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Pseudo and Absolutely Pseudo Irregular Hesitancy Fuzzy Graphs

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Abstract:
We are talked about the idea of Pseudo and absolutely Pseudo irregular HFG, highly& absolutely highly Pseudo irregular HFG, neighbourly& absolutely neighbourly Pseudo irregular HFG and their properties dependent on their Pseudo degree.

Keywords: Hesitancy Fuzzy Graph (HFG), Pseudo irregular Hesitancy Fuzzy Graph, Absolutely Pseudo irregular Hesitancy Fuzzy Graph, Highly Pseudo irregular Hesitancy Fuzzy Graph, Absolutely highly Pseudo irregular Hesitancy Fuzzy Graph, neighbourly Pseudo irregular Hesitancy Fuzzy Graph, Absolutely neighbourly irregular Hesitancy Fuzzy Graph.

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I. INTRODUCTION

In 1965, thought of fuzzy subsets of a set presented by L.A.Zadeh[2] as a method of addressing uncertainty and vagueness.Kaffmann created the principal meaning of fuzzy graph in 1973.In 1975, Azriel Rosenfeld[4] presented the idea of fuzzy graphs. A.NagoorGani and S.R.Latha [5] presented the idea of irregular fuzzygraphs and talked about the portion of its properties. T.Pathinathan and J.JonArockiaraj [8] they have analyzed a portion of the properties of HFG presented the meaning of Hesitancy Fuzzy Graph. Here we are characterized the Pseudo and absolutely Pseudo irregular HFG, highly and absolutely highly pseudo irregular HFG and furthermore talked about the properties of neighbourly and absolutely neighbourly Pseudo irregular HFG.

II. BASIC DEFINITIONS

Definition: 2.1

A Hesitancy Fuzzy Graphs is of the structure$\mathcal{H} = (M, N)$, where $M = \{x_1, x_2, \ldots, x_n\}$such an extent that$\mu_1: M \to [0,1]$,$\gamma_1: M \to [0,1]and\beta_1: M \to [0,1]$ indicate that level of enrollment, non-participation and aversion of the vertex $x_i \in M$separately and $\mu_1(x_i) + \gamma_1(x_i) + \beta_1(x_i) = 1$ for every $x_i \in M$, where $\beta_1(x_i) = 1 - \mu_1(x_i) + \gamma_1(x_i)$and $N \subseteq M \times M$ where $\mu_2: M \times M \to [0,1]$,$\gamma_2: M \times M \to [0,1]and\beta_2: M \times M \to [0,1]$ are an extent that, $\mu_2(x_i, x_j) \le \min[\mu_1(x_i), \mu_1(x_j)]$,$\gamma_2(x_i, x_j) \le max[\gamma_1(x_i), \gamma_1(x_j)]$ and $\beta_2(x_i, x_j) \le \min[\beta_1(x_i), \beta_1(x_j)]$ and $0 \le \mu_2(x_i, x_j) + \gamma_2(x_i, x_j) + \beta_2(x_i, x_j) \le 1$ for each $(x_i, x_j) \in N$

Definition: 2.2
Hesitancy fuzzy graph $\mathcal{H} = (M, N)$ is supposed to be irregular if each vertex of $\mathcal{H}$ have distinct degree.

Definition: 2.3

Let $\mathcal{H} = (M, N)$be an Hesitancy Fuzzy Graph on $\mathcal{H}^*$(M, N). Then, at that point d_2 - degree of a vertex $x \in M$ is characterized by$d_2(x) = \left(d_{2\mu_1}(x), d_{2\gamma_1}(x), d_{2\beta_1}(x)\right)$, where $d_{2\mu_1}(x) = \sum d_{\mu_1}(y)$, $d_{2\gamma_1}(x) = \sum d_{\gamma_1}(y)$ and $d_{2\beta_1}(x) = \sum d_{\beta_1}(y)$, the vertex $x$is adjoining the vertex y.

