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Iterative Computation of Singular and Singularity induced Bifurcation points of Differential Algebraic equations for a Multi-machine Power system Model

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ABSTRACT—In this paper, we present an efficient algorithm tocomputesingularpointsandsingularity-inducedbifurcationpointsofdifferential-algebraicequationsforamulti-machinepower-systemmodel.Powersystemsareoftenmodeledasasetof differential-algebraic equations (DAE) whose algebraic partbringssingularityissuesintodynamicstabilityassessmentofpower systems. Roughly speaking, the singular points are pointsthatsatisfythealgebraicequations,butatwhichthevectorfieldisnot defined. In terms of power-system dynamics, around singularpoints,thegeneratorangles(thenaturalstatesvariables)arenot defined as a graph of the load bus variables (the algebraicvariables).Thus,thecausalrequirementoftheDAEmodelbreaksdownanditcannotpredictsystembehavior.Singularpointsconstitute important organizing elements of power-system DAEmodels.Thispaperproposesaniterativemethodtocomputesingularpointsatanygivenparametervalue.Withalemmapresented in this paper, we are also able to locate singularity in-ducedbifurcationpointsuponidentifyingthesingularpoints.

I. INTRODUCTION

BIFURCATION theory is the commonly used tool to an-alyzevarioustypesofstabilityproblemsinpowersys-tems modeled either as a set of ordinary differential equations(ODEs) or as a set of differential-algebraic equations (DAEs)[1].Intheformercase,theequationsarenotoriouslystiffwhencertain load dynamics are included. As a modeling tool, prob-lemsassociatedwiththisaffectfurtheranalyticalstudiesofthesystem. In order to overcome this problem (as well as the factof the nonexistence of a universally accepted dynamical loadmodel) DAEs have been used based on the approximation oftheserelativelyfastandstableloaddynamicsasalgebraicequa-tions[2]–[8].

Thispaperaddressesthelocalbifurcationsandalgebraicsingularitiesoftheclassicalpowersystemwithacon-stantPQloadmodel,whichismodeledassemi-explicitindex-1DAEs. It is well known that when parameters are subject to varia-tions, the equilibria of the DAE power-system model may ex-hibit three local bifurcations, namely saddle node (SN), Hopf,andsingularityinduced(SI)bifurcations.TheSNandHopfbi-furcations, which are observed in the ODE models of power sys-tems as well, have been extensively studied in power systemsandtheyarelinkedtovoltagecollapseandoscillatoryinstabili-ties,respectively[1].TheSIbifurcationisduetosingularityofthe algebraic equations of the DAE model under some parametervariations.

WithanSIbifurcationtheorem([7,Th.3,p.1999]),animprovedversionofitbasedonthedecompositionofparameter dependent polynomials.More recent work on the SI bifurcations in-cludes the [10] and [11].In [10], Beardmore has extended the SIbifurcationtheoremof[7]toincludenongenericcaseswherebybranchingofequilibriaislocatedatthesingularity,i.e.,[7,As-sumption 2, SI bifurcation Theorem] is removed and applied itto a 3-bus power system, which has been also studied by Kwatnyetal.in[2].In[11],Riazaetal.haveprovidedadetailedstudyon the qualitative nature of singular points of relatively simpleindex-1 DAE examples indicating that in some cases dynamicbehavior of the system is smooth (well-defined vector field) evenatsingularpoints.

Animportantimplicationoftheoccurrenceof theSIbifurca-tionistheexistenceofasingularset(orimpassesurface)intheconstraint manifold containing infinitely many singular pointsat each parameter value, which may play a crucial role in as-sessing the stability of DAE power-system models. The litera-tureinpower-systemstabilityanalysiswithrespecttothealge-braic singularities of the DAEs is rich with references describingvoltage instabilities in terms of the following. Nearness to animpassesurface[12],[13]suddenchangeinvoltages[14],[15]and eventual (or actual) loss of voltage causality [2], [12], tonameafew.In[14]theexis-tence of the impasse surface is closely related to the load models,and for constant load model the DAE model has the propertiesof voltage instability (i.e., sudden reduction in voltages) whenoperatinginthevicinityofimpassepoints(ortrajectoriescoin-cidingwiththeimpassesurface).

More related work have reported been with similar results and using the bifurcation theorytheyhaveshownthatanimportantpartofthestabilityboundaryisformedbytrajectoriesthataretangenttothe singularsurface.More recent studies [16], [17] focus on the direct assessmentof the system stability in the presence of the impasse surfacethatliesonthestabilityboundary.Anewenergyfunctiontech-nique has been presented to compute the critical energy overtherelevantsegmentoftheimpassesurfacethatguaranteesthecausalityifthesystemhaslessenergythenthe criticalvalue.

In spite of the fact that there is no well-established link be-tween algebraic singularity and voltage collapse as in the caseoftheSNbifurcation,mostoftheworksuggeststhat thesystemundergoes some sort of voltage instability when the voltagecausality is lost during a transient. With respect to loss of voltagecausality, it is essential to note that during this, voltages are nolonger implicit functions of dynamic variables when describedby DAE models. To use DAE as a tool, knowledge of wherecausality disappears (or where impasse surface(s) “lie”) can beapplied toward the definition of “limits” of appropriateness fora given model. An underlying issue is that at singular points(including singular equilibria); the DAE model cannot predictthevoltagebehavior.Thus,locationofsingularities,whichcon-stitute important organizing elements of a power-system DAEmodel, is invaluable information for assessing stability of thesystem.Thefamilyofsingularpointsformsaboundaryofwell-defined behavior for a given model. In this work impasse surfaceisasetofsingularpointsthatexhibitslossofvoltagecausality.

Even though many researchers either in the field of power sys-tems[2], [5]–[8], [12]–[17] or in the field of the general DAEtheory [9]–[11] have long recognized the importance of singularpoints(orlossofvoltagecausalityinpowersystems)includingsingularequilibriaintermsofsystemdynamics,thereisnorig-orous method available in the literature for computing their loca-tionsintheparameterspace.Mostoftheeffortfocuseson char-acterizing qualitative description of system dynamics aroundsingularities without providing a systematic method for locatingthem,especiallyforlargerpowersystems.

Our main purpose here is to propose a simple and efficientmethod to identify algebraic singularities (including singularequilibria) of the DAE model of power systems and to visu-alize singularities together with the equilibria and their asso-ciated local bifurcations as a function of the parameters usingtraditional nose curves. The proposed method involves the fol-lowingtwomainsteps.

1) Computing singular points at various parameter valuesalong the nose curve defined by a designated bus injectionchange pattern and illustrating singular points in a two-di-mensional(2-D)nosecurve
2) Developing a lemma showing that any singular point ata given set of generator bus injections is also an equilibriumpoint(thus,itisanSIbifurcationpoint)atanothersetofbusinjections.

In the method for computing singular points, we first usegenerator angles to parameterize the algebraic part of the DAEmodel at any given parameter value (i.e., bus injections) and for-mulate the problem of identifying singular points as a bifurca-tionproblemofasetofalgebraicequationswhoseparameters

The rest of the paper is organized as follows. Section II dis-cuses bifurcations and singularities of the DAE model of theclassical power system. Section II also includes a lemma to iden-tify the SI bifurcations and presents two examples of the DAE(oneofwhichisa5-buspower-systemexample)to

illustratetheapplicationofthelemmaandthekeyconceptofthepaper.Sec-tion III describes methods to compute equilibria and singularpoints of the DAE model in details. Section IV presents the sim-ulation results using voltage stability toolbox (VST)[18], [19]for the IEEE 118-bus system and illustrates singular points inthe nose curve. Finally, Section V summarizes main contribu-tionsofthisworkandsuggestssomeoftherelatedfuture work.

II. DIFFERENTIAL-ALGEBRAIC POWER-SYSTEM MODEL AND SINGULARITIES

A. ClassicalPowerSystemModel

The dynamics of a classical power system with constant PQload buses are commonly described by semi-explicit DAE of theform[2]

$$\dot{\delta}_g = \omega$$

$\dot{\omega}$=−[$M^{-1}D$]ω −M^{-1}[$f_g(\delta_g,\delta_\ell,V)$

−P_g]

$f_\ell(\delta_g,\delta_\ell,V)$−$P_\ell$= 0
Q_ℓ (2.1)

where $\delta_g$is the vector of generators' rotor angles,is the vectorof generators' angular velocities, $\delta_\ell$is the vector of phase anglesof voltages at the load buses,is the vector of voltage magni-tudes, is the inertia matrix, is the damping matrix, $P_g$isthe vector of
$\in \Re^{n+m+k}$ | $\{(x,y,\beta)$ $f(x,y)$ − $\beta_g = 0,$

net real power injections at the generator buses,and finally $P_\ell$and$Q_\ell$are the vectors of net real and reactivepower injections at the load buses, respectively. The differen-tial equation is the swing equation representing generator dy-namics, and algebraic equations are the power flow equations attheloadbuses.Inordertoobtainacompactformof(2.1),let
, $V^T]^T$,β_g[0^T
and ,then,wehave

$= g(x,y)$(2.2)

whereand

Foranetworkconsistingof number of generators n_g andnumberofPQloadbusesthe n_{pq}
P_g parametervectorisintheformofP_g
$= [P_2 \ldots P_{n_g}]^T$
anddenotesnetrealpowerinjectionstothe
$n_g - 1$ number of generators (note that generator bus
#1ischosenastheswingbusofthesystem).Thesetofparam-eters $[P_\ell^T$ $Q_\ell^T]^T$ $P_\ell^T = [P_{n_g+1} \ldots P_{n_g+n_{pq}}]^T$ n_{pq}
denotestheloaddemandsatthenumberofloadbuseswhere and
Q_ℓ are the real and reactive powerdemands, respectively. For the sake of simplicity in the nota-tion,fromthis pointforward,weassumethat $x \in \Re^n, y \in \Re^m$
$\in$ and$\beta$$\Re^k$where$\beta$= $[\beta_g^T\ \beta_\ell^T]^T$
The DAE model of (2.2) has two essential features: 1) ex-plicitparameterdependenceand2)differential-algebraicstruc-ture. The parameter dependence implies that the system equi-libria may exhibit local bifurcations when parameters are sub-jecttovariation.ThesebifurcationsareSN,Hopf,andSI bifur-
cations.TheSIbifurcation,whichisnotobservedfortheODEsmodel of power systems, is due to the algebraic structure. Themain focus of this paper is singularities of DAE model (2.2)including SI bifurcations. In Section II-B, we briefly describethosebifurcationsfocusingon theSIbifurcations.

