \tilde{b}_i, x_j \ge 0, i = 1,2,\dots m, j = 1,2,\dots,n \text{ and } x_j \text{ is integer}\},$ (1-b)

where each objective function has the form:

$$f_{rs}(x,\tilde{\theta}) = \frac{N_{rs}(x,\tilde{\theta})}{D_{rs}(x,\tilde{\theta})} = \frac{(c^{rs} + h^{rs}\tilde{\theta})x + \alpha^{rs}}{d^{rs}x + \beta^{rs}},$$
$r = 1,2,\dots,t \qquad s = 1,2,\dots,k_t.$

In addition, $x = (x_1, x_2, \dots, x_n)^T$ is an integer n-dimension column vector of decision variables, $\tilde{\theta}$ is a single-fuzzy number involved in the objective functions $f_{rs}(x,\tilde{\theta})$ and $D_{rs}(x,\tilde{\theta}) > 0$ for all $x \in X(\tilde{a}_{ij}, \tilde{b}_i), i = 1,2,\dots,m, j = 1,2,\dots n.$

Moreover, $\tilde{a}_{ij}, i = 1,2,\dots,m, j = 1,2,\dots,n$ are fuzzy numbers in the left-hand side of the constraints of problem **F-MMIFP** (1-a)-(1-b), $\tilde{b}_i, i = 1,2,\dots,m$ are fuzzy numbers in the right-hand side of the constraints ofproblem **F-MMIFP** (1-a)-(1-b).

**Definition1.** [25] (Membership Function)
It is appropriate to recall that areal fuzzy numbers $\tilde{\vartheta}$ is a continuous fuzzy subset of the real line whose membership functions $\mu_{\tilde{\vartheta}}(\vartheta)$ is defined by:

$$\mu_{\tilde{\vartheta}}(\vartheta) = \begin{cases} 0 & \vartheta \le a. \\ 1 - \left(\frac{\vartheta - b}{a - b}\right)^2 & a \le \vartheta \le b. \\ 1 & b \le \theta_i \le c. \\ 1 - \left(\frac{\vartheta - c}{d - c}\right)^2 & c \le \theta_i \le d. \\ 0 & \text{otherwise.} \end{cases}$$

**Figure.1** illustrates the graph of a possible shape of a membership function of a fuzzy number $\tilde{\vartheta}$.

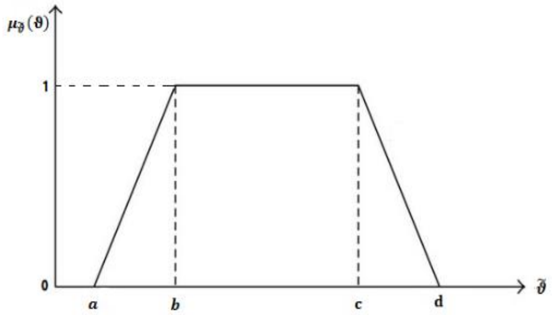


**Figure1.** Trapezoidal fuzzy number.

In order to define problem **(F-MMIFP)**(1-a)-(1-b) mathematically, first, we introduce the concept of α-level set or α-cut [20,21,22,24,25] of the fuzzy parameters $\tilde{\theta}, \tilde{a}_{ij}$ and $\tilde{b}_i$ in the following definition.

**Definition2.** [25] (α-level set)
The α-level set (α-cut) of the fuzzy parameters $\tilde{\theta}, \tilde{a}_{ij}$ and $\tilde{b}_i$ is defined as the ordinary set $L_\alpha(\tilde{\theta}, \tilde{a}_{ij}, \tilde{b}_i)$ for which the degree of their membership function is greater than or equal to the levelα:

$L_\alpha(\tilde{\theta}, \tilde{a}_{ij}, \tilde{b}_i) = \{(\theta, a_{ij}, b_i) \in \mathcal{R}^{2m+n+1} \,|\, \mu_{\tilde{\theta}}(\theta) \ge \alpha, \mu_{\tilde{a}_{ij}}(a_{ij}) \ge \alpha$ and $\mu_{\tilde{b}_i}(b_i) \ge \alpha, i = 1,2,\dots,m,\ j = 1,2,\dots,n\}.$ (2)

It is clear that the level sets have the following property $\alpha_1 \le \alpha_2$ if and only if

$$L_{\alpha_1}(\tilde{\theta}, \tilde{a}_{ij}, \tilde{b}_i) \supset L_{\alpha_2}(\tilde{\theta}, \tilde{a}_{ij}, \tilde{b}_i). \quad (3)$$

We can see from definition1, the α-level set $L_\alpha(\tilde{\theta}, \tilde{a}_{ij}, \tilde{b}_i)$ is the set of the closed intervals depending on the level α.

For a certain degree α, problem (**F-MMIFP)** (1-a)-(1-b) can be understood as the set of the following non-fuzzyα-multi-level multi-objective integer fractional programming problem (**α-MMIFP**).

Problem (**α-MMIFP)** depending on the parameters $(\theta, a_{ij}, b_i) \in L_\alpha(\tilde{\theta}, \tilde{a}_{ij}, \tilde{b}_i)$as:

**(α-MMIFP):**
**[1$^{st}$-level]**

$\underset{x_1}{Max}\, F_1(x,\theta) =$
$\underset{x_1}{Max}\big(f_{11}(x,\theta), f_{12}(x,\theta), \dots, f_{1k_1}(x,\theta)\big),$

*where $x_2, x_3, \dots, x_n$ solves*
**[2$^{nd}$-level]**

$\underset{x_2}{Max}\, F_2(x,\theta) =$
$\underset{x_2}{Max}\big(f_{21}(x,\theta), f_{22}(x,\theta), \dots, f_{2k_2}(x,\theta)\big),$

.
. (4-a)
.