Definition: 2.4 Let$\mathcal{H} = (M, N)$ be a Hesitancy Fuzzy Graph on$\mathcal{H}^*(M, N)$. Then, at that point Pseudo level of a vertex x in Hesitancy Fuzzy Graph $\mathcal{H}$ is characterized by$d_{pa}(x) = (d_{p\mu_1}(x), d_{p\gamma_1}(x), d_{p\beta_1}(x))$, where $d_{p\mu_1}(x) = \frac{d_{2\mu_1}(x)}{d_{\mathcal{G}}^*(x)}$, $d_{p\gamma_1}(x) = \frac{d_{2\gamma_1}(x)}{d_{\mathcal{G}}^*(x)}$and $d_{p\beta_1}(x) =$

$\frac{d_{2\beta_1}(x)}{d^*_{\mathcal{G}}(x)}$.where $d^*_{\mathcal{G}}(x)$is the quantity of edges occurrence at x.

III. PSEUDO AND TOTALLY PSEUDO IRREGULAR HESITANCY FUZZY GRAPHS

Definition: 3.1

Let $\mathcal{H}(M,N)$ be a Hesitancy Fuzzy Graph on $\mathcal{H}^*(M,N)$. Then, at that point,$\mathcal{H}$ is said to be a Pseudo irregular Hesitancy Fuzzy Graph if there exist a vertex which is adjacent to the vertices with distinct pseudo degrees.

Definition: 3.2

Let $\mathcal{H}(M,N)$ be a Hesitancy Fuzzy Graph on $\mathcal{H}^*(M,N)$. Then, at that point,$\mathcal{H}$ is said to be a Pseudo absolutely irregular Hesitancy Fuzzy Graph if there exist a vertex, which is adjacent to the vertices with distinct absolute pseudo degrees.

Example: 3.3

Consider the Hesitancy Fuzzy Graph $\mathcal{H}(M,N)$

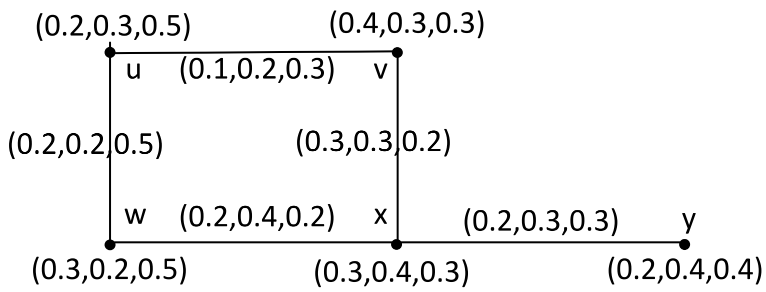

Figure- 1 Pseudo &absolutely Pseudo irregular Hesitancy Fuzzy Graph$\mathcal{H}$

<u>Pseudo degrees:</u>d_{pa}(x) =(0.4,0.55,0.6), d_{pa}(y) = (0.5,0.7,0.75), $d_{pa}(z)$ = (0.5,0.7,0.75), d_{pa}(u) = (0.33,0.47,0.5) and d_{pa}(v) = (0.2,0.3,0.3).

<u>Absolute Pseudo degrees:</u>td_{pa}(x) = (0.6,0.85,1.1), td_{pa}(y) = (0.9,1.0,1.05), td_{pa}(z) = (0.8,0.9,1.25), td_{pa}(u) = (0.63,0.87,0.8) and td_{pa}(v) = (0.4,0.7,0.7).

Proposition:3.4

A pseudo irregular Hesitancy Fuzzy Graph need not be a Pseudo absolutely irregular Hesitancy Fuzzy Graph.

Proof: Consider the $\mathcal{H}(M,N)$ Hesitancy Fuzzy Graph on $\mathcal{H}^*(M,N)$.

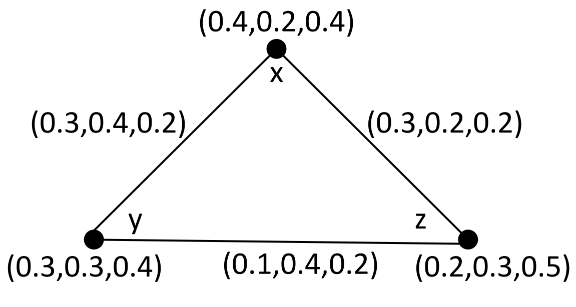

Figure – 2 Pseudo and not absolutely Pseudo irregular Hesitancy Fuzzy Graph$\mathcal{H}$

Pseudo degrees:d_{pa}(x) =(0.4,0.7,0.4), d_{pa}(y) = (0.5,0.6,0.4) and $d_{pa}(z)$ = (0.5,0.7,0.4). Here, a vertex adjacent to the vertices with have distinct Pseudo degrees. Hence $\mathcal{H}$ is Pseudo irregular Hesitancy Fuzzy Graph.

