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A Fixed Point Theorem fora Self-Map On b-Metric Space with New Contraction

Ch. Srinivasa Rao[1], KuradaChinni Naga Tarani[2*] and Nistala V. E. S. Murthy[3]
[1,2,3] *Department of Mathematics, Andhra University, Visakhapatnam-530003, India.*

Abstract.
In this paper we study some properties of self-maps on b-metric spaces. These results are improvements of NagaralPanditSanatammappa, R. Krishnakumar and K. Dinesh [20]. An example is also provided in support of our result.

Keywords*b*-metric space, Fixed point, Cauchy sequence, Convergence.
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I. Introduction and Preliminaries

Building upon Bakthin's groundwork, Czerwik further expanded the theory of b-metric spaces in 1993 [11]. This extension paved the way for subsequent research endeavors aimed at exploring fixed-point existence within the broader context of b-metric spaces. Notable contributions in this direction include works by researchers such as Bota [8], Boriceanu [7], Kapil Jain and JatinderdeepKaur [17], Mehmet Kir [18], and P˘acurar [23].

Inspired by the advancements in b-metric space theory, this paper aims to extend the results established by NagaralPanditSanatammappa, R. Krishnakumar, and K. Dinesh [20]. The extension presented herein contributes to the ongoing discourse on fixed-point existence in b-metric spaces, enriching the understanding of the underlying mathematical structures. Additionally, to illustrate the applicability of our theorem, we provide a pertinent example that lends empirical support to our theoretical findings.

This research not only builds upon the rich legacy of the Banach contraction theorem but also underscores the versatility and efficacy of the *b*-metric space framework in addressing fixed-point problems. Through our contributions, we aim to further enhance the theoretical foundations of fixed-point theory and inspire future investigations in this evolving mathematical discipline.

Expanding on the significance of our extension, it is essential to underscore the broader implications for mathematical applications and problem-solving. The study of fixed points in b-metric spaces not only contributes to the theoretical foundation of mathematics but also finds practical relevance in diverse fields such as computer science, economics, and engineering. The ability to establish the existence of fixed points in these generalized spaces can have profound consequences for the development of algorithms, optimization techniques, and dynamic systems analysis. As such, our extension aims to provide not only theoretical advancements but also to enrich the toolkit available to researchers and practitioners addressing real-world challenges.

Furthermore, the continual evolution of fixed point theory in the realm of b-metric spaces reflects the dynamic nature of mathematical research. The interdisciplinary nature of this field encourages collaborations and the exchange of ideas, leading to novel concepts and innovative methodologies. By building upon the foundations laid by prominent researchers and contributing to this ongoing discourse, our work seeks to foster a deeper understanding of the underlying principles governing fixed point phenomena in b-metric spaces. This collaborative spirit in mathematical research not only enhances the depth of our theoretical knowledge but also cultivates a rich intellectual environment that propels the field forward.

The generalizations of the contraction principle in different directions as well as many new fixed point results with applications have been established by different researchers ([1] − [6], [9], [10], [12] − [16], [21] − [29]).

Notation: Throughout this paper N stands for the set of positive integers.

Definition 1.1.([11],[17]) Let Θ be a nonempty set. Then, a mapping $\Upsilon: \Theta \times \Theta \rightarrow [0, +\infty)$ is called a b-metric if there exists a number $s \geq 1$ such that for all $\theta, \psi, \xi \in \Theta$,

(1) $\Upsilon(\theta, \psi) = 0$if and only if$\theta = \psi$

(2) $\Upsilon(\theta, \psi) = \Upsilon(\psi, \theta)$

(3) $\Upsilon(\theta, \psi) \leq s\big(\Upsilon(\theta, \xi) + \Upsilon(\xi, \theta)\big)$.

Then the pair(Θ, Υ) is called a b-metric space with index$s$.

Clearly, every metric space is a b-metric space with$s = 1$.

Definition 1.2.[7] Let (Θ, Υ) be a b-metric space with index s. Then a sequence $\{\theta_n\}$ in Θ is called a convergent sequence if there exists $\theta \in \Theta$ such that$\Upsilon(\theta_n, \theta) \rightarrow 0 \; as \; n \rightarrow \infty$. That is if there exists $n(\epsilon) \in N$ such that$\forall \; n \geq n(\epsilon)$we have $\Upsilon(\theta_n, \theta) < \epsilon$. In this case we write $lim_{n\rightarrow\infty} \theta_n = \theta$.