B. Local Bifurcations and Singularities of the DAE PowerSystemModel

Localbifurcationsoftheequilibriaassociatedwiththechanges of the parameter βhave been observed in the DAEmodel of power systems. Various types of bifurcations andassociatedcomputationalissuesaresummarizedin [5]and[7].The first step to analyze bifurcations is to compute variousequilibriawhentheparameterβ isvaried.Foragivensetof this parameter, an equilibrium point satisfies two sets ofalgebraicequations.Thesetofallequilibriumpointsisdefinedasfollows:

$g(x,y)$0} .(2.3)

The stability of the DAE systems is more complicated than forsystems described by ODEs due to algebraic structure of themodel. The algebraic part of (2.2) requires that any motion beconstrainedtotheset
on.The vectorfieldmay not be well definedatall points of
M .Atanypoint $(x,y) \in M$,wehave ,andif is
$[D_y(g(x,y) - \beta_\ell)]$ $0 = f(x,y) - \beta_g$
nonsingular, then,is uniquely definedby

$\dot{y}$ (2.6)− $\left[\frac{\partial g}{\partial y}\right]^{-1}$ $\left[\frac{\partial g}{\partial x}\right]$ $[f(x,y) - \beta_g]$.

If $[D_y(g(x,y) - \beta_\ell)]$is singular at a point (x,y), then, thevectorfieldisnotwelldefinedatthatpoint.Typically,suchsingularpoints lie on codimension1 submanifoldsof.

Definition [5]:Suppose M is a regular manifold for allnear ,andthat atapoint $[D_y(g(x,y) - \beta_\ell)] \neq 0$ β^*
.Then, is saidtobe causal. $(x,y) = (x^*,y^*) \in M$ (x^*,y^*,β^*)
Otherwise,itisnoncausal.
Thecausalityofapointcouldbeextendedtothecausalityofaregionasfollows[14],[15]:

$C_p(\beta) = \{(x,y) \in M | \det[D_y(g(x,y) - \beta_\ell)] \neq 0;$
$[D_y g(x,y) - \beta_\ell)]|_{(x,y)}$
hasnegativerealeigenvalues} .(2.7)TheregionC_p
iscalledavoltagecausalregionorsolutionsheet
[13] and p is the voltage causal region index. Within any voltagecausal region, load bus voltages and angles follow generatorangles'behavior.Atanycausalpoint (x^*,y^*,β^*) inthere-gion, the implicit function theorem ensures that there exists afunction $y(x,\beta)$ $= \psi(x^*,\beta^*)$ definedonaneighborhoodof withy^*andthatsatisfies $g(x,\psi(x,\beta)) - \beta_\ell =$
.It
follows that within a voltage causal region, trajectories of theDAEarelocallydefinedbytheODEs (2.8)
$\dot{x} = \psi(x,\beta) = f(x,\psi(x,\beta),\beta)$
Typically,inamajorpartoftheconstraintmanifold,suc

hare-duction is possible and the ODEs uniquely define the dynamicbehavior of DAEs. However, the constraint manifold will, ingeneral, contain noncasual points (or singular points) at whichequivalenceisnotpossible.Thesesingularpoints thatlieintheboundary of voltage causal regions form a singular surface (orimpasse surface)intheconstraint manifold[7],[15]

$\{(x,y) \in \Re^{n+m} | g(x,y) - \beta_\ell = 0, \beta =$ constant$\}$ $=$ $\in$
(2.4)

Typically, we expect M to be composed of one or more dis-connected (differentiable) manifolds [20] called components. In general, when we refer to M, we will mean a particular one ofthese components called the principal component. M is a reg-ularmanifoldofdimension m if

$$\operatorname{rank}\left[\frac{\partial g}{\partial x}\,\frac{\partial g}{\partial y}\right] = m \quad \text{on } M. \qquad (2.5)$$

Thestructureof M depends,ofcourse,ontheparameter β .Evenfor very simple power-system models, (2.5) may not be satisfiedfor some values of β. The manifold M is the state space for thedynamicalsystemdefinedby(2.2)whichinducesavectorfield

$$S(\beta) = \left\{(x,y) \in \Re^{n+m} \,\middle|\, \begin{array}{c} g(x,y)-\beta_\ell = 0, \\ \det[D_y(g(x,y)-\beta_\ell)] = 0 \end{array}\right\} \qquad (2.9)$$

Over casual regions, system dynamic behavior evolves ac-cording to a locally equivalent ODE system representation.However, trajectories that encounter the singular surface typ-ically undergo loss of existence/uniqueness. The DAE modelbreaksdownand fails to predictthesystembehavior.
Localbifurcationanalysesofpowersystemsidentifyqualita-tivechangesinsystemequilibriaofODEssystemof(2.8)suchas number of equilibria and their stability features as the pa-rameters are subject to vary slowly; and these bifurcation con-ceptscanbeeasilyextendedtoDAEssystemsof(2.2)[5] .Thestabilityfeatureofanequilibriumpoint (x_0, y_0) and associatedlocal bifurcations are determined by the eigenvalues of the re-ducedsystemmatrixif $[D_y(g(x,y)-\beta_\ell)]$ is nonsingular

$$[A_{\text{sys}}] = D_x f|_0 - D_y f|_0 \left[D_y g|_0\right]^{-1} D_y g|_0. \qquad (2.10)$$

TheSNbifurcationoccurswhenastableequilibriumpoint(SEP), $x_0^s(\beta)$,meetsatype-1unstableequilibriumpoint(UEP),ataparametervalue $\beta = \beta_{\rm SN}$toformanequilibrium point $x_{\rm SN} = x_0^s(\beta_{\rm SN} = x_0^u(\beta_{\rm SN})$ .Thecorrespondingreducedsystemmatrix $[A_{\rm sys}]$ hasasimpleeigenvalueattheorigin,andcertaintransversalityconditionsaremet[21],[22].Ifthepa-rameter βincreasesbeyondthebifurcationvalue $\beta_{\rm SN}$,then,disappearsandtherearenootherequilibriumpointsnearby. Theconsequenceofthelossofequilibriaisthatthesystemstateschangedynamically.Inparticular,dynamicscanbesuchthatthesystemvoltagesfallinavoltagecollapse.TheSNbifurca-tionhasbecomeawidelyacceptedparadigmforoneimportantformofvoltageinstabilityandlinkedtovoltagecollapse[23],[24].Inanappropriateparameterspacesuchasmegawatt(MW)realpowertransfertheSNbifurcationpoint,alsoknownasthemaximumloadingpointorpointofcollapse,providesinforma-tiononthestaticstabilitymarginofthecurrentoperatingpoint.Hopf bifurcationoccurs whena pair ofcomplex conjugateeigenvaluesmovesfromthelefttorighthalfofthecomplexplane,orviceversa,crossingtheimaginaryaxisatpointsotherthantheorigin.TheimportanceofHopfbifurcationhasbeenincreasinglyrecognized,asitbecameclearthatstabilityoftheequilibriumcouldbelostbythismechanismwellbeforereachingthepointof collapsefor thereal largepowersystems.SuchadetailedanalysisofoscillatoryinstabilityrelatedtoHopfbifurcationforthedisturbanceoccurredonJune12,1992,ontheMidwesternsegmentofUSinterconnectionsystemhasbeenreportedin[25]foraDAEmodelofarealpowersystem.ThelastlocalofbifurcationofinterestistheSIbifurcationthatoccurswhenanequilibriumpoint,say encountersthesingularityofthealgebraicequation at theparameter $\beta_{\rm SI}$ .TheSIbifurcationreferstostabilitychangeduetoaneigenvalueofthereducedsystemmatrixassociated withtheequilibriumpointdivergingtoinfinityfromeither ∞ to,orviceversa[7].Similartypeofinstantaneouschangesintheeigenvaluesofreducedsystemmatrixisalsoobservedinthe case of limit induced (LI) bifurcation that occurs when thecontrollimitssuchaslimitonthefieldvoltagearereached[26].However, in the LI bifurcation case, these changes are smallcompared to those of SI bifurcation case. The set of SI bifur-cationsisdefinedasfollows:

$$\left.\begin{array}{l} \dot{x} = f(x_s, y_s) - \beta_{sg} \neq 0 \\ 0 = g(x_s, y_s) - \beta_{s\ell} \\ \det[D_y(g(x_s, y_s) - \beta_{s\ell})] = 0. \end{array}\right\} \quad (2.11)$$

The singularity of $[D_y(g(x,y) - \beta_\ell)]$ (similarly, unboundedeigenvalueof $[A_{\rm sys}]$)impliesthatthesystemwillexperiencesomesortofinstabilityproblemresultingfromfastinteractionsof network variables. However, it is difficult to predict the natureof instability owing to modeling [27] have proposed a singularly perturbed differential equation(SPDE)asthepower-systemmodelandtheirsimulationresultsindicatethatrapiddeclineinbusvoltagemagnitudesmayoccurif trajectories pass close to the singular surface. More recently,Huangetal.[28]hasalsousedSPDEmodeltoanalyzesystembehaviorandthrougheigenvalueanalysistheyhaveshownthatthe SPDE model will have the same dynamic behavior as thereducedODEsifsomeadjustmentsonthesignofthealgebraicequationsaremade.