*where $x_n$solves*
**[t$^{th}$-level]**

$\underset{x_n}{Max}\, F_t(x,\theta) =$
$\underset{x_n}{Max}\big(f_{t1}(x,\theta), f_{t2}(x,\theta), \dots, f_{tk_t}(x,\theta)\big),$

**Subject to**

$x \in X(a_{ij}, b_i) =$
$\{x \in \mathcal{R}^n | \sum_{j=1}^{n} a_{ij}x_j \le b_i, x_j \ge 0, i = 1,2, \dots m, j = 1,2, \dots, n \text{ and} x_j \text{ is integer}\}.$ (4-b)

$(\theta, a_{ij}, b_i) \in L_\alpha(\tilde{\theta}, \tilde{a}_{ij}, \tilde{b}_i).$ (4-c)

On the basis of the α-level set of the fuzzy numbers, we introduce the concept of α-Pareto-optimal solutions to the problem **(α-MMIFP)** (4-a)-(4-c).

**Definition3.** (α-Pareto Optimal Integer Solution)
$x^* \in X(a_{ij}^*, b_i^*)$ is said to be an $\alpha$-Pareto optimal integer solution to problem (**$\alpha$-MMIFP)**(4-a)-(4-c), if and only if there does not exist another $x \in X(a_{ij}, b_i), (\theta, a_{ij}, b_i) \in L_\alpha(\tilde{\theta}, \tilde{a}_{ij}, \tilde{b}_i)$ such that $f_{rs}(x,\theta) \ge f_{rs}(x^*, \theta^*), r = 1,2, \dots, t,\ s = 1,2, \dots, k_t$with strict inequality holding for at least one $rs$, where the corresponding values of parameters $(\theta^*, a_{ij}^*, b_i^*)$ are called $\alpha$-level optimal parameters.

## III. INTERVAL ANALYSIS

We denote by $I$ the set of all closed and bounded intervals in $\mathcal{R}$. If $A$ is a closed interval, we also adopt the notation $A = [a^L, a^U]$, where $a^L$ and $a^U$ mean the lower and upper bounds of $A$, respectively. Let $A = [a^L, a^U]$ and $B = [b^L, b^U]$ be in $I$, Thenwe have the following operations on $I$[8,9,17]:

**1.** $A + B = \{a + b | a\epsilon A \text{ and } b\epsilon B\}$
$= [a^L + b^L, a^U + b^U] \in I.$
**2.** $kA = \{ka | a\epsilon A\} = [ka^L, ka^U]; if\ k \ge 0$, where k is a real number.
**3.** $kA = \{ka | a\epsilon A\} = [ka^U, ka^L]; if\ k < 0$, where k is a real number.
**4.** $A - B = A + (-B) = [a^L - b^U, a^U - b^L]\epsilon I.$

**Definition4.** [8, 9,17] (Interval-valued function)
A function $F: \mathcal{R}^n \to I$ defined on the Euclidean space $\mathcal{R}^n$ is called an interval-valued function (because $F(x)$ for each $x\epsilon\mathcal{R}^n$ is a closed interval in$\mathcal{R}$). Similar to interval notation, we denote the interval-valued function $F(x)$ with $F(x) = [F^L(x), F^U(x)]$where for every $x\epsilon\mathcal{R}^n, F^L(x), F^U(x)$ are real valued functions and $F^L(x) \le F^U(x)$.

## IV. INTERVAL-VALUAD MULTI-LEVEL MULTI- OBJECTIVE INTEGER FRACTIONAL PROGRAMMING PROBLEM

Form the definition of fuzzy number, it is significant to note that the $\alpha$-level set of fuzzy number can be represented as the closed interval which depends on interval-value of $\alpha$. So, problem (**$\alpha$ -MMIFP)** (4-a)-(4-c) converted to interval-valued multi-level multi-objective integer fractional problem (**IV-MMIFP)** as:

(**IV-MMIFP):**
**[1$^{st}$-level]**

$$\underset{x_1}{Max}\, F_1(x,\theta) = \underset{x_1}{Max}\left(\begin{array}{c} \frac{(c^{11}+h^{11}[\theta^L,\theta^U])x+\alpha^{11}}{d^{11}x+\beta^{11}}, \\ \frac{(c^{12}+h^{12}[\theta^L,\theta^U])x+\alpha^{12}}{d^{12}x+\beta^{12}}, \dots, \\ \frac{(c^{1k_1}+h^{1k_1}[\theta^L,\theta^U])x+\alpha^{1k_1}}{d^{1k_1}x+\beta^{1k_1}} \end{array}\right),$$

where $x_2, x_3, \dots, x_n$ solves
**[2$^{nd}$-level]**

$$\underset{x_2}{Max}\, F_2(x,\theta) = \underset{x_2}{Max}\left(\begin{array}{c} \frac{(c^{21}+h^{21}[\theta^L,\theta^U])x+\alpha^{21}}{d^{21}x+\beta^{21}}, \\ \frac{(c^{22}+h^{22}[\theta^L,\theta^U])x+\alpha^{22}}{d^{22}x+\beta^{22}}, \dots, \\ \frac{(c^{2k_2}+h^{2k_2}[\theta^L,\theta^U])x+\alpha^{2k_2}}{d^{2k_2}x+\beta^{2k_2}} \end{array}\right),$$

.
. (5-a)
.

where $x_n$solves
**[$t^{th}$-level]**

$$\underset{x_n}{Max}\, F_t(x,\theta) = \underset{x_n}{Max}\begin{pmatrix} \frac{(c^{t1}+h^{t1}[\theta^L,\theta^U])x+\alpha^{t1}}{d^{t1}x+\beta^{t1}}, \\ \frac{(c^{t2}+h^{t2}[\theta^L,\theta^U])x+\alpha^{t2}}{d^{t2}x+\beta^{t2}}, \dots, \\ \frac{(c^{tk_t}+h^{tk_t}[\theta^L,\theta^U])x+\alpha^{tk_t}}{d^{tk_t}x+\beta^{tk_t}} \end{pmatrix},$$

**Subject to**

$$x \in X(a_{ij}, b_i) = \{x \in \mathcal{R}^n \mid \textstyle\sum_{j=1}^{n}[a_{ij}^L, a_{ij}^U]x_j \le [b_i^L, b_i^U],\ i = 1,2,\dots,m \text{ and } x_j \text{ is integer}\}, \quad \text{(5-b)}$$

Where the functions $F_1, F_2, \dots, F_t$ are called interval-valued functions, i.e. this functions is closed interval in

$\mathcal{R}$ .Also, $\theta = [\theta^L, \theta^U]$ satisfy $\theta^L \le \theta^U$, $a_{ij} = [a_{ij}^L, a_{ij}^U]$ satisfy $a_{ij}^L \le a_{ij}^U, i = 1,2,\dots,m, j = 1,2,\dots,n$ and $b_i = [b_i^L, b_i^U]$ satisfy $b_i^L \le b_i^U$ for every $\theta, a_{ij}, b_i$.

