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Variance Of Time To Recruitment For A Two Graded Manpower System Involving Different Distributions Thresholds For Independent And Non – Identically Distributed Depletion With Inter-Decision Times Having Order Statistics

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ABSTRACT

In this paper, an organization subjected to a random exit of personnel due to policy decisions taken by the organization is considered; there is an associated loss of manpower if a person quits the organization. As the exit of personnel is unpredictable, a recruitment policy involving two thresholds, optional and mandatory is suggested to enable the organization to plan its decision on appropriate univariate policy of recruitment. Based on shock model approach, two mathematical models are constructed using an appropriate univariate policy of recruitment. The analytical expressions for mean and variance of time to recruitment is obtained for model I when i) loss of manpower forms a sequence of independent and non-identically distributed exponential random variables ii) inter-decision times are order statistics iii) optional and mandatory thresholds having extended exponential distribution. For model II when the optional and mandatory thresholds are SCBZ property.

Keywords - Manpower planning, Shock models, Univariate recruitment policy, Extended Exponential distribution, Hypo-exponential distribution, SCBZ property, Order Statistics.

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I. INTRODUCTION

Exit of personnel which is in other words known as wastage is an important aspect in the study of man power planning. Many models have been discussed using different kinds of wastages and different types of distributions. In [5], for a single grade man power system with a mandatory exponential threshold for the loss of man power, the authors have obtained the system performance measures namely mean and variance of the time to recruitment when the inter-decision times form an order statistics. In [2], for a single grade man power system, the author has introduced the concept of alertness in the recruitment policy which involves two thresholds optional and mandatory, and obtained mean and variance of the time to recruitment under different conditions. In [10],[11] and [12], for a two grade man power system involving optional and mandatory thresholds, the authors have obtained mean and variance of time to recruitment according as the thresholds are exponential random variable or geometric random variable and extended exponential random variable when the inter decision times form an order statistics. In [3], the authors have studied the system characteristics using different univariate policies of recruitment and by assuming different types of thresholds and wastages.

In [6], the authors have obtained mean and variance of time to recruitment for a two graded manpower system with a univariate policy of recruitment involving (i) loss of manpower and inter-decisions times are independent and non-identically distributed exponential random variables (ii) optional and mandatory thresholds follows exponential random variables. In [7], the authors have obtained mean and variance of time to recruitment for a two graded manpower system with a univariate policy of recruitment involving (i) loss of man hours are independent and non-identically distributed exponential random variables (ii) inter-decisions times are order statistics (iii) optional and mandatory thresholds follows exponential random variables. In [8] and [9], the authors have obtained mean and variance of time to recruitment for a two graded manpower system with a univariate policy of recruitment involving loss of manpower and inter-decisions times are independent and non-identically distributed exponential random variables with optional and mandatory thresholds having extended exponential distribution and SCBZ property.

The objectives of the present paper is to study the problem of time to recruitment for a two graded manpower systems and to obtain the mean and variance of time to recruitment using CUM univariate recruitment policy for exponential thresholds with loss of manpower having independent and non-identically distributed exponential random variables and inter-decision times are order Statistics with thresholds having extended exponential distribution and SCBZ property.

II. NOTATIONS

X_i : Loss of manpower due to the i^{th} decision epoch i=1, 2, 3…forming a sequence of independent and non-identically distributed exponential random variables with parameters α_i, (α_i>0).

$M_i(.)$: Distribution function of X_i

$m_i(.)$: Probability density function of X_i with mean $\frac{1}{\alpha_i}$ (α_i>0)

S_k : Cumulative loss of manpower in the first k-decisions (k=1, 2….), i.e. $S_k = \sum_{i=1}^{k} X_i$

$G_k(.)$: Distribution function of sum of k independent and non-identically distributed Exponential random variables.

$g_k(.)$: Probability density function of S_k

$$G_k(t) = \sum_{i=1}^{k} c_i\,(1-e^{-\alpha_i t}), \qquad g_k(t) = \sum_{i=1}^{k} c_i\,\alpha_i e^{-\alpha_i t},$$

$$g_k^*(s) = \sum_{i=1}^{k} c_i \frac{\alpha_i}{\alpha_i + s}$$

where $$C_i = \prod_{\substack{j=1\\ j\neq i}}^{k} \frac{\alpha_j}{\alpha_j - \alpha_i}, i = 1,2...k$$

U_k : A continuous random variables denoting the inter-decision times between $(k-1)^{th}$ and k^{th} decision epochs,k=1, 2,3…forms order statistics

$F_k(.)$: Probability distribution function of U_k .