Definition 1.3. [7] Let (Θ, Υ)be a b-metric space with index s. Then a sequence $\{\theta_n\}$ in Θ is called a Cauchy sequence if $\Upsilon(\theta_n, \theta_m) \rightarrow 0 \; as \; n \rightarrow \infty$. That is given $\epsilon > 0$ there exists $n(\epsilon) \in N$ such that for all$n, m \geq n(\epsilon)$ we have $\Upsilon(\theta_n, \theta_m) < \epsilon$.

Definition 1.4. [7] A b-metric space is complete if every Cauchy sequence is convergent.

Observation:

1. Limit of sequence $\{\theta_n\}$ in a b-metric space (Θ, Υ), if exists, is unique. That is $\theta_n \rightarrow \theta$, $\theta_n \rightarrow \psi \Rightarrow \theta = \psi$.
2. If$\theta_n \rightarrow \theta$and$\psi \in \Theta$. Then $\Upsilon(\theta_n, \psi) \rightarrow \Upsilon(\theta, \psi) \; as \; n \rightarrow \infty$.

2. Main result

Theorem 2.1. Let (Θ, Υ) be a complete b-metric space with index $s \geq 1$ and let $T : \Theta \to \Theta$ be a mapping. Suppose $\exists\ \tau \in [0, 1)$ with $\tau s < \frac{1}{2}$ such that,

(2.1)$\Upsilon(T\theta, T\psi) \leq \tau\, max\{\Upsilon(\theta, \psi), \Upsilon(\theta, T\theta), \Upsilon(\psi, T\psi), \Upsilon(\theta, T\psi), \Upsilon(\psi, T\theta)\} \forall \theta, \psi \in \Theta.$

Then T has a unique fixed point.

(We say that T is a generalized contraction if 2.1 is satisfied)

Proof. Let $\theta_0 \in \Theta$ and define the sequence $\{\theta_n\}_{n=1}^{\infty}$ by$\theta_1 = T\theta_0$and

$\theta_n = T\theta_{n-1} = T^n\theta_0,\ \forall n \geq 1.$

Now,$\Upsilon(\theta_n, \theta_{n+1}) = \Upsilon(T\theta_{n-1}, T\theta_n)$

$$\leq \tau\, max \begin{Bmatrix} \Upsilon(\theta_{n-1}, \theta_n), \Upsilon(\theta_{n-1}, T\theta_{n-1}), \Upsilon(\theta_n, T\theta_n), \\ \Upsilon(\theta_{n-1}, T\theta_n), \Upsilon(\theta_n, T\theta_{n-1}) \end{Bmatrix}$$

$$= \tau\, max \begin{Bmatrix} \Upsilon(\theta_{n-1}, \theta_n), \Upsilon(\theta_{n-1}, \theta_n), \Upsilon(\theta_n, \theta_{n+1}), \\ \Upsilon(\theta_{n-1}, \theta_{n+1}), \Upsilon(\theta_n, \theta_n) \end{Bmatrix}$$

$$\leq \tau\, max \begin{Bmatrix} \Upsilon(\theta_{n-1}, \theta_n), \Upsilon(\theta_n, \theta_{n+1}), \\ s[\Upsilon(\theta_{n-1}, \theta_n) + \Upsilon(\theta_n, \theta_{n+1})] \end{Bmatrix}.$$

$\therefore\ \Upsilon(\theta_n, \theta_{n+1}) \leq \tau s[\Upsilon(\theta_{n-1}, \theta_n) + \Upsilon(\theta_n, \theta_{n+1})].$

$\therefore\ (1 - ks)\Upsilon(\theta_n, \theta_{n+1}) \leq \tau s\, \Upsilon(\theta_{n-1}, \theta_n).$

$\therefore\ \Upsilon(\theta_n, \theta_{n+1}) \leq \left(\frac{\tau s}{1 - \tau s}\right) \Upsilon(\theta_{n-1}, \theta_n)$

(2.2)$\Upsilon(\theta_n, \theta_{n+1}) \leq \Lambda\, \Upsilon(\theta_{n-1}, \theta_n), where\ \Lambda = \frac{\tau s}{1 - \tau s}.$

(2.3) $\therefore\ \Upsilon(\theta_n, \theta_{n+1}) \leq \Lambda\, \Upsilon(\theta_{n-1}, \theta_n).$

By induction,

$\Upsilon(\theta_n, \theta_{n+1}) \leq \Lambda^n \Upsilon(\theta_0, \theta_1).$

Now, we show that $\{\theta_n\}_{n=1}^{\infty}$ is a cauchy sequence in Θ.