NotethatintheDAEof(2.2),theparameter β_g isdecoupledfrom the rest of the equations, and at the singular points (notsingualarequilibria) there exists real power mismatches at thegeneratorbuses (i.e., $f(x_s, y_s) - \beta_{sg} \neq 0$)This decoupled-parameter structure allows us to locate SI bifurca-tion point when a singular point, say (x_s, y_s), belonging to thesingular set of (2.9) and the corresponding nonzero real powermismatches at the generator buses are known. The followinglemma,whichexploitsthisdecoupledstructure,showsthatitispossible to find a new set of parameters such that (x_s, y_s) willbeasingularequilibriumpoint.

Lemma:A singular point of (2.2) (x_s, y_s) at a givenparameter value $\beta = \beta_s$is also an equilibrium point, hence, anSIbifurcationpoint,atanotherparametervalue $\beta = \beta^{\rm new}$.

Proof:Suppose that $(x_s, y_s)$is a singular point of the de-coupledDAEof(2.2)attheparametervalue $\beta = [\beta_g^T\ \beta_\ell^T]^T$ suchthat

$$\dot{x} = f(x_s, y_s) - \beta_{sg} \neq 0$$

$$0 = g(x_s, y_s) - \beta_{s\ell}$$

$$\det[D_y(g(x_s, y_s) - \beta_{s\ell})] = 0. \quad (2.12)$$

Observe that since $(x_s, y_s)$is not (in general) an equilibriumpoint, and we have a nonzero mismatch at the generator buses.Letthismismatchbe $f(x_s, y_s) - \beta_{sg}^T = \neq$.In order to force a zero mismatch at the generator buses, we canalwaysdefineanewsetofinjections $\beta_g^{\rm new} = \beta_{sg} + \Delta\beta_{sg}$ atthegeneratrbusessuchthat

$$\dot{x} = f(x_s, y_s) - (\beta_{sg} + \Delta\beta_{sg})$$

$$= f(x_s, y_s) - \beta_g^{\rm new} = 0$$

$$0 = g(x_s, y_s) - \beta_{s\ell}$$

$$\det[D_y(g(x_s, y_s) - \beta_{s\ell})] = 0. \quad (2.13)$$

predict the natureof instability owing to modeling

$(\beta_g^T\ \beta_\ell^T)^T$

limitations. The DAE modelcannot predict the system behavior and the validity of the model,asacharacterizationofthepowersystem,isquestionable.Itis likely that uncertainties, neglected in the DAE model, nowbecome central to the local behavior of the system. In order toavoidalgebraicsingularityproblems,PraprostandLoparo[13](muchearlierDeMarcoandBergen[12]and Arapostathisetal.
Therefore,asingularpoint atthe parameterβ isanequilibriumpoint.Indeed,itisaSIbifurcationpoint atthenewparameter β^{new}whereβ_g^{new}.
This lemma enables us to identify the SI bifurcation pointsoncesingularpointsandthecorrespondingnonzerorealpowermismatch values at the singular points are available. Note thatthe injections at the load buses remain the same. In order to makea singular point an SI bifurcation point we need to adjust onlythe injections at the generator buses, which are the mechanicalinputtogenerators.Thislemmaassumesthatmechanicalinputto the generators are controllable, which is realistic. This as-sumptionalsoindicatesthatwecancontrolthegenerator angles δ_g, which leads us to propose an iterative method to identify sin-gular pointsandthus,theSI bifurcationpointsbytheprevious

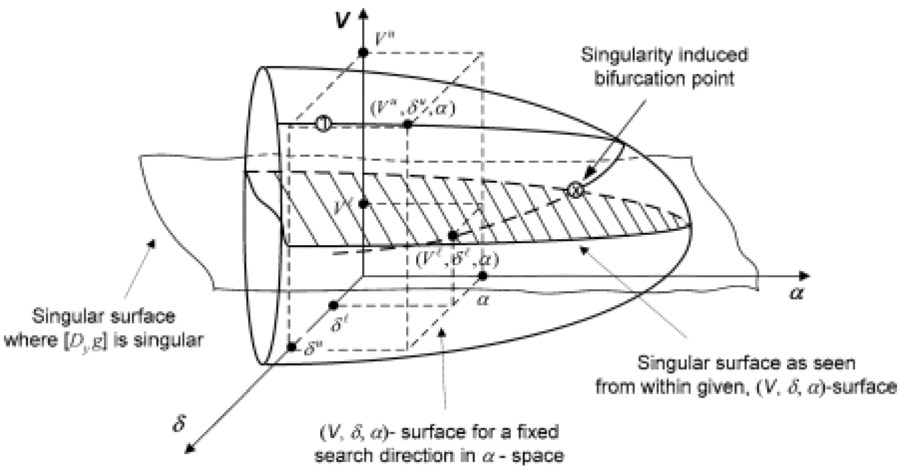

Fig.1.Three-dimensionaldepictionofbifurcationdiagramwithasingularset.

lemma. The application of the lemma and illustration of the con-straint manifold for a 3-machine 5-bus system are presented inSectionII-C.
The main focus of local bifurcation analysis is to determinequalitative changes in the equilibria when the paramaters slowlychange. Recall that the parameter vector βrepresents the bus in-jectionsinthenetwork.Changesinbusinjectionsareachievedthroughparameterizationofbusinjectionswithascalarparam-eterknownasabifurcationparameter βdirectionβ(2.14)
whereβ^0isthebasecasebusinjections, isthescalarbifurca-tion parameter and is the direction $[d_{P_g}^T d_{P_\ell}^T$ vectorintheparameterspace,whichallowsustovarybus injectionsatasinglebusand/orgroupofbuses.Theelementsofdirection$\beta$are

$$d_{P_g} = [\, d_{P_1} \quad \ldots \quad d_{P_{n_g}} \,]^T$$

$$d_{P_\ell} = [\, d_{P_{n_g}} d_{P_{n_g - n_{pq}}} \ldots \,]^T$$

$$d_{Q_\ell} = [\, d_{Q_{n_g+1}} \quad \ldots \quad d_{Q_{n_g+n_{pq}}} \,]^T. \quad (2.15)$$

The elements of d_{P_g}, d_{P_ℓ}, and d_Q can be set to be positive,negative,orzerodependingontheloadincreasescenarioofin-terest. For example, if one wants to increase real power injec-tions into some selected generator buses, and then the corre-sponding elements of $d_{P_g}$are set to be positive. Similarly, inorder to increase real/reactive power demand at some selectedbusesoneneedstosetthecorrespondingentries of$d_{P_\ell}$and d_{Q_ℓ} tobenegative.
The bifurcation diagrams and singular surfaces are multi-di-mensionalevenforrelativelysmallsizedpowersystemsmaking it difficult to visualize them in a multi-dimensionalspace. Therefore, a 2-D or 3-D projection is usually used toillustrate the equilibrium and singular points of (2.2) uponparametervariations.Fig.1illustratesa3-Dequilibriumset(orsurface) with a 2-D singular

surface cutting it. It is worth to statethat there is no reason to expect the singular surface to form asmoothsurfaceasshowninFig.1.However,wedoexp ectittobe a set (not necessarily) connected with a boundary. This 2-Dsingularsurface(shownasplanar)isreallyanapproxi mationofa nonplanar surface that will actually cut the 3-D equilibriumsurface.Notethatanosecurvethatshowsth eevolutionof

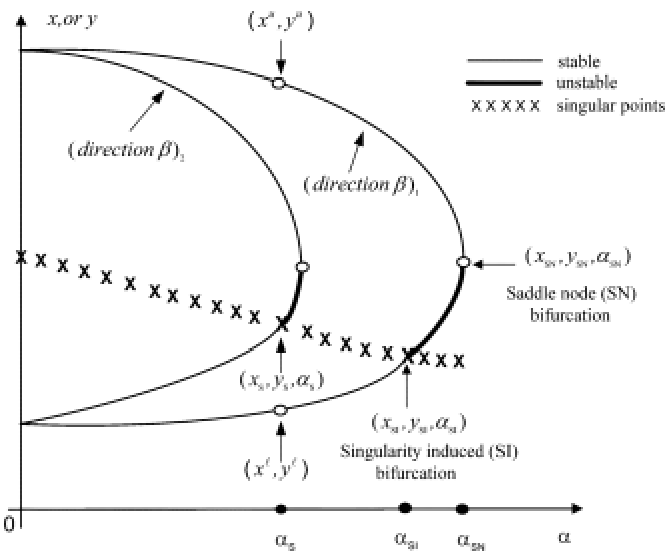

Fig. 2.Illustration of bifurcations of equilibria and singular points in a 2-Dnosecurve.

theequilibria is plotted for a particular load increase pattern.Two equilibrium points of the nose curve at a given parametervaluea are also depicted, namely upper and lower voltagesolutions,andtheyarelabeledas and ,respectively. This nose curve represents the equilibrium set of(2.3) and dashed surface represents the singular surface givenby (2.9). When the nose curve crosses the singular surface, theSIbifurcationoccurs.TheSIbifurcationpointonthe surfaceislabeledby(x) .Itisexpectedthatfordifferentloadincreasepat-terns the nose curve will cross the singular surface at differentpoints indicating other SI bifurcations as can be seen in Fig. 2.Fig. 2shows twonose curves each representinga differentbus injection increase pattern defined by(direction$\beta)_1$and(direction$\beta)_2$ .Notethatalongthenosecurveof(direction$\beta)_1$two local bifurcations, SN, and SI bifurcations and stabilitycharacteristics of the equilibria are illustrated schematically.Note that various singular points are denoted by(x)as thebifurcationparametervaries.