$f_k(.)$: Probability density function of U_k

$U_{(1)}$: Smallest order Statistic

$f_{u(1)}$: Probability density function of $U_{(1)}$

$U_{(m)}$: Largest order Statistic

$f_{u(m)}$: Probability density function of $U_{(m)}$

Y : Continuous random variables denoting the optional threshold.

Z : Continuous random variables denoting the mandatory threshold.

W : Continuous random variable denoting the time to recruitment in the organization.

p : Probability that the organization is not going for recruitment whenever the total loss of manpower crosses the optional threshold Y.

$V_k(t)$: Probability that exactly k-decisions are taken in [0, t)

L(.) : Distribution function of W

l (.) : Probability density function of W

$l^*(.)$: Laplace transform of l(.)

E(W) : Expected time to recruitment

V(W) : Variance of the time to recruitment

CUM policy : Recruitment is done whenever the cumulative loss of manpower crosses the mandatory threshold. The organization may or may not go for recruitment if the cumulative loss of manpower crosses the optional threshold.

III. MAIN RESULTS

The survival function of W is given by

$$P(W>t) = \sum_{k=0}^{\infty} V_k(t)\, P(S_k<Y) + \sum_{k=0}^{\infty} V_k(t)\, P(S_k \geq Y)\, P(S_k<Z)\, p \qquad (1)$$

3.1 Model I : Thresholds follows extended exponential distributed random variables

Y_1, Y_2 : The continuous random variables denoting the optional threshold for grade1 and grade 2 following extended exponential distribution with parameters λ_1, λ_2 respectively.

Z_1, Z_2 : The continuous random variables denoting The mandatory threshold for grade1and grade 2 following extended exponential distribution with parameters μ_1, μ_2 respectively.

It is assumed that $Y_1<Z_1$ and $Y_2<Z_2$ .Y=Max (Y_1, Y_2) and Z=Max (Z_1, Z_2)

Conditioning upon S_k and using the law of total probability it can be shown that

$$P(S_k<Y) = \int_0^{\infty} P(S_k < Y \mid S_k = x)\, g_k(x)\,dx$$

$$= \int_0^{\infty} \left[1-(1-e^{-\lambda_1 x})^2\,(1-e^{-\lambda_2 x})^2\right] g_k(x)\,dx$$

$$P(S_k<Y) = 2g_k^*(\lambda_1) + 2g_k^*(\lambda_2) - g_k^*(\lambda_1+\lambda_2) + 2g_k^*(2\lambda_1+\lambda_2) g_k^*(\lambda_1+\lambda_2) + 2g_k^*(2\lambda_1+\lambda_2) + 2g_k^*(\lambda_1+2\lambda_2) - g_k^*(2\lambda_1) - g_k^*(2\lambda_2) - g_k^*(2\lambda_1+2\lambda_2)$$

$$P(S_k<Y) = 2D_1+2D_2-4D_3+2D_7+2D_8-D_9-D_{10}-D_{11} \quad (2)$$

Similarly

$P(S_k<Z)=2g_k^*(\mu_1)+2g_k^*(\mu_2) - g_k^*(\mu_1+\mu_2))+2g_k^*(2\mu_1+\mu_2)+2g_k^*(\mu_1+2\mu_2) - g_k^*(2\mu_1) - g_k^*(2\mu_2) - g_k^*(2\mu_1+2\mu_2)$

$$P(S_k < Z) = 2D_4+2D_5 - 4D_6+2D_{12} +2D_{13}-D_{14}-D_{15}-D_{16} \quad (3)$$

Using (2) and (3) in (1), we get

$$P(W>t) = \sum_{k=0}^{\infty} V_k(t)\,\{(2D_1+2D_2-4D_3+2D_7+2D_8-D_9 -D_{10}-D_{11})[1-p(2D_4+2D_5-4D_6+2D_{12}+2D_{13} -D_{14}-D_{15}-D_{16})+p(2D_4+2D_5-D_6+2D_{12} +2D_{13}-D_{14}-D_{15}-D_{16})\} \quad (4)$$