By RaduMiculescu and AlexandruMihail ([19], Lemma 2.2),

$\therefore\{\theta_n\}_{n=1}^{\infty}$ is a Cauchy sequence in Θ.

Since (Θ, Υ) is complete, $\{\theta_n\}_{n=1}^{\infty}$ converges to θ^* (say) in Θ.

Now we show that θ^* is a fixed point of T.

$$\Upsilon(\theta_{n+1}, T\theta^*) = \Upsilon(T\theta_n, T\theta^*)$$
$$\leq \tau\, max\{\Upsilon(\theta_n, \theta^*), \Upsilon(\theta_n, T\theta_n), \Upsilon(\theta^*, T\theta^*), \Upsilon(\theta_n, T\theta^*), \Upsilon(\theta^*, T\theta_n)\}$$
$$= \tau\, max\{\Upsilon(\theta_n, \theta^*), \Upsilon(\theta_n, \theta_{n+1}), \Upsilon(\theta^*, T\theta^*), \Upsilon(\theta_n, T\theta^*), \Upsilon(\theta^*, \theta_{n+1})\}.$$

On letting $n \to \infty$, and using observation (2), we get

$$\Upsilon(\theta^*, T\theta^*) \leq \tau\, \Upsilon(\theta^*, T\theta^*).$$

Since $0 \leq \tau < 1$, follows that $\Upsilon(\theta^*, T\theta^*) = 0$.

$\therefore\ \theta^* = T\theta^*$.

Hence, $\theta^*$is a fixed point of T.

Now we show that θ^* is the unique fixed point of T.

Let θ' be a fixed point of$T$ so that $T\theta' = \theta'$.

Then, $\Upsilon(\theta^*, \theta') = \Upsilon(T\theta^*, T\theta')$

$$\leq \tau\, max\{\Upsilon(\theta^*, \theta'), \Upsilon(\theta^*, T\theta^*), \Upsilon(\theta', T\theta'), \Upsilon(\theta^*, T\theta'), \Upsilon(\theta', T\theta^*)\}$$
$$= \tau\, max\{\Upsilon(\theta^*, \theta'), \Upsilon(\theta^*, \theta^*), \Upsilon(\theta^*, \theta^*), \Upsilon(\theta^*, \theta'), \Upsilon(\theta', \theta^*)\}.$$

$$\therefore\ \Upsilon(\theta^*, \theta') \leq \tau\, \Upsilon(\theta^*, \theta').$$

$\therefore\ (1-\tau)\Upsilon(\theta^*, \theta') \leq 0.$

$\therefore\ \Upsilon(\theta^*, \theta') = 0.$

$\therefore \theta^* = \theta'.$

Hence, θ^* is the unique fixed point of T.

The following example is a supporting example to the above theorem.

Example 2.1.Let$\Theta = [0,1]$. Define the b-metric on Θ as$\Upsilon(\theta,\psi) = (\theta-\psi)^2$. Then (Θ, Υ) is a b-metric space with index$s = 2$. Now define and $T: \Theta \to \Theta$ by

$$T\theta = \begin{cases} 0 \ \ if\ \theta \in [0,1] - \left\{\frac{1}{2}\right\} \\ \ \ \frac{1}{16} \quad if\ \theta = \frac{1}{2} \end{cases}$$

Then T satisfies 2.1 with $\tau = \frac{1}{8}$and$\tau s = \frac{1}{4}$. Cleraly,0 is the unique fixed point of$T$.

Now we have the following corollaries to our theorem-2.1.

Corollary 2.1.: Let (Θ, Υ)be a complete metric space and let $T: \Theta \to \Theta$ be a mapping. Suppose$\upsilon \in \left[0, \frac{1}{2}\right)$ such that

$$\Upsilon(T\theta, T\psi) \leq \upsilon\, max\{\Upsilon(\theta,\psi), \Upsilon(\theta, T\theta), \Upsilon(\psi, T\psi), \Upsilon(\theta, T\psi), \Upsilon(\psi, T\theta)\}$$

Then Thas a unique fixed point.

