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The use of fuzzy models in the synthesis problem of soft systems

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Abstract
The paper solves the problem of synthesis in soft systems. The main difficulty in solving the synthesis problem of such systems is that small changes in the initial data can lead to large changes in the result. To represent the initial data, method of the fuzzy gradation proposed by the author is used. The paper shows an example of its advantages over traditional methods. To improve the solution, we use the genetic algorithm and genetic operations as mechanism for generating new solutions. The best solution is determined by the distance method or a convolution of criteria, the choice of which depends on the purposes and limitations of external systems. Calculations are based on the rules of fuzzy arithmetic. Comparison of the proposed algorithm with the traditional approach based on the generalized Zadeh principle shows its advantages. The application of this method does not depend on the specific numerical context, since it uses an order scale. The proposed approach also allows us to select quickly acceptable solutions. Synthesis of treatment methods is considered as an example of the proposed approach.

Keywords and phrases: system theory, synthesis problem, fuzzy arithmetic, genetic algorithm

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I. Introduction

The synthesis problem belongs to the optimization problems according to many criteria. In the general case, it does not allow constructive formalization, decomposition and may not have a unique solution. Therefore, to obtain a unique solution, various simplifications are used. The most developed are the synthesis methods of technical systems, as well as control systems in closed systems described by differential equations [1, 7, 10]. The theoretical foundations of the synthesis of control systems were formulated in [5]. A fuzzy approach to control problems is considered in [8].

Methods for solving the problem of synthesis of soft systems should take into account the features of these systems. Soft systems are open, for them there are not adequate formal models. In addition, in the synthesis of such systems, we have to deal with incomplete, fuzzy information about the system and the conditions for its functioning.

There is a trend towards complication of methods for solving problems in soft systems, in particular, the use of genetic algorithms, artificial intelligence methods [4, 6, 9], neural networks [2, 13, 14]. However, the complication of methods does not lead to an increase in the accuracy and reliability of the results, since these methods are based on the Zadeh generalization principle and the definition of the membership function.

The purpose of this article is to use fuzzy arithmetic in solving the problem of synthesis of soft systems, when the initial data are ill conditioned, i.e. when small data changes have a big influence on the result.

Fuzzy gradations method and rules of fuzzy arithmetic considered by the author in [11 – 13] are used to represent the initial data and perform calculations in the synthesis of such systems. The proposed approach has advantages in solving of problems in soft systems. An important advantage is its independence from the specific numerical context, since it uses a scale of order. This approach also greatly facilitates informal procedures for analyzing a set of acceptable solutions and obtaining the best solution in problems of synthesis of soft systems. To improve the solution, we use the genetic algorithm and genetic operations as mechanism for generating new solutions.

II. Algorithm of synthesis

We formulate the problem as follows. The population (constructor) contains n alternatives (variants), each of which is evaluated according to m criteria. We assume that the criteria are preliminarily ranked by importance. The most

important are functional criteria (group 1), followed by economic criteria (group 2), ergonomic (group 3) and special (group 4). Thus, each element is represented by an ordered set of criteria (indicators) k_1, k_2 ... k_n. This set may contain gaps (zero positions), i.e. not necessarily all criteria are used to evaluate all elements. We assume that the weight of each alternative is determined by the ratio of the number of criteria with the maximum value to the total number of criteria. Constructor can vary (added and/or excluded). It is required to determine the best solution. Of course, such a problem can be solved by an enumeration method, but this is time-consuming. In addition, if we take into account that the set of alternatives can be replenished and the weights of the criteria change, then the complexity of the task increases. We use a genetic algorithm to find a solution.