From renewal theory and using $L(t) = 1- P(W>t)$ and $l(t) = \frac{d}{dt}L(t)$, $l^*(s)$ = Laplace transform of $l(t)$, we get

$$l^*(s) = \sum_{k=0}^{\infty} [f_{k+1}^*(s) - f_k^*(s)]\,\{(2D_1+2D_2- 4D_3+2D_7+2D_8-D_9-D_{10}-D_{11})\,[1-p(2D_4+2D_5- 4D_6+2D_{12}+2D_{13}-D_{14}-D_{15}-D_{16})+p(2D_4+2D_5 - 4D_6+2D_{12}+2D_{13}-D_{14}-D_{15}-D_{16})\} \quad (5)$$

where $D_1 = g_k^*(\lambda_1)$, $D_2 = g_k^*(\lambda_2)$, $D_3 = g_k^*(\lambda_1+\lambda_2)$, $D_4 = g_k^*(\mu_1)$, $D_5 = g_k^*(\mu_2)$, $D_6 = g_k^*(\mu_1+\mu_2)$, $D_7 = g_k^*(2\lambda_1+\lambda_2)$, $D_8 = g_k^*(\lambda_1+2\lambda_2)$, $D_9 = g_k^*(2\lambda_1)$, $D_{10} = g_k^*(2\lambda_2)$, $D_{11} = g_k^*(2\lambda_1+2\lambda_2)$, $D_{12} = g_k^*(2\mu_1+\mu_2)$, $D_{13} = g_k^*(\mu_1+2\mu_2)$, $D_{14} = g_k^*(2\mu_1)$, $D_{15} = g_k^*(2\mu_2)$, $D_{16} = g_k^*(2\mu_1+2\mu_2)$.

Let $U_1, U_2, U_3, U_4, \ldots U_m$ be arranged in an increasing order so that we have sequence $U_{(1)}, U_{(2)}, U_{(3)}, U_{(4)}, \ldots U_{(m)}$. $U_{(r)}$ is the r^{th} order statistics $r=1,2,3,\ldots m$. The random variables $U_{(1)}, U_{(2)}, U_{(3)}, U_{(4)}, \ldots U_{(m)}$ are not independent. For $r=1,2,3\ldots m$, the probability density function of $U_{(r)}$ is given by

$$f_{u(r)}(t) = r({}_mC_r)(F(t))^{r-1} f(t)\,(1-F(t))^{m-r},$$

$r=1,2,3\ldots m$, Where $f(t)= F^1(t)$ (6)

According as probability density function of inter-decision times in (6) if $f_{u(1)}(t)$ (or) $f_{u(m)}(t)$) when the cumulative distribution $F(t)$ of U_i, $i=1,2,3\ldots$ is given by

$$F(t)= 1-e^{-\beta t}, \quad f(t)= \beta e^{-\beta t} \quad (7)$$

From (7), when the probability density function of inter-decision times are that of the first order statistics.

First Order:

Suppose $f(t)=f_{u(1)}(t)$

$$f_{u(1)}(t) = m\beta e^{-m\beta t}, \quad f_{u(1)}^*(s)= \frac{m\beta}{m\beta+s} \quad (8)$$

From (5) , we get

$$l^*(s) = [f^*(s)-1]\, f*(s) \sum_{k=1}^{\infty} (f^*(s))^{k-1}\{(2D_1+2D_2- 4D_3+2D_7+2D_8-D_9-D_{10}-D_{11})[1-p(2D_4+2D_5- 4D_6+2D_{12}+2D_{13}-D_{14}-D_{15}-D_{16}) +p(2D_4+2D_5 - 4D_6+2D_{12} +2D_{13}-D_{14}-D_{15}-D_{16})\} \quad (9)$$

It is known that

$$E(W) = -\left[\frac{d}{ds} l^*(s)\right]_{s=0} \quad (10)$$

$$E(W^2) = \left[\frac{d^2}{ds^2} l^*(s)\right]_{s=0} \quad (11)$$

$$Var(W) = E(W^2) - (E(W))^2 \quad (12)$$

Using (9) in (10) it can be shown that

$$E(W) = \sum_{k=1}^{\infty} \frac{1}{m\beta}\{(2D_1+2D_2-4D_3+2D_7+2D_8 -D_9-D_{10}-D_{11})[1-p(2D_4+2D_5-4D_6 +2D_{12}+2D_{13}-D_{14}-D_{15}-D_{16})+p(2D_4+2D_5 - 4D_6+2D_{12}+2D_{13}-D_{14}-D_{15}-D_{16})\} \quad (13)$$

Equation (13) gives the meantime to recruitment for first order.