Our main idea here is to depict stability limits of operatingpointsinthepresenceofalgebraicsingularitie s.Thetraditional

α

(V^u, δ^u, α) $(V^\ell, \delta^\ell, \alpha)$

α

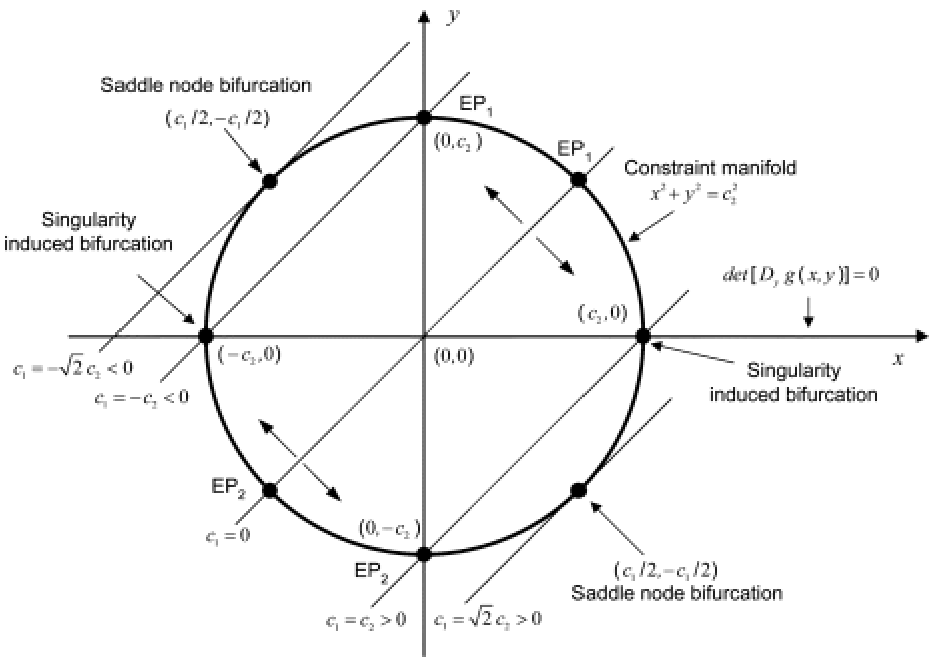

Fig.3.IllustrationofSNandSIbifurcations.

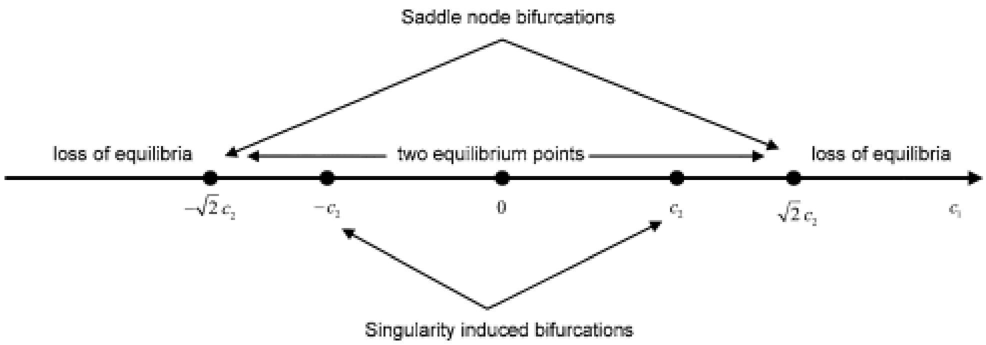

Fig.4. SNandSIbifurcationswithqualitativechangesinthenumberofequilibriumpointsasafunctionofc_1 andc_2.

nosecurves(orPVcurves)areusuallyusedtoindicatest abilitymarginsimposedbyvariouslocalbifurcationsin theparameterspace. We bring singularity information

$$\dot{x} = -x + y + c_1 = f(x, y, c_1)$$

into the nose curve andillustrate changes in both equilibria and singular points as thebifurcation parameterslowly changes. This way of bringingsingularity information gathered from the constraint manifold tothe parameter space gives a visual representation of both staticand dynamic stability boundaries together in the same picture,asshowninFig.2.

$$0 = -c_2^2 = g(x, y)$$

In Section II-C, we provide two illustrative examples of theDAEmodel(oneofwhichisapower-systemexample)inordertoshowthetypesofbifurcatio nsinthesolutionstructureofthesystem equilibria, singularities of the constraint manifold, andtheapplicationofthelemma.

C. TwoIllustrativeExamples

ExampleI:Considerthefollowingparameter-dependentDAE:

$$= x^2 + y^2 \quad (2.16)$$

whereis the dynamic state variable,is the algebraic variableand,andaretheparameters. NotethattheDAEof(2.16)isintheformof(2.2)wherepa-rametersandaredecoupledfromtherestoftheequations.Observethatforanygivenparametervalues,equilibriumpointsaretheintersectionofthetwocurves:1) ,aline,and 2) c_2^2 ,acirclecenteredattheoriginwitharadius .Fig.3showstheconstraintmanifold(i.e.,$x^2 y^2$ c_2^2)andvariousequilibriaaswellastheirbifurcationsdependingontheparameter .For ,therearetwodynamicSEPslabeledas and for in Fig. 3. However, c_1

whenissubjecttovaryineitherpositiveornegativedirectionweobservebifurcationsoftheequilibria.Firstbifurcationoccursattheparameters c_1 c_2 $\pm c_2$ whenoneofthestableequilibriafor increasinginpositivedirectionor for in-creasinginnegativedirection)coincideswiththesingularitiesofthe constraintmanifold. Thisisan SI bifurcation. Observe thattheconstraintmanifoldhassingularitiesat $y = x - c_1$ $x + y =$ c_2 $\{(-c_2, 0); (c_2; 0)\}$ $-c_2 < c_1 < c_2$ forwhichtheJacobianmatrixofthealgebraicequationof(2.16)(i.e., EP_1 EP_2 $c_1 = 0$ c_1 $2y$)hasasimple eigenvalueat theorigin.Furtherincreasein causesoneofthestableequilibriatocrossthesingularsurfaceandtobecomeatype-1UEP.Thesecond (EP_2) c_1 EP_2 c_1 $[D_y g(x, y, c_2)] =$ c_1

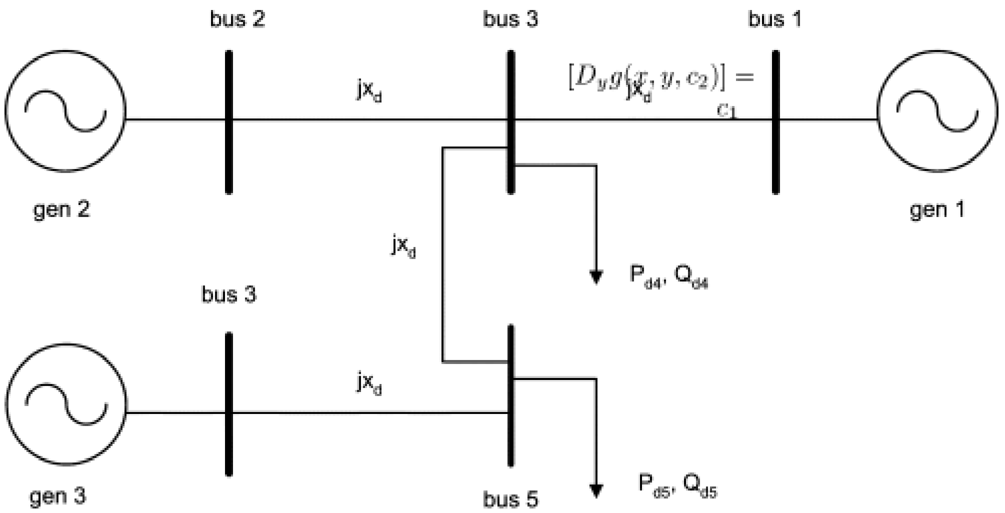

Fig.5.Five-bussystemwiththreegeneratorsandtwoloadbuses.

bifurcationisobservedatparameters $c_1 = \pm\sqrt{2}c_2$ forwhichtwoequilibriumpoints, and meetat ,which is an SN bifurcation EP_1 EP_2 as shown $(c_1/2, -c_1/2)$ in Fig. 3. Finally, beyondtheparameter theDAEof(2.16)doesnothaveanyequilibrium.TheoccurrenceofSNandSIbifurcationswiththe qualitative changes in the number of system equilibria as afunctionofisalsosummarizedinFig.4. $c_1 = \pm\sqrt{2}c_2$

The singular set of (2.16) separates the constraint manifoldinto tworegions thatare connectedthroughthe singularpoints$(-c_2, 0)$and $(c_2, 0)$. These two regions, which are the half cir-clesinFig.3,aredefinedasfollows:

$$C_0 = \left\{(x, y) \mid -c_2 < x < c_2 \text{ and } y = \sqrt{c_2^2 - x^2}\right\} \quad (2.17)$$

$$C_1 = \left\{(x, y) \mid -c_2 < x < c_2 \text{ and } y = -\sqrt{c_2^2 - x^2}\right\}$$

(2.18)