J. Holland proposed the original version of the genetic algorithm in his theory of adaptation [3]. We generalize it for the case of fuzzy data. The genetic algorithm uses the operators of crossover (crossing-over), mutation and inversion. The crossover operator is essential for creating new structures. It takes two structures, randomly selects a breakpoint (where the components are split) on those structures, and swaps the sequences of components to the right of the breakpoint. For example, if two structures $<a_1, a_2, a_3, a_4>$ and $<b_1, b_2, b_3, b_4>$ intersect between the second and the third positions, then the new structures will be $<a_1, a_2, b_3, b_4>$ and $<b_1, b_2, a_3, a_4>$. This operator works with currently available structural populations. We mainly use the crossover operation in the selection of solution variants. We supplement it with operations of intersection and union of sets-populations, as limiting cases of the crossover operation. We apply the intersection operation to two variants of unequal dimensions, when one of them lacks some of the elements (incomplete dimension). The union operation is used when both variants have incomplete dimensions (some of the positions are zero), and the filled positions complement each other. To take into account and introduce new information into the existing population, we use the mutation operator, which arbitrarily changes one or more components of the selected structure. The probability of using this operator is very small, and it ensures that all points in the search space are reachable. We mainly use the mutation operation to select methods for obtaining a solution. Its use is associated with the unsuitability of the method under consideration and the need for its modification. The inversion operator changes the nature of the relationship between the components of the structure. He takes one structure, randomly selects two break points on it, and places in reverse order the elements located between these points. For example, inversion of the structure $<a_1, a_2, a_3, a_4>$ with break points between the first and second and between the third and fourth elements gives a new structure $<a_1, a_3, a_2, a_4>$. We apply the inversion operation when our preferences change in evaluating variants, for example, if the criterion in the first position has ceased to play a dominant role and we need to replace it. The mutation operator does not affect the choice of structures and is used when it is not possible to build a good population. The crossover operator effectively affects structures containing a large number of elements. The mutation operator, on the contrary, is more effective for small structures.

We use the Pareto method to select admissible alternatives. For this, a fuzzy Pareto set is constructed. First, we select alternatives, each of which has a maximum score on at least one criterion among the valid alternatives. Then they are compared to each other. The incomparable alternatives remain, and the rest are discarded (excluded). A new population is formed from the remaining variants (Pareto set 1). So the Pareto set consists of alternatives that are no worse than the excluded ones in all criteria and better in at least one criterion.

We supplement this set using crossover, inversion and mutation operations that are applied to the elements of the structure of different variants. We take the completed set as the original one, and we form from its variants the Pareto 2 set.

After this, we compare the variants of sets 1 and 2. Then if it is possible, the contraction is performed, and the remaining variants form a new Pareto set 3. This set is replenished again, and the procedure is performed until the Pareto set stops improving (expanding).

Decision-making methods are used to select the best solution from the obtained Pareto set (convolution method, main criterion method, threshold criteria method, distance method, etc.). A genetic algorithm can also be used to select an adequate method of decision-making. The method is considered as applicable if its information request I_m corresponds to the conditions of problem I_0, i.e. $I_m \subseteq I_0$. Since the population of methods is small, the mutation operation is the main one for their transformation and selection.

III. Example of study

Consider the synthesis problem by the example of constructing methods of treatment. Medicines and procedures are used as elements of

the "constructor" for the synthesis of treatment methods. We divide the general task into tasks of a smaller dimension, which are solved at the element level (individual drugs and procedures), at the level of subsystems (for example, treatment courses), and at the level of techniques. The solution is evaluated by the set of criteria K_j, given in the form of fuzzy gradations VL, L, M, H, VH (see below). Criteria take into account patient characteristics and disease, resource limitations and performance characteristics. On the upper level (methods) can be used two generalized criterion "possibility of use" and "result of application". At lower levels it is necessary to preserve all information and description are held around a set of criteria that take into account, in particular, the consequences, the negative effects of illness and treatment, the compatibility of the individual components, etc. Initial data on elemental level are given in table 1. We assume that all criteria are of equal importance and are evaluated in direct scale and the elements are compatible. The data in table 1 are obtained by following way. The initial quantitative and qualitative information about objects and criteria, obtained using measurements and expert methods, is transformed into fuzzy gradations. To do this, each named variable is associated with a standardized (normalized) variable that takes values in the range [0, 1], and then fuzzy gradations are associated with the values of the standardized variable. In this case, the value 0 corresponds to the gradation VVL (the lowest value), and the value 1 corresponds to the gradation VVH (the highest value). A value of 0.1 corresponds to the modal value of the VL gradation (very low value); similarly a value of 0.3 – gradation of L (low value); a value of 0.5 – gradation M (middle value); the value of 0.7 – gradation H (high value), the value of 0.9 – gradation VH (very high value). The transition from physical to standardized variable is determined by the ratio $x = (z - z_{min})/(z_{max} - z_{min}) \pm 0.1$, where the plus sign corresponds to the value of z_{min}, and the minus sign to the value of z_{max}. Here x is a standardized variable from the interval (0, 1); z is a "physical" variable, determined by measurement or expert method, which takes values in the interval $[z_{min}, z_{max}]$. Therefore, named numbers or dimensionless estimates represent the values of a physical variable.