Absolute Pseudo degrees:td_{pa}(x) = (0.8,0.9,10.8), td_{pa}(y) = (0.8,0.9,0.8) andtd_{pa}(z) = (0.7,1.0,0.9). Here, a vertex $z$is adjacent to the vertices$x$ and $y$are have the same absolute Pseudo degree.Hence $\mathcal{H}$ is not Pseudo absolutely irregular Hesitancy Fuzzy Graph.

Proposition:3.5

A Pseudo absolutely irregular Hesitancy Fuzzy Graph $\mathcal{H}$need not be a Pseudo irregular Hesitancy Fuzzy Graph.

Theorem: 3.6

Let $\mathcal{H}(M,N)$ be a Hesitancy Fuzzy Graph on $\mathcal{H}^*(M,N)$. In the event that$M(x)$ is a steady capacity, Then the following condition are same. (i) $\mathcal{H}$ is a pseudo irregular Hesitancy Fuzzy Graph . (ii) $\mathcal{H}$ is a Pseudo absolutely irregular Hesitancy Fuzzy Graph.

Proof : Let $M(x) = (\mu_1(x), \gamma_1(x), \beta_1(x))$be a steady capacity.Let $M(x) = (f_1, f_2, f_3)$, for all$x \in M$. Assume$\mathcal{H}$ is a pseudo irregular Hesitancy Fuzzy Graph. Then, at that point,no less than one vertex of$\mathcal{H}$ which is adjacent to the vertices with distinct pseudo degree. Let p_1 andp_2 be the adjacent vertices of p_3 with distinct pseudo degrees (x_1, x_2, x_3) and (y_1, y_2, y_3)respectively. Then, at that point$(x_1, x_2, x_3) \neq (y_1, y_2, y_3)$. Suppose $\mathcal{H}$is not a Pseudo irregular Hesitancy Fuzzy Graph.Then, at that point, each vertex of $\mathbb{G}$ which is adjacent to the vertices with same pseudo total degree $td_{pa}(v_1) = td_{pa}(v_2)$,$\Longrightarrow$ (x_1, x_2, x_3)+ (f_1, f_2, f_3)=(y_1, y_2, y_3)+ (f_1, f_2, f_3),$\Longrightarrow$ (x_1, x_2, x_3)=(y_1, y_2, y_3).Which is a contradiction to $(x_1, x_2, x_3) \neq (y_1, y_2, y_3)$. Subsequently$\mathcal{H}$ is a Pseudoirregular Hesitancy Fuzzy Graph. Hence$(i) \Longrightarrow (ii)$ is demonstrated. Presently, Suppose $\mathcal{H}$ is a Pseudo irregular Hesitancy Fuzzy Graph. Then, at that point, at least one vertex of $\mathcal{H}$ which is adjacent to the vertices with the distinct pseudo total degree.Let p_1 and p_2 be the adjacent vertices ofp_3 with distinct pseudo degrees$(z_1, z_2, z_3)$and(w_1, w_2, w_3)separately. Presently,$td_{pa}(p_1) \neq td_{pa}(p_2), \Longrightarrow d_{pa}(p_1) + M(p_1) \neq d_{pa}(p_2) + M(p_2), \Longrightarrow (z_1, z_2, z_3) \neq (w_1, w_2, w_3)$. Thus$\mathcal{H}$ is a pseudo irregular Hesitancy Fuzzy Graph. Thus $(ii) \Longrightarrow (i)$ is demonstrated. Hence (i) and (ii) are Equivalent .

IV. HIGHLY PSEUDO IRREGULAR HESITANCY FUZZY GRAPH

Definition: 4.1

Let $\mathcal{H}(M,N)$ be an Hesitancy Fuzzy Graph on $\mathcal{H}^*(M,N)$. Then, at that point,$\mathcal{H}$ is said to be a Highly pseudo irregular Hesitancy Fuzzy Graph if each vertex of $\mathcal{H}$ is adjacent to the verticeswith distinct pseudo degrees .