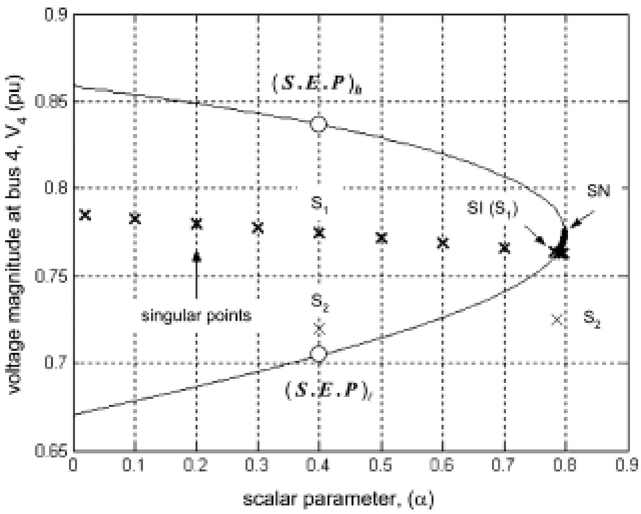

Fig.6.Voltagemagnitudeatbus4(V)andsingularpointsversusparameter

Within the region, C_0or C_1, the DAE example of (2.16) couldbe reduced to the locally equivalent ODEs given below, andthe dynamics of the DAE is uniquely determined by the cor-respondingODEineachregion

$$\dot{x} = -x + \sqrt{c_2^2 - x^2} + c_1 \text{ on } C_0$$

(2.19) $\dot{x} = -x - \sqrt{c_2^2 - x^2} + c_1$ on C_1.

ThisexampleillustratesthatevenasimpleDAEmodel mayex-hibit local bifurcations. In the next example, we study bifurca-tionsandsingularitiesofaDAEmodelofa5-buspowersystemandillustratetheapplicationofthelemmaforlocatingSIbifur-cationpointspresentedinSectionII-B.

Example II:We now present a 5-bus power-system exampleto study bifurcations and demonstrate the application of thelemma for identifying SI bifurcations. The 5-bus system, whoseone-line diagram is shown in Fig. 5, has three generators andtwo constant PQ load buses [13]. The base case bus injectionsinperunit pu)witha100-MWbaseareasfollows:

, $= [-10, -5]^T$ $[P_4 \;\; P_5]^T = -2]^T$ and .

Generators,whichareundampedhavetheinternalvoltages $1.2]^T$ puthatareequaltoterminalvoltages alpha().

sincethereactance $x_d =$ 0.1puincludesthetransientreac-tances of the generator and transmission line. Generator 1 ischosenastheswingbuswithzeroangleandalltheother phaseangles are relativeto the swingbus. In orderto determine aset of equilibrium points including the SI bifurcation, we varymechanicalinputstothegenerators2and3($P_2$and P_3);andreal/reactivepowerdemandatbus4.Theresulting searchdirec-tioninthebusinjectionspaceisasfollows:

$d_{P_g} = [0.50.5]^T$

$d_{P_l} = [0.50]^T$

$d_{Q_l} = [-0.150]^T$.

Fig.6illustrateshowtheequilibriafor thevoltagemagnitudeat bus 4 $[V_4]$ and their corresponding stability characteristicschange with parameter variations. Observe that as the param-eter α varies, the system equilibria undergo SI and SN bifurca-tionslabeledasSI,SN (S_1).Asthebusinjectionsareincreasedthrough the scalar parameter α, both the high-voltage equdilib-rium SEP_h andlow-voltageequilibrium SEP_ℓ are dynamicallystable.However,at $\alpha_{SI} = 0.785$ low-voltageequilibriumpointundergoesastabilityexchange(stableunstable)duetoanSI

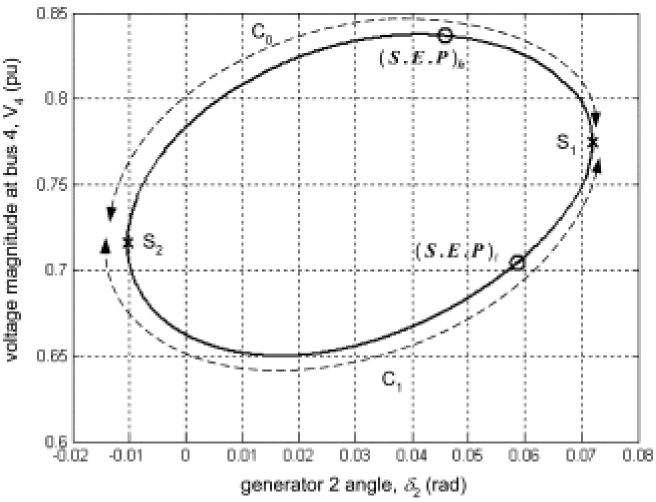

Fig.7.Criticaleigenvaluesofthesystemmatrix A_{sys} astheparameteralpha (α)variesindicatingtheSIandSNbifurcations.

bifurcation and it becomes a type-1 UEP. Further increase in theparametercausesthehigh-andlow-voltageequilibriatomeetatanSNbifurcationfor α. $\alpha_{SN} = 0.8$ TheSNandSIbifurcationsaredetectedbymonitoringtheeigenvalues ofsystemmatrix (2.10)as thesystemmoves fromoneequilibriumpointtoanotherwithchangesinthebifurca-tionparameters α $\alpha = 0.773$ .Fig.7showshowtwocriticaleigenvaluesofthesystemmatrixmoveas α changesfrom toalongthelowerbranchofnosecurve;whichleadto SIandSNbifurcations.Thearrowsindicatethedirectionofin-creaseintheparameter α . JustbeforetheSI bifurcation;say at 0.773 ,thecriticaleigenvalues(pleasenotethatnoncrit-icalonesarenotshowninFig.7)arelocatedinthelefthalf plane,whichimpliesstability.Astheparameterchangesfromto $\alpha_{SI} = 0.785$ oneofthecomplexeigenvaluemoves (in jump fashion) to the right half plane and becomes a largepositivenumberwhiletheothereigenvaluestaysinthelefthalfplane but it becomes a large negative real number. Therefore,stability feature of the equilibria undergoes an instantaneouschange from stable to unstable with exactly one eigenvalue. ThisstabilityexchangeisduetoanSIbifurcationatwhichtheJaco-bianmatrix $[D_y g(x, y)]$ hasasimpleeigenvalueattheoriginandoneoftheeigenvaluesofsystemmatrix $[A_{sys}]$ becomes un-bounded [7]. A clear picture of the occurrence of the SI bifur-cationwithamuchlargerrealeigenvaluecanbeobtainedattheexpense of simulation time [18]. As α increases further, an SNbifurcationoccursat $\alpha_{SN} = 0.8$ andoneofthecriticaleigen-valueof $[A_{sys}]$ becomeszerowhiletheotheroneremainsintheleft half plane. The SN bifurcation corresponds to the point ofmaximumloadingforthisparticularloadincreasepatternoftheDAEsystem.

Fig.6alsoshowssingularpointsatvariousvaluesof α alongthenosecurve,whicharedepictedby (x) andlabeledas S_1 .InSectionIII,wewillpresentamethodtocomputethesesin-gularpoints.Itisworthmentioningherethattherearemultiplesingularpointsatanygivenparameter.However, weare in-terestedinthosethateventuallymeetwithoneoftheequilibrialocated in the lower branch of the nose curve as the parameterissubjecttovaryasillustratedinFig.6.Notethatthesingular

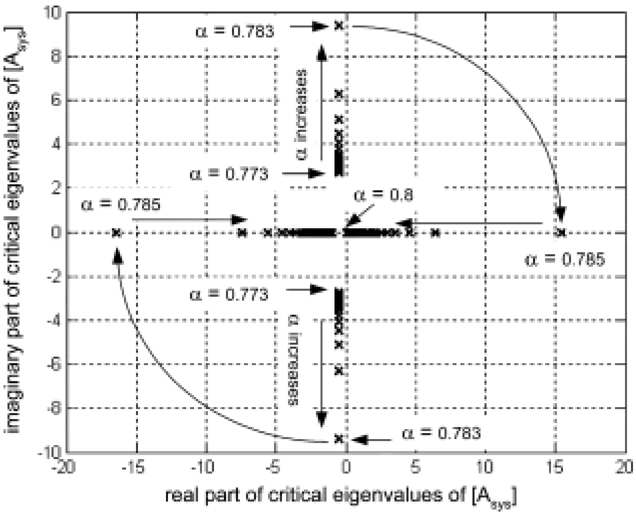

Fig.8.Constraintmanifoldprojectionontothe(V;)-space at =0:4. 4 2

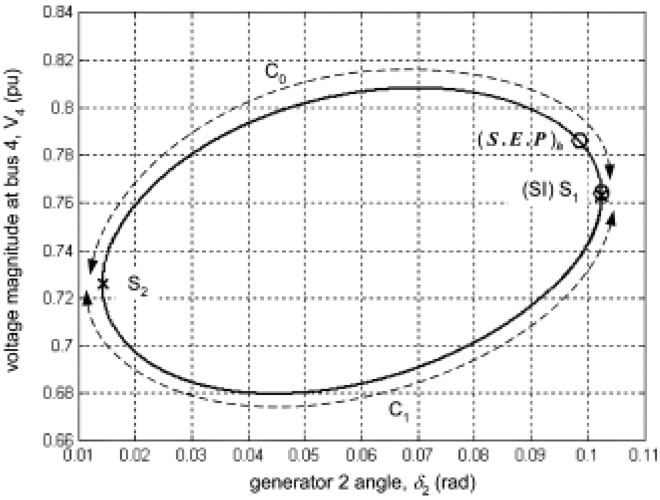

Fig.9.Constraintmanifoldprojectionontothe(V;)-spaceat =0:785 4 2
illustratingtheoccurrenceofaSIbifurcation.

pointS_1at coincideswithlow-voltageequilibriumindicatingaSIbifurcation.InFig.6, wealsodepictanothersin-gularpoint for and astoclearlyshow the relative locations of other singular points that are notassociatedwiththeSIbifurcation.