We accept that the error of the values in table 1 is approximately one gradation (does not exceed one gradation).The alternatives in table 1 form a Pareto set. Each of the alternatives has a maximum value of at least one criterion, so none of them can be excluded. The condition of certainty (reliability) at the elemental level has the form

$$\min_j K_j > \nu, \quad (1)$$

where the index of fuzziness ν can be selected at the level of the gradations M (weak condition) or H (strong condition) depending on the taken strategy [11, 12]. Define the fuzziness index as the interval of gradations

$$\nu \in (\min_j \min(K_j, \bar{K}_j), \max_j \min(K_j, \bar{K}_j)). \quad (2)$$

Table 1
The initial data of elemental level

Alternatives (Elements)	Values of the criteria							
	K_1	K_2	K_3	K_4	K_5	K_6	K_7	K_8
x_1	VH	H	M	M	M	M	M	M
x_2	H	M	VH	M	H	L	M	M
x_3	M	M	M	H	VH	H	M	M
x_4	M	M	H	VH	M	M	H	M
x_5	H	VH	M	M	M	M	M	H
x_6	M	M	M	M	H	VH	M	M
x_7	H	H	H	M	H	M	H	M
x_8	H	M	H	H	M	H	M	H
x_9	H	H	M	M	H	M	H	H

x_{10}	H	M	M	H	M	M	VH	M
x_{11}	M	H	H	M	M	M	M	VH
x_{12}	H	H	H	H	H	H	H	H
x_{13}	M	M	M	M	M	M	M	M
x_{14}	VH	M	H	M	M	M	M	M
x_{15}	M	M	M	M	M	M	H	VH

We use decision-making methods to determine the best alternative. The choice of method depends on the information available and the requirements of external systems. Calculations are performed according to the rules of fuzzy arithmetic. The set of the best solutions is determined by the expression

$$x^* = \arg\max_x K(x),$$

(3)

where the expression for $K(x)$ depends on the chosen method. If the main criterion is known, then the main criterion method is used. In this case, $K(x) = K_0(x)$, where $K_0(x)$ is the main criterion. For example, if the main criterion is K_1, then $x^* = \{x_1, x_{14}\}$. If the main criterion is K_2, then $x^* = \{x_5\}$. If the main criterion is K_3, then $x^* = \{x_2\}$. If the main criterion is K_4, then $x^* = \{x_4\}$; if the main criterion is K_5, then $x^* = \{x_3\}$; if the main criterion is K_6, then $x^* = \{x_6\}$; if the main criterion is K_7, then $x^* = \{x_{10}\}$; if the main criterion is K_8, then $x^* = \{x_{11}, x_{15}\}$. If we need to get the most reliable solution, then convolution by worst criterion should be applied. In this case, we have $K(x) = \min_j K_j(x)$. Calculations according to (3) using the data in table 1 give $K(x_2)$ = L, $K(x_{12})$ = H, $K(x_1) = K(x_3) = \ldots = K(x_{15})$ = M. So $x^* = \{x_{12}\}$. If additive convolution is used, then $K(x) = \sum_j K_j(x)/m$, where m is the number of criteria; in our case m = 8. For simplicity, we assume that all criteria are equally important. In this case, calculations give $K(x_1)$ = (9VL + 7VL + 6·5VL)/8 = 46VL/8 ≈ 6VL = M-H. Similarly, we obtain $K(x_2) = K(x_6)$ = 46VL/8 = M-H. For $x_3 \ldots x_5$, x_{10} and x_{11}, $K(x)$ = 48VL/8 = M-H. For $x_7 \ldots x_9$ and x_{13}, $K(x)$ = 40VL/8 = M; $K(x_{12})$ = H, $K(x_{14}) = K(x_{15})$ = 46VL/8 = (M-H). So $x^* = \{x_{12}\}$. If convolution by the best criterion is applied, then $K(x) = \max_j K_j(x)$. Calculations give for $x_1 \ldots x_6$, x_{10}, x_{11}, x_{14} and x_{15}, $K(x)$ = VH; for $x_7 \ldots x_9$ and x_{12}, $K(x)$ = H; $K(x_{13})$ = M. So $x^* = \{x_1 \ldots x_6, x_{10}, x_{11}, x_{14}, x_{15}\}$. If it is possible to determine the "ideal" solution, for example, theoretically, then the distance method can be applied. In this case, the set of best solutions is determined by the expression

$$x^* = \arg\min_x d(x),$$

(4)