Using (9) in (11) it can be shown that

$$E(W^2) = \frac{2}{(m\beta)^2}\sum_{k=1}^{\infty} (k+1)\{(2D_1+2D_2-D_3 +2D_7+2D_8-D_9-D_{10}-D_{11})[1-p(2D_4+2D_5 - 4D_6+2D_{12}+2D_{13}-D_{14}-D_{15}-D_{16}) +p(2D_4+2D_5 - 4D_6+2D_{12}+2D_{13} -D_{14}-D_{15}-D_{16})\} \quad (14)$$

Using (13) and (14) in (12), we get variance of time to recruitment for first order.

m^th Order:

Suppose $f(t)= f_{u(m)}(t)$

From (6), we get $f_{u(m)}(t)= m(F(t))^{m-1} f(t)$ (15)

By using the fourier Transforms, we get

$$f^*_{u(m)}(s)= \frac{m!\beta^m}{(s+\beta)(s+2\beta)(s+3\beta)\ldots\ldots(s+m\beta)} \quad (16)$$

$$l^*(s) = \left(\frac{m!\beta^m}{B} - 1\right) \sum_{k=1}^{\infty} \left[\frac{m!\beta^m}{B}\right]^k\{(2D_1+2D_2 -4D_3+2D_7+2D_8-D_9-D_{10}-D_{11}) [1-p(2D_4+2D_5- 4D_6+2D_{12}+2D_{13}-D_{14} - D_{15}-D_{16})+p(2D_4+2D_5 - 4D_6+2D_{12} +2D_{13}-D_{14}-D_{15}-D_{16})\} \quad (17)$$

where $B=(s+\beta)(s+2\beta)(s+3\beta)\ldots\ldots(s+m\beta)$

Using (17) in (9) it can be shown that

$$E(W) = \frac{\sum_{n=1}^{m}\frac{1}{n}}{\beta} \sum_{k=1}^{\infty} \{(2D_1+2D_2-4D_3+2D_7 +2D_8-D_9-D_{10}-D_{11})[1-p(2D_4+2D_5 - 4D_6+2D_{12}+2D_{13}-D_{14}-D_{15}-D_{16}) +p(2D_4+2D_5 - 4D_6+2D_{12}+2D_{13} -D_{14}-D_{15}-D_{16})\} \quad (18)$$

Equation (18) gives meantime to recruitment for m th order.

Using (17) in (10) it can be shown that

$E(W^2)=$

$$\left(\sum_{n=1}^{m}\frac{1}{n}\right)^2\left[\sum_{k=1}^{\infty}\frac{2k+1}{\beta^2}\right]+\frac{\sum_{k=1}^{\infty}\left(\sum_{n=1}^{m}\frac{1}{n^2}\right)}{\beta^2}$$

$\{(2D_1+2D_2-4D_3+2D_7+2D_8-D_9-D_{10}-D_{11})$
$[1-p(2D_4+2D_5-4D_6+2D_{12}+2D_{13}-D_{14}-D_{15}-D_{16})$
$+p(2D_4+2D_5-4D_6+2D_{12}+2D_{13}-D_{14}-D_{15}-D_{16})\}$ (19)

Using (18), (19) in (12), we get variance of time to recruitment for n^{th} order statistics.

3.1.1 Numerical Illustrations

The analytical expressions for the performance measures namely mean and variance of time to recruitment are analyzed numerically by varying a parameter at a time and keeping other parameters fixed. The effect of nodal parameters of loss of man power ($\alpha_1, \alpha_2, \alpha_3\ \alpha_4, \alpha_5$) and inter-decision time β, m the number of decision epochs in(0,t] and p on the performance measures are shown in the following tables.

Table 1
First Order

Varying the parameters of inter-decision time β and m the number of decision epochs in (0,t] and fixing the parameters of loss of man-hours ($\alpha_1, \alpha_2, \alpha_3\ \alpha_4, \alpha_5$) and p.

$\alpha_1 = 0.001, \alpha_2 = 0.002, \alpha_3 = 0.003,$
$\alpha_4 = 0.004, \alpha_5 = 0.005$, p = 0.05, λ_1=0.01, λ_2=0.013, μ_1=0.02, μ_2=0.025, β =0.6

m / β		0.6	0.62	0.64	0.66	0.68
1	E(W)	0.3593	0.3177	0.33679	0.3267	0.3171
	V(W)	2.6178	2.4516	2.3008	2.1635	2.0381
2	E(W)	0.1797	0.1739	0.1684	0.1633	0.1585
	V(W)	0.6545	0.6129	0.5752	0.5409	0.5095
3	E(W)	0.1198	0.1159	0.1123	0.1089	0.1057
	V(W)	0.2909	0.2724	0.2556	0.2404	0.2265
4	E(W)	0.0898	0.0869	0.0842	0.0817	0.0793
	V(W)	0.1636	0.1532	0.1438	0.1352	0.1274
5	E(W)	0.0719	0.0695	0.0674	0.0635	0.0634
	V(W)	0.1047	0.0981	0.0920	0.0865	0.0815

m^{th} Order

Table 2

Varying the parameters of inter-decision time β and m the number of decision epochs in (0,t] and fixing the parameters of loss of man-hours ($\alpha_1, \alpha_2, \alpha_3\ \alpha_4, \alpha_5$) and p.