Definition:4.2

Let $\mathcal{H}(M,N)$ be an Hesitancy Fuzzy Graph on $\mathcal{H}^*(M,N)$. Then, at that point,$\mathcal{H}$ is said to be a Highly pseudo totally irregular Hesitancy Fuzzy Graph if every vertex of $\mathcal{H}$ is adjacent to the vertices with distinct total pseudo degrees .

Example: 4.3

Consider an Hesitancy Fuzzy Graph $\mathcal{H}(M,N)$ on $\mathcal{H}^*(M,N)$.

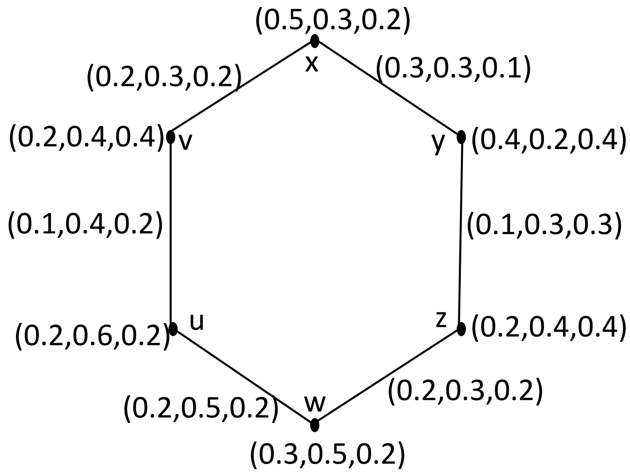

Figure-3: Highly & Highly absolutely Pseudo irregular HFG

Pseudo degrees: d_{pa}(x) =(0.35,0.65,0.4), d_{pa}(y) = (0.4,0.6,0.4), $d_{pa}(z)$ = (0.4,0.7,0.4),d_{pa}(w) = (0.3,0.75,0.45) ,d_{pa}(u) = (0.35,0.75,0.4) and d_{pa}(v) = (0.4,0.75,0.35).

Absolute Pseudo degrees: td_{pa}(x) = (0.85,0.95,0.6), td_{pa}(y) = (0.8,0.8,0.8), td_{pa}(z) = (0.6,1.1,0.8),td_{pa}(w) = (0.6,1.25,0.65), td_{pa}(u) = (0.55,1.35,0.6) and td_{pa}(v) = (0.6,1.15,0.75).

Theorem: 4.4

Let $\mathcal{H}(M,N)$ be an Hesitancy Fuzzy Graph on $\mathcal{H}^*(M,N)$. In the event that$M(x)$is a steady capacity, then the following conditions are same. $(i)\mathcal{H}$ is a highly pseudo irregular Hesitancy Fuzzy Graph. (ii) $\mathcal{H}$is a highly pseudo absolutely irregular Hesitancy Fuzzy Graph .

Proof: Assume that $M(x) = (\mu_1(x), \gamma_1(x), \beta_1(x))$ is a steady capacity. Let $M(x) = (f_1, f_2, f_3)$, for all $x \in M$. Assume$\mathcal{H}$ is an highly pseudo irregular Hesitancy Fuzzy Graph. Then, at that point, every vertex of $\mathcal{H}$ which is adjacent to the vertices with distinct pseudo degree. Let v_1 and v_2 be the adjacent vertices of v_3 with distinct pseudo degrees (a_1, a_2, a_3) and (b_1, b_2, b_3)separately. Then, at that point$(a_1, a_2, a_3) \neq (b_1, b_2, b_3)$. Assume $\mathcal{H}$ is not a highly Pseudo absolutely irregular Hesitancy Fuzzy Graph.Then each vertex of $\mathcal{H}$ which is adjacent to the vertices with same pseudo total degree $td_{pa}(v_1) = td_{pa}(v_2)$,$\Longrightarrow$ $d_{pa}(v_1) + M(v_1) = d_{pa}(v_2) + M(v_2)$, $\Longrightarrow$ (a_1, a_2, a_3)=(b_1, b_2, b_3). Which is a logical inconsistency to (a_1, a_2, a_3)≠ (b_1, b_2, b_3). Hence $\mathcal{H}$ is anhighly Pseudo totally irregular Hesitancy Fuzzy Graph. In this manner $(i) \Longrightarrow (ii)$ is demonstrated. Presently, Suppose $\mathcal{H}$ is a highly Pseudo absolutely irregular Hesitancy Fuzzy Graph. Then, at that point, every vertex of $\mathcal{H}$ which is adjacent to the vertices with the distinct pseudo total degree.Let v_1 and v_2 be the adjacent vertices of v_3 with distinct pseudo degrees (p_1, p_2, p_3) and (q_1, q_2, q_3) separately . Presently, $td_{pa}(v_1) \neq td_{pa}(v_2)$,$\Longrightarrow (p_1, p_2, p_3)$≠ (q_1, q_2, q_3).Hence $\mathcal{H}$ is a highly pseudo irregular Hesitancy Fuzzy Graph. Hence $(ii) \Longrightarrow (i)$ is demonstrated. Subsequently (i) and (ii) are Equivalent.