**Definition5.** [10]
To interpret the meaning of optimization of interval-valued functions, we introduce a partial ordering $\preccurlyeq$ over $I$ . Let $A = [a^L, a^U], B = [b^L, b^U]$ be two closed, bounded, real intervals $(A, B \in I)$, then we say that $A \preccurlyeq B$, if and only if $a^L \preccurlyeq b^L, a^U \preccurlyeq b^U$.

**Definition6.** [10]
$x^* \in X(a_{ij}, b_i)$ is a non-dominated solution of problem **(IV-MMIFP)** if these exist no feasible solution $x$ such that $f_r(x) \preccurlyeq f_r(x^*)$, $r = 1,2,\dots,t$, so we say that $f_r(x^*)$ is the non-dominated objective value.

Problem **(IV-MMIFP)** can be treated using the nonnegative weighed sum approach [10,11,13,18,22] and will be converted to the following problem with a single-objective functions as in problem **(IV-MSIFP)**.

**(IV-MSIFP):**
**[$1^{st}$-level]**

$$\underset{x_1}{Max}\, F_1(x,\theta) = \underset{x_1}{Max}\begin{pmatrix} \omega_{11}\frac{(c^{11}+h^{11}[\theta^L,\theta^U])x+\alpha^{11}}{d^{11}x+\beta^{11}} \\ +\omega_{12}\frac{(c^{12}+h^{12}[\theta^L,\theta^U])x+\alpha^{12}}{d^{12}x+\beta^{12}} + \cdots \\ +\omega_{1k_1}\frac{(c^{1k_1}+h^{1k_1}[\theta^L,\theta^U])x+\alpha^{1k_1}}{d^{1k_1}x+\beta^{1k_1}} \end{pmatrix},$$

where $x_2, x_3, \dots, x_n$ solves
**[$2^{nd}$-level]**

$$\underset{x_2}{Max}\, F_2(x,\theta) = \underset{x_2}{Max}\begin{pmatrix} \omega_{21}\frac{(c^{21}+h^{21}[\theta^L,\theta^U])x+\alpha^{21}}{d^{21}x+\beta^{21}} \\ +\omega_{22}\frac{(c^{22}+h^{22}[\theta^L,\theta^U])x+\alpha^{22}}{d^{22}x+\beta^{22}} + \cdots \\ +\omega_{2k_2}\frac{(c^{2k_2}+h^{2k_2}[\theta^L,\theta^U])x+\alpha^{2k_2}}{d^{2k_2}x+\beta^{2k_2}} \end{pmatrix},$$

.
.
. (6-a)

where $x_n$ solves
**[$t^{th}$-level]**

$$\underset{x_n}{Max}\, F_t(x,\theta) = \underset{x_n}{Max}\begin{pmatrix} \omega_{t1}\frac{(c^{t1}+h^{t1}[\theta^L,\theta^U])x+\alpha^{t1}}{d^{t1}x+\beta^{t1}} \\ +\omega_{t2}\frac{(c^{t2}+h^{t2}[\theta^L,\theta^U])x+\alpha^{t2}}{d^{t2}x+\beta^{t2}} + \cdots \\ +\omega_{tk_t}\frac{(c^{tk_t}+h^{tk_t}[\theta^L,\theta^U])x+\alpha^{tk_t}}{d^{tk_t}x+\beta^{tk_t}} \end{pmatrix},$$

**Subject to**

$$x \in X(a_{ij}, b_i) = \{x \in \mathcal{R}^n \mid \textstyle\sum_{j=1}^{n}[a_{ij}^L, a_{ij}^U]x_j \le [b_i^L, b_i^U], i = 1,2,\dots,m \text{ and } x_j \text{ is integer}\}, \quad \text{(6-b)}$$

where;

$$\omega_{rs} \in [0,1], \quad \textstyle\sum_{r=1}^{t}\omega_{rs} = 1, \quad s = 1,2,\dots,k_t. \quad \text{(6-c)}$$

Problem **(IV-MSIFP)** can be converted to real-valued multi-level single-objective integer fractional programming problem (**RV-MSIFP**) by applying the concept of problem interval-valued optimization problem on the objective function(see[8]) along with the of convex linear combination on the constrains (see[2,3,4,5,6])as follows:

**(RV-MSIFP):**
**[$1^{st}$-level]**

$$\underset{x_1}{Max}\, F_1(x,\theta) = \underset{x_1}{Max}\begin{pmatrix} \frac{\omega_{11}(c^{11}+h^{11}\theta^L+h^{11}\theta^U)x+\omega_{11}\alpha^{11}}{d^{11}x+\beta^{11}} \\ +\frac{\omega_{12}(c^{12}+h^{12}\theta^L+h^{12}\theta^U)x+\omega_{12}\alpha^{12}}{d^{12}x+\beta^{12}} + \cdots \\ +\frac{\omega_{1k_1}(c^{1k_1}+h^{1k_1}\theta^L+h^{1k_1}\theta^U)x+\omega_{1k_1}\alpha^{1k_1}}{d^{1k_1}x+\beta^{1k_1}} \end{pmatrix},$$

where $x_2, x_3, \dots, x_n$ solves
**[$2^{nd}$-level]**

$$\underset{x_2}{Max}\, F_2(x,\theta) = \underset{x_2}{Max}\begin{pmatrix} \frac{\omega_{21}(c^{21}+h^{21}\theta^L+h^{21}\theta^U)x+\omega_{21}\alpha^{21}}{d^{21}x+\beta^{21}} \\ +\frac{\omega_{22}(c^{22}+h^{22}\theta^L+h^{22}\theta^U)x+\omega_{22}\alpha^{22}}{d^{22}x+\beta^{22}} + \cdots \\ +\frac{\omega_{2k_2}(c^{2k_2}+h^{2k_2}\theta^L+h^{2k_2}\theta^U)x+\omega_{2k_2}\alpha^{2k_2}}{d^{2k_2}x+\beta^{2k_2}} \end{pmatrix},$$