$\alpha_1 = 0.001, \alpha_2 = 0.002, \alpha_3 = 0.003$,
$\alpha_4 = 0.004, \alpha_5 = 0.005$, p = 0.05,
λ_1=0.01, λ_2=0.013, μ_1=0.02, μ_2=0.025, β =0.6

m / β		0.6	0.62	0.64	0.66	0.68
1	E(W)	0.3593	0.3477	0.3369	0.3267	0.3171
	V(W)	1.8901	1.7701	1.6612	1.5620	1.4715
2	E(W)	0.5390	0.5216	0.5053	0.4900	0.4756
	V(W)	3.6538	3.4218	3.2113	3.0196	2.8446
3	E(W)	0.6588	0.6375	0.6176	0.5989	0.5813
	V(W)	5.1549	4.8277	4.5307	4.2603	4.0134
4	E(W)	0.7486	0.7245	0.7018	0.6805	0.6605
	V(W)	6.4567	6.0468	5.6748	5.3361	5.0268
5	E(W)	0.8205	0.7940	0.7692	0.7459	0.7239
	V(W)	7.6082	7.1253	6.6869	6.2878	5.9234

3.2 Thresholds follows SCBZ property

Y_1, Y_2 : The continuous random variables denoting the optional thresholds for the grade 1 and grade 2 follows SCBZ property with parameters ($\theta_1, \theta_2, \theta_3, \theta_4, \lambda_1, \lambda_2$) respectively.

Z_1, Z_2 : The continuous random variables denoting the mandatory thresholds for the grade 1 and grade 2 follows SCBZ property with parameters ($\theta_5, \theta_6, \theta_7, \theta_8, \mu_1, \mu_2$) respectively.

It is in assumed that $Y_1<Z_1$ and $Y_2<Z_2$.

Y=Max (Y_1, Y_2) and Z=Max (Z_1, Z_2)

$H_1(.)$: Distribution function of Y_1 ,
$H_2(.)$:Distribution function of Y_2
$H_3(.)$:Distribution function of Z_1,
$H_4(.)$:Distribution function of Z_2

As in Rao and Talwalkar (1990), In this section, the distribution of optional Y and mandatory Z are given by

$$H_1(y)=1-p_1e^{-(\theta_1+\lambda_1)y}-q_1e^{-\theta_2 y} \qquad (20)$$

where $p_1=\frac{\theta_1-\theta_2}{\theta_1-\theta_2+\lambda_1}$, $q_1=\frac{\lambda_1}{\theta_1-\theta_2+\lambda_1}=1-p_1$,

$p_1+q_1=1$

Similarly $H_2(y)$ is a distribution function of Y_2 have SCBZ property with parameter ($\theta_3, \theta_4, \lambda_2$).

$$H_2(y)=1-p_2e^{-(\theta_3+\lambda_2)y}-q_2e^{-\theta_4 y} \quad (21)$$

where $p_2=\frac{\theta_3-\theta_4}{\theta_3-\theta_4+\lambda_2}$, $q_2=\frac{\lambda_2}{\theta_3-\theta_4+\lambda_2}=1-p_2$,

$p_2+q_2=1$

$H_3(z)$ is a distribution function of Z_1 have SCBZ property with parameter $(\theta_5,\theta_6,\mu_1)$

$$H_3(z)=1-p_3e^{-(\theta_5+\mu_1)z}-q_3e^{-\theta_6 z} \quad (22)$$

where $p_3=\frac{\theta_5-\theta_6}{\theta_5-\theta_6+\mu_1}$ $q_3=\frac{\mu_1}{\theta_5-\theta_6+\mu_1}=1-p_3$,

$p_3+q_3=1$

$H_4(z)$ is a distribution function of Z_2 have SCBZ property with parameter $(\theta_7,\theta_8,\mu_2)$.