The relative location of singular points with respect to equi-libria and SI bifurcation point can be clearly seen using 2-Dprojections of the constraint manifold. Fig. 8 shows a 2-D pro-jectionoftheconstraintmanifoldontothe -spacefor .Theconstraintmanifoldconsistsoftwovoltagecausalregions(C_0andC_1)separatedbysingularpointsS_1andS_2 .Note that each voltage causal region contains dynamically SEPslabeledas and .Theseequilibriumpointscorre-spondtohigh-andlow-voltageequilibriumpointsat shown in Fig. 6. Singular points $S_1$and $S_2$are the same onesshowninFig.6at $\alpha_{SI} = 0.785$,andtheyindicatethebifurcationsofthe algebraic variables (i.e., load bus voltage magnitudes and an-gles) when generator angles are considered as parameters. It willbe informative to illustrate the occurrence of the SI bifurcationby using the constraint manifold projection. Fig. 9 shows thesame2-Dprojectionontothe -spacefor .Thistime,however,thelow-voltageequilibriumpoint

(S_1) $\alpha = 0.4$ $\alpha_{SI} = 0.785$

(V_4, δ_2)

$\alpha = 0.4$

SEP_h SEP_ℓ

$\alpha = 0.4$

movesalongtheregion C_1 as $-\beta_{\ell 0}$ increasesfrom 0.785 toandcoincideswiththesingularpoint S_1 while thehighvoltagebone, SEP, staysintheregion C_0. Notethatfor this load increase pattern, both equilibria move toward thesingularpoint S_1 nottoward S_2 alongtheregionsastheparam-eter varies. Therefore, S_2 or any other singular points ratherthan S_1 arenotassociatedwiththeSIbifurcation. We now illustrate the application of the lemma to this power-system example. According to the lemma, recall that we canfind a new set of bus injections such that any of the singularpoints along the nose curve shown in Fig. 6 can be an SI bi-furcation point. For illustrative purposes, we choose the sin-gularpoint S_1 at $= 0.4$. Forthe5-bussystem,thevec-tors of β_g andcontain net power injections at the buses.Specifically, $= [P_g^T \; Q_\ell^T]^T$ where P_g, $[P_2 \ldots]$ and $Q_\ell = [Q_4 \; Q_5]^T$, and .

Attheparameter $=$, thebusinjectionvectors wouldbe pu, $-5]^T$ puand $Q_{s\ell} =$ pu. The corresponding mismatch vector at thesingularpoint is where $\Delta P_{sg} =$ pu. We can adjust theinjectionofthebuses2and3suchthatthesingularpoint S_1 will be an SI bifurcation point. The new injection at the gener-atorbuseswouldbe $\beta_g^{new} = [0^T \; (P_g^{new})^T]^T$ where $P_g^{new} = [5.115 \;\; 5.541]^T$ $P_3^{new}]^T$ pu. Note that injectionsattheloadbuses(buses4and5)remainunchangedandthecor-respondingswingbusinjectionis $P_1 = 5.5$ puobtainedby $-(P_2 + P_3 + P_4 + P_5)$.

Afterhavingillustratedlocalbifurcationsandsingularitiesoftherelativelysimple DAEs,wearenowatastageof presentingmethods for computing them. In Section III, we first brieflysummarizeacommonlyusedalgorithmtocomputeequilibriumpoints and their associated bifurcations, and then we present asearch method for computing singular points at any given pa-rameteralongthenosecurve.

III. IDENTIFICATION OF EQUILIBRIUM AND SINGULAR POINTS

A. IdentificationofSystemEquilibria

In this section, we summarize the method implemented inVST for computing equilibria and their associated static bifur-cationsastheparametervaries.Thestartingpointforthe bifur-cation analysis of the power-system model (2.2) is the identifi-cationofsystemequilibria.Foragivensetofparameters β_0, anequilibriumpoint (x_0, y_0, β_0) satisfiestwoalgebraicequations

$f(x_0, y_0)$ 0 $g(x_0, y_0)$ (3.1)

Load flow analysis is basically the identification of the set ofequilibriumpointsof(3.1).TheVSTimplementsloadflowcal-culations that function up to the point of collapse (SN bifur-cation point). Conventional numerical methods for computingequilibria, such as the NR method, must be modified in orderto obtain reliable results near bifurcation points. Two methodshave been applied to power-system analysis: the continuation(orhomotopy)[29]methodandthedirect(orpointofcollapse)method[30].ThedirectmethodproposedbySeydeltocompute thebranchpointsinsingle-parameternonlinearalgebraicequa-tions has proved remarkably effective in power-system appli-cations. Many investigators have implemented variants of thisapproach, imaginatively tailored to the special features and re-quirementsofpowersystems[31]–[34].Werefertotheseasa group as the NRS method. We describe our implementationof the Seydel's direct method. For convenience, we collect thedependent variablesandinto a single vector which we de-noteby z (i.e., $z = [x^T \; y^T]^T$). Similarly,wecollectthepairoffunctions $f(x, y, \alpha)$ and $g(x, y, \alpha)$ intothesinglefunction $F(z, \alpha) = [f(x, y, \alpha) \; g(x, y, \alpha)]^T$. Notethatthevector of parameters $\beta = [\beta_g^T \; \beta_\ell^T]^T$ is replaced by the scalar bifur-cation parameter α that parameterize β through (2.14). We seektoinvestigatethezerosof F, $z \in \Re^N$ (equilibria) as a functionofthebifurcation parameter $\alpha \in \Re$ where $N = n + m$

$F(z, \alpha)$ (3.2)

ThestandardNRmethodappliedto (3.2)is $= 0$

(3.3)

$[D_z F(z_i, \alpha)] \Delta z = -F(z_i, \alpha)$, $z_{i+1} = z_i + \Delta z$

where $[D_z F]$ istheloadflowJacobianmatrix. However, the NR method breaks down near (static) bifurca-tion points,i.e., when $[D_z F]$ issingular($< N$). Ingenericone-parameterfamiliesthedimensionof atabifurcationpointispreciselyone,i.e., 1

. Thus,tolocatesuchapointweseekvaluesfor ,

andnontrivial v or $w \in \Re^N$ whichsatisfy

$F(z, \alpha)$ (3.4a)

$= 0$, or $[D_z F(z_i, \alpha)] v = 0$ $w^T [D_z F(z_i, \alpha)] = 0.$ (3.4b)

Therequirement fornontrivialityof v w, maybestatedby

$\|v\| = 1$, or $\|w\| = 1$ (3.4c)

Onebasicapproachtofindingbifurcationpointsistoapply the NRmethodto(3.4).ThisistheNRSmethod.Data thatsatisfies(3.4)willbedenotedas z_b, α_b, v_b, w_b andwedesignatetheJacobian $J_b = [D_z F(z_b, \alpha_b)]$. Notethatthevectors

,havespecialsignificance.Theyare,respectively,therightandlefteigenvectorscorrespondingtothezeroeigenvalueof J_b.Theeigenvector spansthekernelof J_b and spansthekernelof J_b^T. Once a bifurcation point is located, it is feasible to modifytheabovemethodtocomputepointsaroundthefold(nose)oftheequilibriumsurface

(3.5)
forvaluesof $[-\varepsilon_1\varepsilon_2]$with .

This allows computation of equilibrium points close to the bi-furcation point where the conventional NR calculations $\lambda = 0$ wouldfail.Ofcourse correspondstotheSNbifurcationpoint.The above method can be effective but it has the disadvantagethatissignificantlymorecomputationallyintensivethanastan-dardloadflow.Itinvolvessolving $2N+1$ N equationsasop- F posedto,anditrequirescomputingsecond- $[D_zF(z,\alpha)E(\lambda,F)] \equiv 0$ orderderivativesof.However,itpossibletodevisepotentiallymoreefficient $\varepsilon_1, \varepsilon_2 > 0$

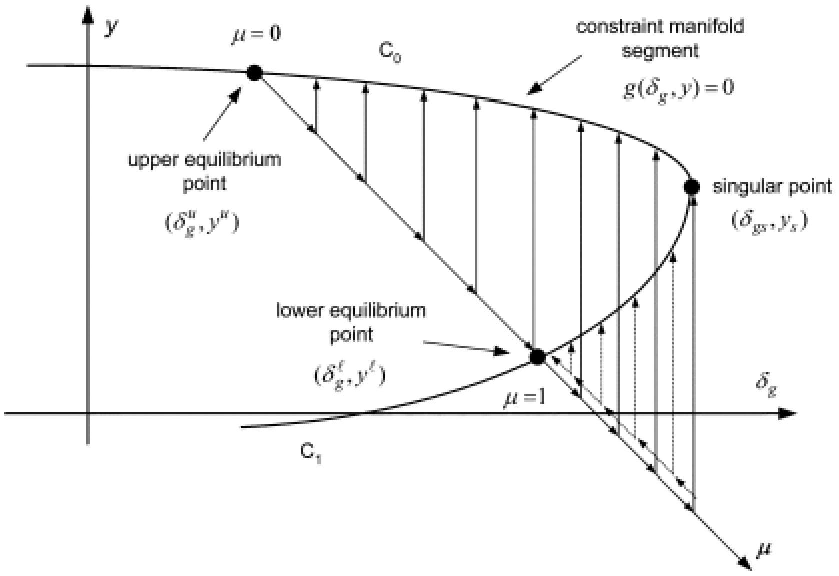

Fig.10.Graphicalillustrationofthemethodforcomputingsingularpoints.

methodsthatexploitthefactthat(3.4b)and(3.4c)arelinearinand[33].
w
In VST, governing equations of the classical model andJacobian matrix including the second derivatives have beenconstructedsymbolicallyandathree-stageloadflowmethod,
$\rightarrow$, NR $\rightarrow$ NRS NR has been implemented to computeequilibria and bifurcations. First, the standard NR method isused until it fails to converge. Then, it automatically switchesto the NRS method to find load flow solutions at and aroundSN bifurcation point. After passing through the SN bifurcationpoint, the standard NR method is switched back to computelow-voltagesolutions.