.
.
. (7-a)

where $x_n$ solves

**[$t^{th}$-level]**

$$\underset{x_n}{Max}\, F_t(x,\theta) = \underset{x_n}{Max}\begin{pmatrix} \frac{\omega_{t1}(c^{t1}+h^{t1}\theta^L+h^{t1}\theta^U)x+\omega_{t1}\alpha^{t1}}{d^{t1}x+\beta^{t1}} \\ +\frac{\omega_{t2}(c^{t2}+h^{t2}\theta^L+h^{t2}\theta^U)x+\omega_{t2}\alpha^{t2}}{d^{t2}x+\beta^{t2}}+\cdots \\ +\frac{\omega_{tk_t}(c^{tk_t}+h^{tk_t}\theta^L+h^{tk_t}\theta^U)x+\omega_{tk_t}\alpha^{tk_t}}{d^{tk_t}x+\beta^{tk_t}}\end{pmatrix},$$

**Subject to**

$$X(a_{ij},b_i) = \{x\in\mathcal{R}^n \mid \textstyle\sum_{j=1}^n(\lambda_p a_{ij}^L+(1-\lambda_p)a_{ij}^U)x_j \le (\lambda_q b_i^L+(1-\lambda_q)b_i^U), x_j \text{ is integer}\}, \quad (7\text{-b})$$

where;

$$\lambda_p,\lambda_q\epsilon[0,1],\ p=1,3,5\ldots,n \text{ and } q=2,4,6,\ldots,n+1. \quad (7\text{-c})$$

Now to deal with multi-level problem, firstly solve the first level decision making **$1^{st}$-level** problem, in which represented as:

$$\underset{x_1}{Max}\, F_1(x,\theta) = \underset{x_1}{Max}\begin{pmatrix} \frac{\omega_{11}(c^{11}+h^{11}\theta^L+h^{11}\theta^U)x+\omega_{11}\alpha^{11}}{d^{11}x+\beta^{11}} \\ +\frac{\omega_{12}(c^{12}+h^{12}\theta^L+h^{12}\theta^U)x+\omega_{12}\alpha^{12}}{d^{12}x+\beta^{12}}+\cdots \\ +\frac{\omega_{1k_1}(c^{1k_1}+h^{1k_1}\theta^L+h^{1k_1}\theta^U)x+\omega_{1k_1}\alpha^{1k_1}}{d^{1k_1}x+\beta^{1k_1}}\end{pmatrix}, \quad (8\text{-a})$$

**Subject to**

$$X(a_{ij},b_i) = \{x\in\mathcal{R}^n \mid \textstyle\sum_{j=1}^n(\lambda_p a_{ij}^L+(1-\lambda_p)a_{ij}^U)x_j \le (\lambda_q b_i^L+(1-\lambda_q)b_i^U), x_j \text{ is integer}\}, \quad (8\text{-b})$$

where;

$\lambda_p,\lambda_q\epsilon[0,1],\ p=1,3,5\ldots,n \text{ and } q=2,4,6,\ldots,n+1$

Obtain the $\alpha$-Pareto optimal integer solution $X_1^*=(x_1^F,x_2^F,x_3^F,\ldots,x_n^F)$ and the optimal value $F_1^*$(solved by **LINGO** software package, together with the branch and bound method [16]).

Secondly, solve the second level decision making **$2^{nd}$-level** problem, which can be represented as:

$$\underset{x_2}{Max}\, F_2(x,\theta) = \underset{x_2}{Max}\begin{pmatrix} \frac{\omega_{21}(c^{21}+h^{21}\theta^L+h^{21}\theta^U)x+\omega_{21}\alpha^{21}}{d^{21}x+\beta^{21}} \\ +\frac{\omega_{22}(c^{22}+h^{22}\theta^L+h^{22}\theta^U)x+\omega_{22}\alpha^{22}}{d^{22}x+\beta^{22}}+\cdots \\ +\frac{\omega_{2k_2}(c^{2k_2}+h^{2k_2}\theta^L+h^{2k_2}\theta^U)x+\omega_{2k_2}\alpha^{2k_2}}{d^{2k_2}x+\beta^{2k_2}}\end{pmatrix}, \quad (9\text{-a})$$

**Subject to**

$$X(a_{ij},b_i) = \{x\in\mathcal{R}^n \mid \textstyle\sum_{j=1}^n(\lambda_p a_{ij}^L+(1-\lambda_p)a_{ij}^U)x_j \le (\lambda_q b_i^L+(1-\lambda_q)b_i^U), x_1=x_1^F, x_j \text{ is integer}\}, \quad (9\text{-b})$$

where;

$\lambda_p,\lambda_q\epsilon[0,1],\ p=1,3,5\ldots,n\ andq=2,4,6,\ldots,n+1.$

Obtain the $\alpha$ -Pareto optimal integer solution $X_2^*=(x_1^F,x_2^S,x_3^S,\ldots,x_n^S)$ and the optimal value $F_2^*$, (solved by **LINGO** software package, together with the branch and bound method [16]).