$$H_4(z)=1-p_4e^{-(\theta_7+\mu_2)z}-q_4e^{-\theta_8 z} \quad (23)$$

where $p_4=\frac{\theta_7-\theta_8}{\theta_7-\theta_8+\mu_2}$, $q_4=\frac{\mu_2}{\theta_7-\theta_8+\mu_2}=1-p_4$,

$p_4+q_4=1$

Conditioning upon S_k and using the law of total probability it can be shown that

$$P(S_k<Y)=\int_0^\infty[1-H_1(x)\,H_2(x)\,]g_k(x)dx$$

$$\int_0^\infty[1-(1-p_1e^{-(\theta_1+\lambda_1)x}-q_1e^{-\theta_2 x})(1-p_2e^{-(\theta_3+\lambda_2)x}-q_2e^{-\theta_4 x})]g_k(x)dx$$

$$P(S_k<Y)=p_1g_k^*(\theta_1+\lambda_1)+p_2g_k^*(\theta_3+\lambda_2)+q_1g_k^*(\theta_2)+q_2g_k^*(\theta_4)-p_1p_2g_k^*(\theta_1+\theta_3+\lambda_1+\lambda_2)-p_1q_2g_k^*(\theta_1+\theta_4+\lambda_1)-q_1p_2g_k^*(\theta_2+\theta_3+\lambda_2)-q_1q_2g_k^*(\theta_2+\theta_4)$$

$$P(S_k<Y)=p_1N_1+p_2N_2+q_1N_3+q_2N_4-p_1p_2N_5-p_1q_2N_6-q_1p_2N_7-q_1q_2N_8 \quad (24)$$

Similarly

$$P(S_k<Z)=p_3g_k^*(\theta_5+\mu_1)+p_4g_k^*(\theta_7+\mu_2)+q_3g_k^*(\theta_6)+q_4g_k^*(\theta_8)-p_3p_4g_k^*(\theta_5+\theta_7+\mu_1+\mu_2)-p_3q_4g_k^*(\theta_5+\theta_6+\mu_1)-q_3p_4g_k^*(\theta_6+\theta_7+\mu_2)-q_3q_4g_k^*(\theta_6+\theta_8)$$

$$P(S_k<Z)=p_3N_9+p_4N_{10}+q_3N_{11}+q_4N_{12}-p_3p_4N_{13}-p_3q_4N_{14}-q_3p_4N_{15}-q_3q_4N_{16} \quad (25)$$

Using (24) and (25) in (1) it can be shown that

$$P(W>t)=\sum_{k=0}^{\infty}V_k(t)\{(p_1N_1+p_2N_2+q_1N_3+q_2N_4-p_1p_2N_5-p_1q_2N_6-q_1p_2N_7-q_1q_2N_8)(1-p(p_3N_9+p_4N_{10}+q_3N_{11}+q_3N_{12}-p_3p_4N_{13}-p_3q_4N_{14}-q_3p_4N_{15}-q_3q_4N_{16}))+p(p_3N_9+p_4N_{10}+q_3N_{11}+q_3N_{12}-p_3p_4N_{13}-p_3q_4N_{14}-q_3p_4N_{15}-q_3q_4N_{16})\} \quad (26)$$

From renewal theory and using $L(t)=1-P(W>t)$ and $l(t)=\frac{d}{dt}L(t)$, $l^*(s)$ = Laplace transform of $l(t)$, we get

$$l^*(s)=\sum_{k=0}^{\infty}[f_{k+1}^*(s)-f_k^*(s)]\{(p_1N_1+p_2N_2+q_1N_3+q_2N_4-p_1p_2N_5-p_1q_2N_6-q_1p_2N_7-q_1q_2N_8)(1-p(p_3N_9+p_4N_{10}+q_3N_{11}+q_3N_{12}-p_3p_4N_{13}-p_3q_4N_{14}-q_3p_4N_{15}-q_3q_4N_{16}))+p(p_3N_9+p_4N_{10}+q_3N_{11}+q_3N_{12}-p_3p_4N_{13}-p_3q_4N_{14}-q_3p_4N_{15}-q_3q_4N_{16})\} \quad (22)\ (27)$$

Proceeding in the same way as in model I, from the (6) to (12), it can be shown that

First Order:

$$E(W)=\sum_{k=1}^{\infty}\frac{1}{m\beta}\{(p_1N_1+p_2N_2+q_1N_3+q_2N_4-p_1p_2N_5-p_1q_2N_6-q_1p_2N_7-q_1q_2N_8)(1-(p_3N_9+p_4N_{10}+q_3N_{11}+q_4N_{12}-p_3p_4N_{13}-p_3q_4N_{14}-q_3p_4N_{15}-q_3q_4N_{16}))+p(p_3N_9+p_4N_{10}+q_3N_{11}+q_3N_{12}-p_3p_4N_{13}-p_3q_4N_{14}-q_3p_4N_{15}-q_3q_4N_{16})\} \quad (28)$$