Proof: Let $\theta_0 \in \Theta$ and define the sequence $\{\theta_n\}_{n=1}^{\infty}$ by $\theta_1 = T\theta_0$ and

$\theta_n = T\theta_{n-1} = T^n\theta_0,\ \forall n \geqslant 1.$

Now, $\Upsilon(\theta_n, \theta_{n+1}) = \Upsilon(T\theta_{n-1}, T\theta_n)$

$$\leq \upsilon\, max \begin{Bmatrix} \Upsilon(\theta_{n-1}, \theta_n), \Upsilon(\theta_{n-1}, T\theta_{n-1}), \Upsilon(\theta_n, T\theta_n), \\ \Upsilon(\theta_{n-1}, T\theta_n), \Upsilon(\theta_n, T\theta_{n-1}) \end{Bmatrix}$$

$$= \upsilon\, max \begin{Bmatrix} \Upsilon(\theta_{n-1}, \theta_n), \Upsilon(\theta_{n-1}, \theta_n), \Upsilon(\theta_n, \theta_{n+1}), \\ \Upsilon(\theta_{n-1}, \theta_{n+1}), \Upsilon(\theta_n, \theta_n) \end{Bmatrix}$$

$$\leq \upsilon\, max\{\Upsilon(\theta_{n-1}, \theta_n), \Upsilon(\theta_n, \theta_{n+1}), [\Upsilon(\theta_{n-1}, \theta_n) + \Upsilon(\theta_n, \theta_{n+1})]\}$$

$$\leq \upsilon\big(\Upsilon(\theta_{n-1}, \theta_n) + \Upsilon(\theta_n, \theta_{n+1})\big)$$

$$(1-\upsilon)\Upsilon(\theta_n, \theta_{n+1}) \leq \upsilon\Upsilon(\theta_{n-1}, \theta_n)$$

$$\Upsilon(\theta_n, \theta_{n+1}) \leq \left(\frac{\upsilon}{1-\upsilon}\right)\Upsilon(\theta_{n-1}, \theta_n)$$

$$\Upsilon(\theta_n, \theta_{n+1}) \leq \Omega\, \Upsilon(\theta_{n-1}, \theta_n), \qquad where\ \Omega = \frac{\upsilon}{1-\upsilon}$$

$\therefore\ \Upsilon(\theta_n, \theta_{n+1}) \leq \Omega\, \Upsilon(\theta_{n-1}, \theta_n)$

By induction, $\Upsilon(\theta_n, \theta_{n+1}) \leq \Omega^n \Upsilon(\theta_0, \theta_1)$

Now we show that $\{\theta_n\}_{n=1}^{\infty}$is a Cauchy sequence in Θ.

$$\Upsilon(\theta_n, \theta_{n+k}) \leq \Upsilon(\theta_n, \theta_{n+1}) + \Upsilon(\theta_{n+1}, \theta_{n+2}) + \ldots. + \Upsilon(\theta_{n+k-1}, \theta_{n+k})$$

$$\leq \Omega^n \Upsilon(\theta_0, \theta_1) + \Omega^{n+1}\Upsilon(\theta_0, \theta_1) + \ldots + \Omega^{n+k-1}\Upsilon(\theta_0, \theta_1)$$

$$\leq \Omega^n \Upsilon(\theta_0, \theta_1)[1 + \Omega + \Omega^2 + \cdots + \Omega^{k-1}]$$

$$\leq \frac{\Omega^n(1-\Omega^{k+1})}{1-\Omega}\Upsilon(\theta_0, \theta_1)$$

$\therefore\ \Upsilon(\theta_n, \theta_{n+k}) \leq \frac{\Omega^n(1-\Omega^{k+1})}{1-\Omega}\Upsilon(\theta_0, \theta_1) \to 0\ as\ n \to \infty$

$\therefore \{\theta_n\}$is a Cauchy sequence in Θ.

Since (Θ, Υ) is complete, $\{\theta_n\}_{n=1}^{\infty}$ converges to θ^* (say) in Θ.

$i.e., lim_{n\to\infty} \theta_n = \theta^*$.