Finally, solve the third, fourth,…,$t$decision making **$t^{th}$-level** problem, which can be represented as:

$$\underset{x_n}{Max}\, F_t(x,\theta) = \underset{x_n}{Max}\begin{pmatrix} \frac{\omega_{t1}(c^{t1}+h^{t1}\theta^L+h^{t1}\theta^U)x+\omega_{t1}\alpha^{t1}}{d^{t1}x+\beta^{t1}} \\ +\frac{\omega_{t2}(c^{t2}+h^{t2}\theta^L+h^{t2}\theta^U)x+\omega_{t2}\alpha^{t2}}{d^{t2}x+\beta^{t2}}+\cdots+ \\ \frac{\omega_{tk_t}(c^{tk_t}+h^{tk_t}\theta^L+h^{tk_t}\theta^U)x+\omega_{tk_t}\alpha^{tk_t}}{d^{tk_t}x+\beta^{tk_t}}\end{pmatrix}, \quad (10\text{-a})$$

**Subject to**

$$X(a_{ij},b_i) = \{x\in\mathcal{R}^n \mid \textstyle\sum_{j=1}^n(\lambda_p a_{ij}^L+(1-\lambda_p)a_{ij}^U)x_j \le (\lambda_q b_i^L+(1-\lambda_q)b_i^U), x_1=x_1^F, x_2=x_2^S,\ldots,x_{n-1}=x_{n-1}^{N-1}, x_j \text{ is integer}\}, \quad (10\text{-b})$$

where;

$\lambda_p,\lambda_q\epsilon[0,1],\ p=1,3,5\ldots,n \text{ and } q=2,4,6,\ldots,n+1.$

Obtain the $\alpha$ -Pareto optimal integer solution $X_n^*$=$(x_1^F,x_2^S,\ldots,x_n^N)$ and the optimal value $F_t^*$.

## V. A SOLUTIONALGORITHM

In this section, a solution algorithm to solve problems (**F-MMIFP)** is described in a series of steps. The suggested algorithm can be summarized in the following manner:

**Step (1):** Formulate problem **(F-MMIFP)** as in (1-a)-(1-b).

**Step (2):** Set a certain degree $\alpha=\alpha^*\epsilon[0,1]$, acceptable for all decision makers.

**Step (3):** Convert problem (**F-MMIFP)** to the form of problem (**$\alpha$-MMIFP)** (4-a)-(4-c).

**Step (4):** Convert problem(**$\alpha$-MMIFP)** into problem **(IV-MMIFP)** (5-a)-(5-b).

**Step (5):** Use the nonnegative weighting sum approach to convert problem **(IV-MMIFP)** to problem **(IV-MSIFP)** (6-a)-(6-c).

**Step (6):** Convert problem **(IV-MSIFP) in**to problem **(RV-MSIFP)** (7-a)-(7-c) by applying the concept of interval-valued optimization problem on the objective function and the concept of convex linear combination on the constrains.

**Step (7):** Solve the first level decision making problem (8-a)-(8-b) (by using the **LINGO** software package, together with the branch and bound method [16]) to obtain $\alpha$ -Pareto optimal solution $X_1^* = (x_1^F, x_2^F, x_3^F, \dots, x_n^F)$.

**Step (8):** Solve the second level decision making problem (9-a)-(9-b) with $x_1 = x_1^F$, to obtain $\alpha$ -Pareto optimal solution $X_2^* = (x_1^F, x_2^S, x_3^S, \dots, x_n^S)$.

**Step (9):** Solve the third, fourth, …,$t$ decision making problem where $t$decision making problem (10-a)-(10-b), given $x_1 = x_1^F$, $x_2 = x_2^S$, $\dots, x_{n-1} = x_{n-1}^{N-1}$ to the obtain $\alpha$-Pareto optimal integer solution $X_n^*$=$(x_1^F, x_2^S, \dots, x_n^N)$.

**Step (10):** Let $X_n^* = (x_1^F, x_2^S, \dots, x_n^N)$ is non-dominated solution of problem **(F-MMIFP)** then go to step 11.

**Step (11):** Stop.

## VI. ANILLUSTRATIVE EXAMPLE

We provide a numerical example to illustrate the solution algorithm described in the previous section. For this purpose, let us consider the following problem**:**

**(F-MMIFP)**:

**[1$^{st}$-level]**

$$\underset{x_1}{Max}\, F_1(x_1,x_2,x_3,\tilde{\boldsymbol{\theta}}) =$$

$$\underset{x_1}{Max}\begin{Bmatrix} f_{11}(x_1,x_2,x_3,\tilde{\boldsymbol{\theta}}) = \frac{2\tilde{\boldsymbol{\theta}}x_1+x_2+(1-\tilde{\boldsymbol{\theta}})x_3+5}{x_1+2x_2+4x_3}, \\ f_{12}(x_1,x_2,x_3,\tilde{\boldsymbol{\theta}}) = \frac{6x_1+(4-\tilde{\boldsymbol{\theta}})x_2-\tilde{\boldsymbol{\theta}}x_3}{x_1+2x_2+x_3} \end{Bmatrix},$$

where$x_2, x_3$ solves

**[2$^{nd}$-level]**

$$\underset{x_2}{Max}\, F_2(x_1,x_2,x_3,\tilde{\boldsymbol{\theta}}) =$$

$$\underset{x_2}{Max}\begin{Bmatrix} f_{21}(x_1,x_2,x_3,\tilde{\boldsymbol{\theta}}) = \frac{(2-\tilde{\boldsymbol{\theta}})x_1+3\tilde{\boldsymbol{\theta}}x_2-2x_3+4}{2x_1+x_2+x_3}, \\ f_{22}(x_1,x_2,x_3,\tilde{\boldsymbol{\theta}}) = \frac{3\tilde{\boldsymbol{\theta}}x_1-(3+\tilde{\boldsymbol{\theta}})x_2+4x_3+1}{x_1+2x_2+x_3} \end{Bmatrix}$$

where$x_3$solves

**[3$^{rd}$-level]**

$$\underset{x_3}{Max}\, F_3(x_1,x_2,x_3,\tilde{\boldsymbol{\theta}}) =$$

$$\underset{x_3}{Max}\begin{Bmatrix} f_{31}(x_1,x_2,x_3,\tilde{\boldsymbol{\theta}}) = \frac{7x_1+(\tilde{\boldsymbol{\theta}}-1)x_2-3x_3}{x_1+3x_2+2x_3}, \\ f_{32}(x_1,x_2,x_3,\tilde{\boldsymbol{\theta}}) = \frac{6\tilde{\boldsymbol{\theta}}x_1-(2+\tilde{\boldsymbol{\theta}})x_2+3x_3+2}{x_1+3x_2} \end{Bmatrix}$$