B. IdentificationofSingularPoints

In this section, we present an algorithm to compute singularpoints of the DAE model of (2.2) at any given parameter valuealong the nose curve. The method is an iterative technique thatcombines well-known NR and NRS methods, which are com-monlyusedtocomputeSNbifurcationsinpowersystemsasex-plainedinSectionIII-A.Theproposedalgorithmbenefitsfromthe knowledge of the system equilibria and the occurrence ofthe SI bifurcation. Generator angles are parameterized througha scalar parameter in the constraint manifold. Then, at any givenparametervalue,theidentificationofasingularpointisformu-lated as a bifurcation problem of a set of algebraic equationswhoseparametersarethegeneratorangles.In thefollowing,weexplainwhyweparameterizegeneratoranglesandhowthispa-rameterizationisachieved.

1) ParameterizationofGeneratorAngles:Recall thattheal-gebraic part of the DAE model of (2.2) represents the real andreactivepowerequationsatthePQloadbuses
$g(\delta_g, y)$(3.6)
Atafixedparametervalue,theconstraintmanifoldconsistsofaset of points (δ_g, y)satisfying (3.6). As explained and illustratedin Section II (see Figs. 8 and 9 of Example II) the constraintmanifoldcontainsvoltagecausalregionsand singularpoints

connecting them. Fig. 10 hypothetically illustrates a magnifiedsegmentoftheconstraintmanifoldcomposedoftwovoltagecausalregions, C_0andC_1 ,andasingularpoint (δ_{gs}, y_s).Notethat theregionC_0 contains the upper equilibrium pointwhiletheregion C_1containsthelowerequilibrium point . These equilibria correspond to the high- andlow-voltage solutions at a given parameter value along thenose curve (see Fig. 6 for an example), which are known to usfrom the computaion of equilibria and bifurcations explainedinSectionIII-A.

Observe that for any given generator angle $\delta_g$there are twocorresponding solutions for the algebraic variablethat repre-sents the load bus voltage magnitude and phase angle. As thegenerator angle increases, these two solutions move along theregions C_0andC_1until theymeetat thesingularpoint .

Atthesingularpoint,theJacobianmatrix becomes singular and there is no solution forifis futherincreased. This observation indicates that algebraic variablesshow a nose curve type of behavior as the generator angles vary,and they undergo an SN bifurcation at the singular point. ThisbehaviorissimilartotheSNbifurcationoftheequilibriaasthebus injections change. This observation leads us to use gener-ator angles as parameters and to seek methods to compute theSN bifurcations of algebraic varibles, which is a singular pointoftheDAEmodel.

A recent work by Singh and Hiskens[16] on the characteriza-tion of the stability boundary of the DAE model has illustratedthefactthatsingularsurfaceslieontheboundaryofthestabilityregion of a SEP and they contain infinitely many singular points.However, as illustrated in Figs. 6–9 of Section II, we are onlyinterestedincomputingthosesingularpointsthateventuallyin-tersect with an SI bifurcation point for a given bus injection pat-tern.Specifically,wealsoassumethatwespecifyaprioriwhichinjections will changeto create othersingular points.

In order to trace the corresponding segment of the manifoldandtocomputethesingularpointshowninFig. 10weneedtoimplementaniterativemethodthatinitiatesatapointinC_0 andendsupatanotherpointinC_1 passingthroughthesingularpoint (δ_{gs}, y_s). The upper and lower equilibrium points are theobvious choice for the starting and ending points of the algo-rithm since they are available to us from the equilibriacompu-tation. The following parameterization of the generator angleswillachievethatpurpose:

$$\left[(1-\mu)\delta_g^u + \mu\delta_g^\ell\right] \quad \delta_g \qquad (3.7)$$

where and $\delta^\ell$are $(n1)$-dimensional vectors representingthe generator angles at the upper and lower equilibrium pointsat a given parameter value [or equivalentlyby (2.14)], re-spectively,andisanewscalarbifurcation parameter. With this parameterization, the identification of the singularpoint of the constraint manifold at a fixed parameterreducesto a single parameter bifurcation problem of the following equa-tion:

$$g(y, \mu) = 0 \qquad (3.8)$$

Note that we drop the parameter $\beta_\ell$in (3.8) for the sake ofsimplicity in the notation. Clearly, the SN bifurcation of thealgebraic variablesas the bifurcation parameterchangeswould be a singular point of the constraint manifold at the cor-respondingparameter.InSectionIII-B-2,wedescribeatwo-staged algorithm that implements the NR and NRS methods tolocate the singular points. A similar method has also been re-portedin[16]tocomputethesingularpointontheimpassesur-face that has the minimum potential energy as to characterizethe stability boundary for the case when the boundary does notcontain any unstable equilibria and/orperiodicorbits.

2) Combined NR and NRS Method:As we have explainedinSectionIII-B-I,asingularpointoftheDAEmodelatagivenparameter is a static bifurcation point of the load bus voltagemagnitudeandphaseangleswhenthegeneratoranglesaresub-jecttovary.Thus,theproblemofcomputingasingularpointis equivalent to identification of the SN bifurcation of the al-gebraic (3.8) as the scalar parametervaries [thus,changesthrough(3.7)].Therefore,weseekasingularpoint (y, μ) in theconstraintmanifoldsuchthat $\text{rank}[D_y(g(y,\mu)-g(\ell))] = m-1$.

Inotherwords,thesingularpointsmustbelongtotheconstraintmanifold and the Jacobian matrix must have a simple eigenvalueattheorigin.Wecanrewritetheseconditions asfollows:

$$g(y, \mu) = 0 \qquad (3.9)$$

$$[D_y g(y, \mu)] v_y = 0 \qquad (3.10)$$

$$\|v_y\|_2 = 1 \qquad (3.11)$$

where $y \in \mathbb{R}^m$ isthealgebraicvariables(loadbusvoltagemag-nitudeandphaseangles), $[D_y g(y, \mu)] \in \mathbb{R}^{m\times m}$ istheJaco-bianmatrixofthealgebraicequations, $v_y \in \mathbb{R}^m$ istherighteigenvector corresponding to the zero eigenvalue of the Jaco-bian matrix, and

$\Re^1$is the bifurcation parameter used tovary the generator angles. Observe that (3.11) assures that theeigenvectoris nontrivial. Equation (3.10) together with
(3.11)establishesthesingularityofJacobianmatrix.
The conventional NR method is the most common iterativetechnique to compute the roots of nonlinear algebraic
equations.Thismethodcanbeappliedto(3.8)asfollows:

(3.12)

Theabove iterativeschemeworkswellalmosteverypointintheconstraintmanifold.However,itwillfailtoconvergearoundasingularpointsincetheJacobianmatrixisclosetothesingularity.TheNRSmethodhasbeeneffectivelyusedtocompute static bifurcation points in power systems. In order toapplytheNRSmethodto(3.9)–(3.11),arealeigenvalue ofisintroducedasan independentvariable. Thatwill
make it possible to implement an iterative scheme that goesaroundthesingularpoint

$h_2 = [D_y g(y,\mu) - \lambda I] v_y = 0$
$h_3 \|v_y\|_2 - 1$ (3.13)
Thereareatotalof
in(3.13)andthesamenumberofunknownvariableswhileistheindependentvariable.Foragiven
,(3.13)canbesolvedfortheunknowns

$\hat{z} = [\, y \quad v_y$

$\hat{z}_i$(3.14)

where $h_2^T h_3^T\,]^T$and is the corre-spondingextendedJacobianmatrixof(3.13).
TheNRSalgorithm,likeanyotherNewton-iterativemethod,needs a good initial condition, that is a point in the constraintmanifold close enough to the singular point along with thesmallestrealeigenvalueof
andthecorrespondingright eigenvector. Otherwise, we may experience conver-gence problems. Therefore, we first use the NR method. TheNR computations proceed starting at the upper equilibriumpoint$0)$
alongtheconstraintmanifolduntilitfailstocon-verge.
The last successful NR data point is used to implementan inverse iteration method [35] for estimating the eigenvalueof nearest , and the corresponding righteigenvector. These data are then used to initiate an NRSprocedure using (3.14) to compute around the singular pointforvaluesof $[-\varepsilon_1, \varepsilon_2]$with .Thevalue isalwaysincludedanddataatthesingularpointisthereby

obtained.
In order to compute singular points at various parametersalong the nose curve and depict them together with the equi-libria in a 2-D nose curve, the following procedure, which isalsographicallyillustratedinFig.11,isimplementedinVST.
Step 1) Choose a load (bus injections) increase pattern.Step2)Computeequilibriumpoints(nosecurve),thesta-
bilitypropertiesoftheequilibriaandlocatelocalbifurcations.Step3)Chooseaparameter alongthe nosecurve and fixit.
Step4)Computethesingularpointatthisparameter.
Step 5)Repeat the steps 3–4 as many times as desired up totheparameter .
Step6)Depict singular points in the nose curve.