Equation (28) gives the meantime to recruitment for first order.

$$E(W^2)=\frac{2}{(m\beta)^2}\sum_{k=1}^{\infty}(k+1)\{(p_1N_1+p_2N_2+q_1N_3+q_2N_4-p_1p_2N_5-p_1q_2N_6-q_1p_2N_7-q_1q_2N_8)(1-p(p_3N_9+p_4N_{10}+q_3N_{11}+q_3N_{12}-p_3p_4N_{13}-p_3q_4N_{14}-q_3p_4N_{15}-q_3q_4N_{16}))+p(p_3N_9+p_4N_{10}+q_3N_{11}+q_3N_{12}-p_3p_4N_{13}-p_3q_4N_{14}-q_3p_4N_{15}-q_3q_4N_{16})\} \quad (29)$$

Using (28) and (29) in (12), we get variance of time to recruitment for first order.

mth Order:

$$E(W)=\frac{\sum_{n=1}^{m}\frac{1}{n}}{\beta}\sum_{k=1}^{\infty}\{(p_1N_1+p_2N_2+q_1N_3+q_2N_4-p_1p_2N_5-p_1q_2N_6-q_1p_2N_7-q_1q_2N_8)(1-p(p_3N_9+p_4N_{10}+q_3N_{11}+q_3N_{12}-p_3p_4N_{13}-p_3q_4N_{14}-q_3p_4N_{15}-q_3q_4N_{16}))+p(p_3N_9+p_4N_{10}+q_3N_{11}+q_3N_{12}-p_3p_4N_{13}-p_3q_4N_{14}-q_3p_4N_{15}-q_3q_4N_{16})\} \quad (30)$$

Equation (30) gives meantime to recruitment for mth order.

$$E(W^2)=\left(\sum_{n=1}^{m}\frac{1}{n}\right)^2\left(\sum_{k=1}^{\infty}\frac{2k+1}{\beta^2}\right)+\left(\frac{\sum_{k=1}^{\infty}\left(\sum_{n=1}^{m}\frac{1}{n^2}\right)}{\beta^2}\right)\{(p_1N_1+p_2N_2+q_1N_3+q_2N_4$$

$- p_1p_2N_5-p_1q_2N_6-q_1p_2N_7-q_1q_2N_8)$
$(1-p(p_3N_9+p_4N_{10}+q_3N_{11}+q_3N_{12}- p_3p_4N_{13}$
$-p_3q_4N_{14}-q_3p_4N_{15}-q_3q_4N_{16}))$
$+p(p_3N_9+p_4N_{10}+q_3N_{11}+q_3N_{12}- p_3p_4N_{13}$
$-p_3q_4N_{14}-q_3p_4N_{15}-q_3q_4N_{16})\}$ (31)

where $N_1 = g_k^*\left(\theta_1 + \lambda_1\right)$, $N_2 = g_k^*\left(\theta_3 + \lambda_2\right)$,
$N_3 = g_k^*(\theta_2)$, $N_4 = g_k^*(\theta_4)$,
$N_5 = g_k^*(\theta_1 + \theta_3 + \lambda_1 + \lambda_2)$,
$N_6 = g_k^*(\theta_1 + \theta_4 + \lambda_1)$, $N_7 = g_k^*(\theta_2 + \theta_3 + \lambda_2)$,
$N_8 = g_k^*(\theta_2 + \theta_4)$, $N_9 = g_k^*(\theta_5 + \mu_1)$,
$N_{10} = g_k^*(\theta_7 + \mu_2)$, $N_{11} = g_k^*(\theta_6)$, $N_{12} = g_k^*(\theta_8)$,
$N_{13} = g_k^*(\theta_5 + \theta_7 + \mu_1 + \mu_2)$,
$N_{14} = g_k^*(\theta_5 + \theta_6 + \mu_1)$,
$N_{15} = g_k^*(\theta_6 + \theta_7 + \mu_2)$, $N_{16} = g_k^*(\theta_6 + \theta_8)$.

Using (30) and (31) in (12), we get variance of time to recruitment for mth order statistics.