V.NEIGHBOURLY PSEUDO IRREGULAR HESITANCY FUZZY GRAPH

Definition: 5.1

Let $\mathcal{H}(M,N)$ be a Hesitancy Fuzzy Graph on $\mathcal{H}^*(M,N)$. Then, at that point,$\mathcal{H}$ is said to be a neighbourly pseudo irregular Hesitancy Fuzzy Graph if every two adjacent vertices of $\mathcal{H}$ have distinct pseudo degrees .

Definition: 5.2

Let $\mathcal{H}(M,N)$ be an Hesitancy Fuzzy Graph on $\mathcal{H}^*(M,N)$. Then, at that point,$\mathcal{H}$ is said to be a Neighbourly pseudo absolutely irregular Hesitancy Fuzzy Graph if every two adjacent vertices of $\mathcal{H}$ have distinct total pseudo degrees .

Example: 5.3 Consider an Hesitancy Fuzzy Graph $\mathcal{H}(M,N)$ on $\mathcal{H}^*(M,N)$.

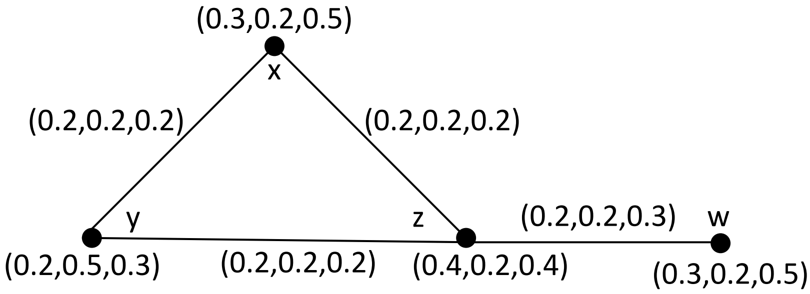

Figure- 4: Neighbourly &Neighbourly absolutelyPseudo irregular HFG

<u>Pseudo degrees:</u>d_{pa}(x) =(0.8,0.8,0.55), d_{pa}(y) = (0.75,0.85,0.55), $d_{pa}(z)$ = (0.5,0.57,0.37)and $d_{pa}(w)$ = (0.9,0.9,0.7).Here, every two adjacent vertex of $\mathcal{H}$have distinct Pseudo degrees. Hence $\mathcal{H}$ is neighbourly Pseudo irregular Hesitancy Fuzzy Graph.

<u>Absolute Pseudo degrees:</u>td_{pa}(x) = (1.1,1.2,0.85),td_{pa}(y) = (1.15,1.25,0.75), td_{pa}(z) = (1.0,0.77,0.67) andtd_{pa}(w) = (1.2,1.1,1.2). Here , every two adjacent vertices of $\mathcal{H}$have distinct absolute Pseudo degree.Hence $\mathcal{H}$ is neighbourly Pseudo absolutely irregular Hesitancy Fuzzy Graph.

VI. CONCLUSION

We have talked about the idea of highly Pseudo and absolutely highly Pseudo irregular in Hesitancy Fuzzy Graph and outlined a few models. Additionally examined the connection among neighbourly and absolutely neighbourly irregular HFG.

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