Now we show that θ^* is a fixed point of$T$.

$$\Upsilon(\theta^*, T\theta^*) \leq \Upsilon(\theta^*, \theta_{n+1}) + \Upsilon(\theta_{n+1}, T\theta^*)$$

$= \Upsilon(\theta^*, \theta_{n+1}) + \Upsilon(T\theta_n, T\theta^*)$

$\leq \Upsilon(\theta^*, \theta_{n+1}) + \upsilon\ max\begin{Bmatrix}\Upsilon(\theta_n, \theta^*), \Upsilon(\theta_n, \theta_{n+1}), \Upsilon(\theta^*, T\theta^*), \\ \Upsilon(\theta_n, T\theta^*), \Upsilon(\theta^*, \theta_{n+1})\end{Bmatrix}$

On letting $n \to \infty$

$\Upsilon(\theta^*, T\theta^*) \leq \upsilon\ max\{\Upsilon(\theta^*, T\theta^*), \Upsilon(\theta^*, T\theta^*)\}$

$\Upsilon(\theta^*, T\theta^*) \leq \upsilon\ \Upsilon(\theta^*, T\theta^*)$

$(1-\upsilon)\Upsilon(\theta^*, T\theta^*) \leq 0$

$\Rightarrow \Upsilon(\theta^*, T\theta^*) = 0$

$\therefore\ \theta^* = T\theta^*$

$\therefore\ \theta^*$is a fixed point of$T$.

Now we show that θ^* is the unique fixed point of T.

Let $\theta^{'}$ be a fixed point of Tso that $T\theta^{'} = \theta\,'$.

Suppose $\theta^* \neq \theta^{'}$

Then, $\Upsilon(\theta^*, \theta^{'}) = \Upsilon(T\theta^*, T\theta^{'})$

$$\leq \upsilon\ max\{\Upsilon(\theta^*, \theta^{'}), \Upsilon(\theta^*, T\theta^*), \Upsilon(\theta^{'}, T\theta^{'}), \Upsilon(\theta^*, T\theta^{'}), \Upsilon(\theta^{'}, T\theta^*)\}$$

$$= \upsilon\ max\{\Upsilon(\theta^*, \theta^{'}), \Upsilon(\theta^*, \theta^*), \Upsilon(\theta^{'}, \theta^{'}), d(\theta^*, \theta^{'}), \Upsilon(\theta^{'}, \theta^*)\}$$

$\Upsilon(\theta^*, \theta^{'}) \leq \upsilon\ \Upsilon(\theta^*, \theta^{'})$

$(1-\upsilon)\Upsilon(\theta^*, \theta^{'}) \leq 0$

$\therefore\ \Upsilon(\theta^*, \theta^{'}) = 0$

$\Rightarrow \theta^* = \theta^{'}$.

$\therefore\ \theta^*$is the one and only fixed point of T.

Corollary 2.2.**[20]** Let (Θ, Υ) be a complete b-metric space with index $s \geq 1$. Define the sequence $\{\theta_n\}_{n=1}^{\infty} \subset \Theta$ by the iteration $\theta_n = T\theta_{n-1} = T^n\theta_0$where $T: \Theta \to \Theta$ is a mapping and $\theta_0 \in \Theta$. Suppose

$$(2.4)\Upsilon(T\theta, T\psi) \leq \alpha_1\Upsilon(\theta, \psi) + \alpha_2[\Upsilon(\theta, T\theta) + \Upsilon(\psi, T\psi)] + \alpha_3[\Upsilon(\theta, T\psi) + \Upsilon(\psi, T\theta)]$$

Where $\alpha_1, \alpha_2, \alpha_3 \geq 0$and $\alpha_1 + 2\alpha_2 + 2\alpha_3 < \frac{1}{2s}, \forall\ \theta, \psi \in \Theta$. Then there exists $\theta^* \in \Theta$ such that $\theta_n \to \theta^*$ is a unique fixed point of$T$.

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II. Conclusion:

In summary, our paper explores the properties of self-maps on b-metric spaces, building upon the work of

NagaralPanditSanatammappa, R. Krishnakumar, and K. Dinesh [20]. We present improvements and offer insights supported by an illustrative example. These findings contribute to a deeper understanding of b-metric spaces and self-maps, advancing knowledge in this field.

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