**Subject to**

$\tilde{\boldsymbol{a}}_{\boldsymbol{11}}x_1 + x_2 + x_3 \le 2\tilde{\boldsymbol{b}}_{\boldsymbol{1}}$,

$x_1 - 2x_2 + \tilde{\boldsymbol{a}}_{\boldsymbol{23}}x_3 \le 3\tilde{\boldsymbol{b}}_{\boldsymbol{2}}$,

$x_1 + \tilde{\boldsymbol{a}}_{\boldsymbol{32}}x_2 \ge \tilde{\boldsymbol{b}}_{\boldsymbol{3}}$,

$x_1, x_2, x_3 \ge 0$and integer.

Where $\tilde{\theta}$ is a fuzzy numbers in the objective functions, $\tilde{a}_{11}, \tilde{a}_{23}, \tilde{a}_{32}$ are the fuzzy numbers in the left hand side and $\tilde{b}_1, \tilde{b}_2, \tilde{b}_3$ are the fuzzy numbers in the right hand side constraints.

| | $\boldsymbol{\theta}$ | $\boldsymbol{a_{11}}$ | $\boldsymbol{a_{23}}$ | $\boldsymbol{a_{32}}$ | $\boldsymbol{b_1}$ | $\boldsymbol{b_2}$ | $\boldsymbol{b_3}$ |
|---|---|---|---|---|---|---|---|
| $\boldsymbol{a}$ | 1 | 2 | 3 | 1 | 2 | 1 | 1 |
| $\boldsymbol{b}$ | 2 | 5 | 5 | 6 | 7 | 5 | 3 |
| $\boldsymbol{c}$ | 4 | 8 | 7 | 11 | 12 | 9 | 7 |
| $\boldsymbol{d}$ | 6 | 11 | 9 | 16 | 17 | 13 | 10 |

Let$\boldsymbol{\alpha = 0.5}$ , the equivalent non-fuzzy problems (**$\boldsymbol{\alpha}$-MMIFP)** take the form:

**[1$^{st}$-level]**

$$\underset{x_1}{Max}\, F_1(x_1,x_2,x_3,\boldsymbol{\theta}) =$$

$$\underset{x_1}{Max}\begin{Bmatrix} f_{11}(x_1,x_2,x_3,\boldsymbol{\theta}) = \frac{2\boldsymbol{\theta}x_1+x_2+(1-\boldsymbol{\theta})x_3+5}{x_1+2x_2+4x_3}, \\ f_{12}(x_1,x_2,x_3,\boldsymbol{\theta}) = \frac{6x_1+(4-\boldsymbol{\theta})x_2-\boldsymbol{\theta}x_3}{x_1+2x_2+x_3} \end{Bmatrix},$$

where$x_2, x_3$ solves

**[2$^{nd}$-level]**

$$\underset{x_2}{Max}\, F_2(x_1,x_2,x_3,\boldsymbol{\theta}) =$$

$$\underset{x_2}{Max}\begin{Bmatrix} f_{21}(x_1,x_2,x_3,\boldsymbol{\theta}) = \frac{(2-\boldsymbol{\theta})x_1+3\boldsymbol{\theta}x_2-2x_3+4}{2x_1+x_2+x_3}, \\ f_{22}(x_1,x_2,x_3,\boldsymbol{\theta}) = \frac{3\boldsymbol{\theta}x_1-(3+\boldsymbol{\theta})x_2+4x_3+1}{x_1+2x_2+x_3} \end{Bmatrix},$$

where$x_3$solves

**[3$^{rd}$-level]**

$$\underset{x_3}{Max}\, F_3(x_1,x_2,x_3,\boldsymbol{\theta}) =$$

$$\underset{x_3}{Max}\begin{Bmatrix} f_{31}(x_1,x_2,x_3,\boldsymbol{\theta}) = \frac{7x_1+(\boldsymbol{\theta}-1)x_2-3x_3}{x_1+3x_2+2x_3}, \\ f_{32}(x_1,x_2,x_3,\boldsymbol{\theta}) = \frac{6\boldsymbol{\theta}x_1-(2+\boldsymbol{\theta})x_2+3x_3+2}{x_1+3x_2} \end{Bmatrix},$$

**Subject to**

$a_{11}x_1 + x_2 + x_3 \le 2b_1$,

$x_1 - 2x_2 + a_{23}x_3 \le 3b_2$,

$x_1 + a_{32}x_2 \ge b_3$,

$1.5 \le \theta \le 5$,

$3.5 \le a_{11} \le 9.5$,

$4 \le a_{23} \le 8$,

$3.5 \le a_{32} \le 13.5$,

$4.5 \le b_1 \le 14.5$,

$3 \le b_2 \le 11$,

$2 \le b_3 \le 8.5$,

$x_1, x_2, x_3 \ge 0$ and integer.