$[D_y g(y,\mu)]$

$h_1 = g(y,\mu) = 0$

$= \qquad = 0.$

$(2m+1)$
λ

λ
$\mu]^T$

$[D_{\hat{z}} H(\hat{z}_i)]\Delta\hat{z} = -H(\hat{z}_i), \quad \hat{z}_{i+1} = \quad + \Delta\hat{z}$

$H = [\,h_1^T$ $[D_{\hat{z}} H(\hat{z})]$

$[D_y g(y,\mu)]$
v_y

$(\mu =$

$[D_y g(y,\mu)]$ $\lambda = 0$
v_y

$\lambda \in$ $\varepsilon_1, \varepsilon_2 > 0$

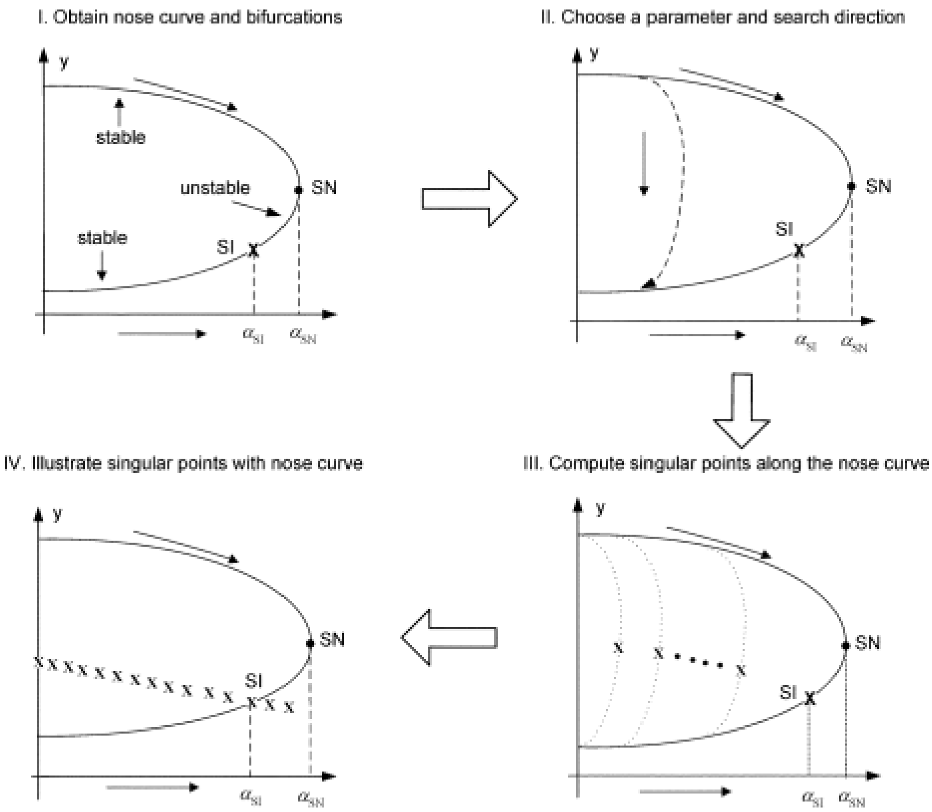

Fig. 11.Graphicalillustrationof the procedurefor computing singularpoints anddepictingthemalongthe nose curve.

InSectionIV,weillustratetheapplicationofourmethodthatincludesthestepsgivenabovetotheIEEE118-bussystem.

IV. SIMULATIONRESULTS

In this section, we present results on the SI bifurcation andsingular points for the IEEE 118-bus test system. The real andreactive powers at bus 75 have been increased according to(2.14). Fig. 12 illustrates how the voltage magnitude at bus 75changes with parameter variations. As can be seen, two kindsof bifurcations are identified, namely SN and SI bifurcations.As the parameter increases both upper and lower parts ofthenosecurvearedynamicallystable.At $\alpha_{SI} = 7.92$,thesystem undergoes a stability exchange associated with the SIbifurcation and the stability feature of the lower equilibriumpoints changes qualitatively, from stable to unstable. As theparameter further increases, one stable (upper part) and onetype-1 unstable (lower part) equilibrium point meet at an SNbifurcationpointfor $\alpha_{SN} = 8.86$,whichisthetipofthenosecurve. Beyond the SN bifurcation point, there is no feasiblesolution to the load flow equations. The stability properties ofthelowerpartofthenosecurvearecertainlymodeldependent.When the load dynamics are included the entire lower part ofthenosecurvemightbeunstable. Fig.12alsodepictssingularpointsatdifferentparametervalues.Theparametervalue $\alpha_{SI} = 7.92$ isespeciallyimpor-tantinthesensethatitenablesustocheckwhetherthesin-gularpointsearchmethodgivescorrectresults.RecallthatattheSIbifurcationoccursandallthestateinfor-mationatthisparametervalueisavailabletousfromequilibria

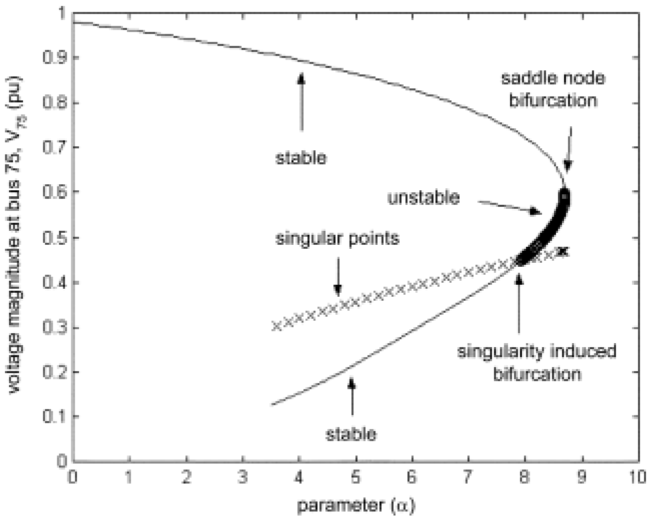

Fig. 12.Voltage magnitude at bus 75 versus parameter alpha () with singularpointswhenthe realandreactive poweratbus75increase.

and bifurcation analysis. The SI bifurcation point also belongsto the singular set defined by (2.9). It is expected that the pro-posedmethodshouldgivethesameresultasthatofthebifurca-tionanalysisat $\alpha_{SI} = 7.92$ .AsseenfromFig.12,thisisindeedthe case. Observe that for the voltage magnitude at bus 75 sin-gularpointsateachparametervalueliesbetweenthehigherandlower voltage solutions until the SI occurs. Note that the lowerpartofthenosecurvemaynotbepracticaloperatingpointsduetothelow-voltageprofile.However,thisisnotthegeneralcaseasshowninFig.13thatdepictsthevoltagemagnitudeatbus 63

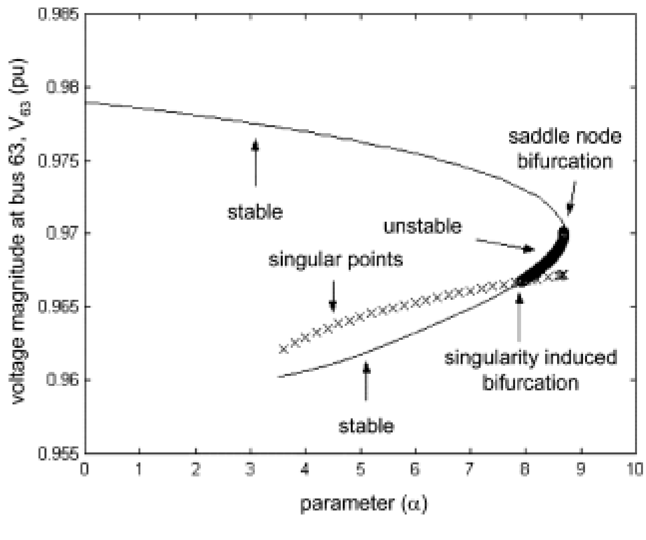

Fig. 13.Voltage magnitude at bus 63 versus parameter alpha () with singularpointswhentherealandreactivepoweratbus75increase.

for the same load increase pattern. The lower part of the nosecurve and singular points including the SI bifurcation point lieabove 0.95 pu, which is usually considered to be the low-voltagethreshholdvalueforanormaloperationofthepowersystems.

V. CONCLUSION AND FUTURE WORK

In this paper, we have presented an

iterative method to lo-cate and identify singular points of the DAE model of powersystem.Inthemethod,weusegeneratoranglesto parameterizethe algebraic part of the DAE model and we identify the sin-gularpointsasbeingtheSNbifurcationofthealgebraicpartofthe DAE model. We have shown by a lemma that any singularpoint at a given set of bus injections is an SI bifurcation pointatanothersetofbusinjections.Wehavecombined staticinfor-mationfromtheSNanddynamicinformationfromthesingularpoint together in order to provide a comprehensive picture of thesystem stability. We have updated the VST to include singularpoint computations. Simulation results on a 5-bus system andthe IEEE 118-bus system have been presented. We have illus-trated singular points with the traditional nose curve for differentloadchangescenarios.

As future work, an energy function approach should be im-plemented in order to provide a dynamic security index thatconsidersthesingularpoints.Specifically,thedynamicstabilitymarginofagivenoperatingpointcouldbecomputedastheen-ergydifferencebetweenthecurrentoperatingpointandthesin-gularpoint.Thisscalarenergyvaluewouldbethe dynamicse-curityindex.Moreover,inordertocompletethepicture astaticsecurity index as being the energy difference between the cur-rent operating point and the point of collapse should be com-binedwiththedynamicsecurityindex.

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