where $d(x)$ is the distance of the alternative x to the "ideal" solution. The expression for $d(x)$ depends on the choice of the distance measure. We will carry out calculations for the Hamming measure, as well as measures by the largest and smallest difference. We do not consider the Euclidean measure, since the results are obtained close to the Hamming measure. Determine the ideal solution according to the data in table 1. It corresponds to the highest values of all criteria, namely, $K_j(x_0)$ = VH for all $j = 1 \ldots 8$. In the case of the Hamming measure, we have $d(x) = \sum_j \left| K_j(x) - K_j(x_0) \right| / m$, where m is the number of criteria; in our case m = 8. Calculations give $d(x_1)$ = (0 + 2VL + 6·4VL)/8 = 26VL/8 ≈ 3VL = L. For x_2, x_6, x_{14} and x_{15}, $d(x)$ = 26VL/8 = L. For x_3, x_4, x_5, x_{10} and x_{11}, $d(x)$ = 24VL/8 = L. For x_7, x_8 and x_9, $d(x)$ = 22VL/8 ≈ 3VL = L. For x_{12}, $d(x_{12})$ = 2VL = VL-L. For x_{13}, $d(x_{13})$ = 4VL = L-M. So $x^* = \{x_{12}\}$. For the measure by the smallest difference $d(x) = \min_j \left| K_j(x) - K_j(x_0) \right|$. Calculations give for x_7, x_8, and x_{12}, $d(x)$ = 2VL = VL-L. For x_{13}, $d(x_{13})$ = 4VL = L-M. For $x_1 \ldots x_6$, x_{10}, x_{11}, x_{14} and x_{15}, $d(x)$ = 0. So $x^* = \{x_1 \ldots x_6, x_{10}, x_{11}, x_{14}, x_{15}\}$. For the measure by the largest difference we have $d(x) = \max_j \left| K_j(x) - K_j(x_0) \right|$. Calculations give for x_{12}, $d(x_{12})$ = 2VL = VL-L. For all other x, we obtain $d(x)$ = 4VL = L-M. So $x^* = \{x_{12}\}$. The results obtained show that the solution can change depending on the chosen decision-making method and significantly differs from the ideal one.

We evaluate the certainty (reliability) of solutions at an elementary level. The calculations according

to (2) show that for elements x_1 … x_6, x_{10}, x_{11}, x_{14}, x_{15} the index of fuzziness is in the range (VL, M); for the elements x_7, x_9 , it is in the range (L, M); for the element x_{12}, it is equal to L; for x_{13}, it is M. Therefore, the condition of validity (1) allows us to exclude x_2, x_{13}; the other elements are divided into four groups. The elements of the first group, namely x_3, x_4, x_5, x_{10}, x_{11} contain two gradations H and one gradation VH. The second group of elements x_1, x_6, x_{14}, x_{15}, contains one gradation VH and one gradation H. The third group of elements x_7, x_8, x_9 contains five gradations H. The fourth group consists of one element x_{12}, which has all gradations H.

We consider the alternatives from table 1 as elements from which a general solution to the synthesis problem is built. Further actions depend on the purpose and the taken strategy. If separate criteria are important, we can take into account only the elemental level, selecting the alternatives for which these criteria are maximal, or at least not less than the gradation H. Since we are interested in the general solution, we must identify the chains of elements that leads to the best outcome, which in turn depends on the purpose.

To reduce the time of solution search and selection of valid connection of elements we use three conditions, namely conditions of necessity, minimal sufficiency, and the strongest combination (unification) [11]. The condition of necessity is that elements are combined, for which the relation of reliability (1) is valid. Therefore, we should consider only chains of elements with gradations at least not less than the gradation H, i.e. VH or H. The condition of minimal sufficiency allows us to exclude duplication. From this, it follows that the elements with the most degree of consistency can be interchangeable for the required level of gradations. The condition of strongest combination consists in that the elements combine to maximize the values of the criteria. From this it follows that, first, it is necessary to combine the elements, which have the smallest degree of consistency, i.e., the distance between them is largest. In this case, first this condition is applied to the elements having the greatest estimate VH by criteria. Then, if the elements with the gradation VH are exhausted, the elements with the gradation H are considered, and so on.