Convert ($\boldsymbol{\alpha}$**-MMIFP)** problem into problem **(IV-MMIFP)** as follows:

**[$1^{st}$-level]**

$$\underset{x_1}{Max}\, F_1(x_1,x_2,x_3,\boldsymbol{\theta}) = \underset{x_1}{Max}\left\{\begin{array}{l} f_{11} = \frac{[\mathbf{3,10}]x_1+x_2+x_3-[\mathbf{1.5,5}]x_3+5}{x_1+2x_2+4x_3}, \\ f_{12} = \frac{6x_1+4x_2-[\mathbf{1.5,5}]x_2-[\mathbf{1.5,5}]x_3}{x_1+2x_2+x_3} \end{array}\right\},$$

where $x_2, x_3$ solves

**[$2^{nd}$-level]**

$$\underset{x_2}{Max}\, F_2(x_1,x_2,x_3,\boldsymbol{\theta}) = \underset{x_2}{Max}\left\{\begin{array}{l} f_{21} = \frac{2x_1-[\mathbf{1.5,5}]x_1+[\mathbf{4.5,15}]x_2-2x_3+4}{2x_1+x_2+x_3}, \\ f_{22} = \frac{[\mathbf{4.5,15}]x_1-3x_2-[\mathbf{1.5,5}]x_2+4x_3+1}{x_1+2x_2+x_3} \end{array}\right\},$$

where $x_3$ solves

**[$3^{rd}$-level]**

$$\underset{x_3}{Max}\, F_3(x_1,x_2,x_3,\boldsymbol{\theta}) = \underset{x_3}{Max}\left\{\begin{array}{l} f_{31} = \frac{7x_1+[\mathbf{1.5,5}]x_2-x_2-3x_3}{x_1+3x_2+2x_3}, \\ f_{32} = \frac{[\mathbf{9,30}]x_1-2x_2-[\mathbf{1.5,5}]x_2+3x_3+2}{x_1+3x_2} \end{array}\right\},$$

**Subject to**

$$[\mathbf{3.5},\mathbf{9.5}]x_1 + x_2 + x_3 \le [\mathbf{9},\mathbf{29}],$$
$$x_1 - 2x_2 + [\mathbf{4},\mathbf{8}]x_3 \le [\mathbf{9},\mathbf{33}],$$
$$x_1 + [\mathbf{3.5},\mathbf{13.5}]x_2 \ge [\mathbf{2},\mathbf{8.5}],$$
$$x_1, x_2, x_3 \ge 0 \text{ and integer.}$$

Now, by using the weighting method, and let $\omega_{11} = 0.4, \omega_{12} = 0.6, \omega_{21} = 0.3, \omega_{22} = 0.7, \omega_{31} = 0.1$ and $\omega_{32} = 0.9$ the problem **(IV-MMIFP)** becomes a single-objective fractional programming problem **(IV-MSIFP)**, which takes the form:

**[$1^{st}$-level]**

$$\underset{x_1}{Max}\, F_1(x_1,x_2,x_3,\boldsymbol{\theta}) = \underset{x_1}{Max}\left\{\begin{array}{l} \frac{[\mathbf{1.2,4}]x_1+0.4x_2+0.4x_3-[\mathbf{0.6,2}]x_3+2}{x_1+2x_2+4x_3} \\ + \frac{3.6x_1+2.4x_2-[\mathbf{0.9,3}]x_2-[\mathbf{0.9,3}]x_3}{x_1+2x_2+x_3} \end{array}\right\},$$

where $x_2, x_3$ solves

**[$2^{nd}$-level]**

$$\underset{x_2}{Max}\, F_2(x_1,x_2,x_3,\boldsymbol{\theta}) = \underset{x_2}{Max}\left\{\begin{array}{l} \frac{0.6x_1-[\mathbf{0.45,1.5}]x_1+[\mathbf{1.35,4.5}]x_2-0.6x_3+1.2}{2x_1+x_2+x_3} \\ + \frac{[\mathbf{3.15,10.5}]x_1-2.1x_2-[\mathbf{1.05,3.5}]x_2+2.8x_3+0.7}{x_1+2x_2+x_3} \end{array}\right\},$$

where $x_3$ solves

**[$3^{rd}$-level]**

$$\underset{x_3}{Max}\, F_3(x_1,x_2,x_3,\boldsymbol{\theta}) = \underset{x_3}{Max}\left\{\begin{array}{l} \frac{0.7x_1+[\mathbf{0.15,0.5}]x_2-0.1x_2-0.3x_3}{x_1+3x_2+2x_3} \\ + \frac{[\mathbf{8.1,27}]x_1-1.8x_2-[\mathbf{1.35,4.5}]x_2+2.7x_3+1.8}{x_1+3x_2} \end{array}\right\},$$

**Subject to**

$$[\mathbf{3.5},\mathbf{9.5}]x_1 + x_2 + x_3 \le [\mathbf{9},\mathbf{29}],$$
$$x_1 - 2x_2 + [\mathbf{4},\mathbf{8}]x_3 \le [\mathbf{9},\mathbf{33}],$$
$$x_1 + [\mathbf{3.5},\mathbf{13.5}]x_2 \ge [\mathbf{2},\mathbf{8.5}],$$
$$x_1, x_2, x_3 \ge 0 \text{ and integer.}$$

Convert problem **(IV-MSIFP)** into real-valued multi-level single-objective integer fractional programming problem(**RV-MSIFP**) as follows:

**[$1^{st}$-level]**

$$\underset{x_1}{Max}\, F_1(x_1,x_2,x_3,\boldsymbol{\theta}) = \underset{x_1}{Max}\left\{\begin{array}{l} \frac{5.2x_1+0.4x_2-2.2x_3+2}{x_1+2x_2+4x_3} \\ + \frac{3.6x_1-1.5x_2-3.9x_3}{x_1+2x_2+x_3} \end{array}\right\},$$

where $x_2, x_3$ solves

**[$2^{nd}$-level]**

$$\underset{x_2}{Max}\, F_2(x_1,x_2,x_3,\boldsymbol{\theta}) = \underset{x_2}{Max}\left\{\begin{array}{l} \frac{-1.35x_1+5.85x_2-0.6x_3+1.2}{2x_1+x_2+x_3} \\ + \frac{13.65x_1-6.65x_2+2.8x_3+0.7}{x_1+2x_2+x_3} \end{array}\right\},$$

where $x_3$ solves

**[$3^{rd}$-level]**

$$\underset{x_3}{Max}\, F_3(x_1,x_2,x_3,\boldsymbol{\theta}) = \underset{x_3}{Max}\left\{\begin{array}{l} \frac{0.7x_1+0.55x_2-0.3x_3}{x_1+3x_2+2x_3} \\ + \frac{35.1x_1-7.65x_2+2.7x_3+1.8}{x_1+3x_2} \end{array}\right\},$$