3.2.1 Numerical Illustrations

The analytical expressions for the performance measures namely mean and variance of time to recruitment are analyzed numerically by varying a parameter at a time and keeping other parameters fixed. The effect of nodal parameters of loss of man power ($\alpha_1, \alpha_2, \alpha_3\ \alpha_4, \alpha_5$) and inter-decision time β, m the number of decision epochs in(0,t] and p on the performance measures are shown in the following tables.

First Order

Table 3

Varying the parameters of inter-decision time β and m the number of decision epochs in (0,t] and fixing the parameters of loss of man-hours ($\alpha_1, \alpha_2, \alpha_3\ \alpha_4, \alpha_5$) and p.
$\theta_1 = 0.09,\ \theta_2 = 0.092,\ \theta_3 = 0.07,\ \theta_4 = 0.072,$
$\theta_5 = 0.05, \theta_6 = 0.053, \theta_7 = 0.04, \theta_8 = 0.043$
p = 0.05, λ_1=0.01, λ_2=0.013, μ_1= 0.02, μ_2=0.025,
β =0.6,m=5, $\alpha_1 = 0.01, \alpha_2 = 0.02,$
$\alpha_3 = 0.03, \alpha_4 = 0.04, \alpha_5 = 0.05$.

m / β		0.6	0.62	0.64	0.66	0.68
1	E(W)	1.4943	1.4461	1.4009	1.3584	1.3185
	V(W)	13.2781	12.4353	11.6702	10.9737	10.3377
2	E(W)	0.7471	0.7230	0.7004	0.6792	0.6592
	V(W)	3.3195	3.1088	2.9176	2.7434	2.5844
3	E(W)	0.4981	0.4820	0.4670	0.4528	0.4395
	V(W)	1.4753	1.3817	1.2967	1.2193	1.1486
4	E(W)	0.8299	0.7772	0.7294	0.6859	0.6461
	V(W)	0.8299	0.7772	0.7294	0.6859	0.6461
5	E(W)	0.2989	0.2892	0.2802	0.2717	0.2637
	V(W)	0.5311	0.4974	0.4668	0.4389	0.4135

mth Order

Table 4

Varying the parameters of inter-decision time β and m the number of decision epochs in (0,t] and fixing the parameters of loss of man-hours ($\alpha_1, \alpha_2, \alpha_3\ \alpha_4, \alpha_5$) and p.
$\theta_1 = 0.09,\ \theta_2 = 0.092,\ \theta_3 = 0.07, \theta_4 = 0.072,$
$\theta_5 = 0.05, \theta_6 = 0.053, \theta_7 = 0.04, \theta_8 = 0.043$
p = 0.05, λ_1=0.01, λ_2=0.013,
μ_1= 0.02, μ_2=0.025, β =0.6,
$\alpha_1 = 0.001, \alpha_2 = 0.002, \alpha_3 = 0.003,$
$\alpha_4 = 0.004, \alpha_5 = 0.005$, p = 0.05,

m / β		0.6	0.62	0.64	0.66	0.68
1	E(W)	0.1070	0.1036	0.1004	0.0973	0.0945
	V(W)	0.2930	0.2744	0.2575	0.2422	0.2281
2	E(W)	0.1606	0.1554	0.1505	0.1460	0.1417
	V(W)	0.4808	0.4503	0.4226	0.3974	0.3744
3	E(W)	0.1963	0.1899	0.1840	0.1784	0.1732
	V(W)	0.6280	0.5881	0.5520	0.5190	0.4889
4	E(W)	0.2230	0.2158	0.2091	0.2027	0.1968
	V(W)	0.7514	0.7037	0.6604	0.6210	0.5850

5	**E(W)**	**0.2444**	**0.2365**	**0.2291**	**0.2222**	**0.2157**
	V(W)	**0.8586**	**0.8041**	**0.7546**	**0.7096**	**0.6684**

IV. CONCLUSION

We observe the following from the above numerical illustrations

From tables 1 and 3, we know that

1.The mean and variance of the time to recruitment decreases as the number of decisions m and inter-decision times β are increases simultaneously.

2.If m, the number of decision epochs in (0,t] increases, while the mean and variance of the time to recruitment decreases .

3.Mean and variance of the time to recruitment decreases as inter-decision times increases and on an average the inter-decision times decreases.

From tables 2 and 4, we know that

1.The mean and variance of the time to recruitment increases as the number of decisions m and inter-decision times β are increases simultaneously.

2. If m, the number of decision epochs in (0,t] increases, while the mean and variance of the time to recruitment increases .

3.As β, Mean and variance of the time to recruitment decreases as inter-decision times increases and on an average the inter-decision times decreases.

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