**Subject to**

$$7.7x_1 + x_2 + x_3 \le 15,$$
$$x_1 - 2x_2 + 6x_3 \le 21,$$
$$x_1 + 9.5x_2 \ge 4.6,$$
$$x_1, x_2, x_3 \ge 0 \text{and integer.}$$

Solve the following first level decision making problem:

**[$1^{st}$-level]**

$$\underset{x_1}{Max}\, F_1(x_1,x_2,x_3,\boldsymbol{\theta}) = \underset{x_1}{Max}\left\{\begin{array}{l} \frac{5.2x_1+0.4x_2-2.2x_3+2}{x_1+2x_2+4x_3} \\ + \frac{3.6x_1-1.5x_2-3.9x_3}{x_1+2x_2+x_3} \end{array}\right\},$$

**Subject to**

$$7.7x_1 + x_2 + x_3 \le 15,$$
$$x_1 - 2x_2 + 6x_3 \le 21,$$
$$x_1 + 9.5x_2 \ge 4.6,$$
$$x_1, x_2, x_3 \ge 0 \text{ and integer.}$$

The optimal value the objective function is $\boldsymbol{F_1^*} = \mathbf{3.233}$ and the $\alpha$-Pareto optimal integer solution is $\boldsymbol{X_1^*} = (\boldsymbol{x_1^F}, \boldsymbol{x_2^F}, \boldsymbol{x_3^F}) = (\mathbf{1}, \mathbf{1}, \mathbf{0})$.

In the same way, solve the following second level decision making problem:

**[$2^{nd}$-level]**

$$\underset{x_2}{Max}\, F_2(x_1, x_2, x_3, \boldsymbol{\theta}) =$$

$$\underset{x_2}{Max} \left\{ \begin{array}{l} \frac{-1.35x_1+5.85x_2-0.6x_3+1.2}{2x_1+x_2+x_3} \\ + \frac{13.65x_1-6.65x_2+2.8x_3+0.7}{x_1+2x_2+x_3} \end{array} \right\},$$

**Subject to**

$$7.7x_1 + x_2 + x_3 \leq 15,$$
$$x_1 - 2x_2 + 6x_3 \leq 21,$$
$$x_1 + 9.5x_2 \geq 4.5,$$
$$\mathrm{x}_1^{\mathrm{F}} = 1,$$
$$\mathrm{x}_1, \mathrm{x}_2 \geq 0 \text{ and integer.}$$

The optimal value the objective function is $\mathbf{F_2^*} = \mathbf{4.466}$ and the α-Pareto optimal integer solution is $\boldsymbol{X_2^*} = (\boldsymbol{x_1^F}, \boldsymbol{x_2^S}, \boldsymbol{x_3^S}) = (\mathbf{1}, \mathbf{1}, \mathbf{0}\,)$.

Finally, solve the third level decision making problem, which take form:

**[$3^{rd}$-level]**

$$\underset{x_3}{Max}\, F_3(x_1, x_2, x_3, \boldsymbol{\theta}) =$$

$$\underset{x_3}{Max} \left\{ \begin{array}{l} \frac{0.7x_1+0.55x_2-0.3x_3}{x_1+3x_2+2x_3} \\ + \frac{35.1x_1-7.65x_2+2.7x_3+1.8}{x_1+3x_2} \end{array} \right\},$$

**Subject to**

$$7.7x_1 + x_2 + x_3 \leq 15,$$
$$x_1 - 2x_2 + 6x_3 \leq 21,$$
$$x_1 + 9.5x_2 \geq 4.6,$$
$$x_1^F = 1,$$
$$x_2^S = 1,$$
$$x_3 \geq 0 \text{ and integer.}$$

The optimum value of the objective function is $\mathbf{F_3^*} = \mathbf{9.372}$ and the α-Pareto optimal integer solution is $\boldsymbol{X_3^*} = (\boldsymbol{x_1^F}, \boldsymbol{x_2^S}, \boldsymbol{x_3^T}) = (\mathbf{1}, \mathbf{1}, \mathbf{3})$.

So the non-dominated solution of problem **(F-MMIFP)** is given as: $(\boldsymbol{x_1^F}, \boldsymbol{x_2^S}, \boldsymbol{x_3^T}) = (\mathbf{1}, \mathbf{1}, \mathbf{3})$ and the non-dominated objective value of the functions are $\mathbf{F_1^*} = \mathbf{3.180}$, $\mathbf{F_2^*} = \mathbf{1.806}$ and $\mathbf{F_3^*} = \mathbf{4.591}$.

## VII. CONCLUDING REMARKS

In this paper, we introduced a method to solve problem **(F-MMIFP)**. By using the concept of the $\alpha$-level set of fuzzy number, this problem has been transformed to an interval-valued problem. The problem (**IV-MSIFP)** was converted into problem **(RV-MSIFP)** by using the concept of interval-valued optimization with the concept of convex linear combination of the first and last point of the interval in the constrains. An algorithm to obtain the non-dominated solution of problem **(F-MMIFP)** has been described in finite steps. Finally, an illustrative numerical example to demonstrate the algorithm was given.

However, there are many open points for discussion in future, which should be explored and studied in the area of multilevel multi-objective non-linear integer fractional optimization such as:

- Interval-valued rough multi-level multi-objective quadratic fractional integer programming problems.
- Interval-valued stochastic multi-level multi-objective quadratic fractional integer programming problems.
- Interactive approach for bi-level non-linear fractional programming integer problem with rough in the objective functions; in the constraints and in both.
- Interactive approach for bi-level non-linear fractional programming integer problem with stochastic in the objective functions; in the constraints and in both.

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