Using these conditions, we fulfil a preliminary analysis of the data in table 1 to determine permissible limits of change of criteria for elements combining in the chain. Note that no one set of elements in table 1 does not allow us to get all graduation VH. The maximum number of gradations VH, namely seven has a chain $< x_1, x_5, x_4, x_3, x_6, x_{10}, x_{11} >$ and its modification obtained by the replacement by replacement, for example, x_1 on x_{14}, x_{11} on x_{15} (we consider only the overlapping gradations VH). Elements in a chain are connected sequentially. If it is sufficient to have the result at the level of the gradation criteria not less than H, we can take into account only the element x_{12}. If we want one or more criteria to have gradation VH, it imposes a limit on the length of the chain defined by the condition of minimal sufficiency. For example, the chain $< x_1, x_4, x_6, x_{11} >$ and its modification obtained by the replacement x_{11} with on x_{15} or x_{11} on x_5, have 4 gradation VH with the minimal possible number of elements. Chains $< x_3, x_5, x_4 >$ and $< x_4, x_5, x_6 >$ have 3 gradations VH at the minimum possible number of elements. Chains $< x_1, x_4, x_6, x_8 >$, $< x_1, x_4, x_6, x_9 >$ and $< x_1, x_4, x_6, x_{12} >$, also have 3 graduations VH, but are redundant in number of elements. Chains $< x_1, x_4, x_{12} >$, $< x_3, x_5, x_{12} >$ and $< x_3, x_4, x_{12} >$ with two gradations VH have the excess of the number of elements. In this case, a chain is selected that has the required number of gradations VH and contains the smallest number of elements.

Now we consider an approach to obtain the best solution. We use a strong condition of certainty in (1), which allows us to significantly reduce the number of possible combinations of elements (chains).We introduce X_i – a set of elements which have the maximal consistency with the element x_i. In our case, the degree of consistency of element x_i with arbitrary element from X_i equals $\alpha = [\mathrm{H}, \mathrm{VH}]$; the distance *d* between x_i and any element from X_i is the smallest allowed, namely $d = \overline{\alpha} = [\mathrm{L}, \mathrm{VL}]$. Define the set $\overline{X}_i$ as the set of elements farthest from the element x_i. In our case, the distance between x_i and arbitrary element from $\overline{X}_i$ equals $d = \overline{\alpha} = [\mathrm{M}, \mathrm{VL}]$, where $\overline{\alpha}$ – opposite value, for example if $\alpha = \mathrm{VH}$, then

$\overline{\alpha} = \mathrm{VL}$, etc. Based on the initial data of table 1, the degree of consistency of elements is determined by the matrix of correspondence given in the table 2.

Table 2
Matrix of correspondence

Fuzzy gradations	VL	VL-L	L	L-M	M	M-H	H	H-VH	VH
	$\alpha(x, y)$								
VL	VH	H-VH	H	M-H	M	L-M	L	VL-L	VL
VL-L		VH	H-VH	H	M-H	M	L-M	L	VL-L
L			VH	H-VH	H	M-H	M	L-M	L
L-M				VH	H-VH	H	M-H	M	L-M
M					VH	H-VH	H	M-H	M
M-H						VH	H-VH	H	M-H
H							VH	H-VH	H
H-VH								VH	H-VH
VH									VH

Note. Since the relation of correspondence is symmetric, $\alpha(x, y) = \alpha(y, x)$.

With respect to an arbitrarily selected element x_i, all other elements belong to X_i or $\overline{X}_i$ at a given level reliability. Elements from the set X_i are, in a certain sense, duplicating elements for the element x_i, and elements from $\overline{X}_i$ are additional to it. The results of the calculations are given in table 3. As the degree of consistency $\alpha(x_i, x_k)$ symmetric with respect to the indices i, k then in the sets X_i and $\overline{X}_i$ given in table 3, we show only elements with numbers $k > i$. For example, if $x_4 \in X_3$, then also $x_3 \in X_4$, so the element x_3 is not shown in X_4, etc.

Table 3
The sets X_i and $\overline{X}_i$ for initial elements x_i

Element, x_i	X_i	$\overline{X}_i$
x_1	$\{x_5, x_7, x_8, x_9, x_{12}, x_{14}\}$	$\{x_3, x_4, x_6, x_{10}, x_{11}, x_{15}\}$
x_3	$\{x_4, x_6, x_7, x_9, x_{12}\}$	$\{x_5, x_8, x_{10}, x_{11}, x_{14}, x_{15}\}$
x_4	$\{x_6, x_8, x_{10}, x_{12}\}$	$\{x_5, x_7, x_9, x_{11}, x_{14}, x_{15}\}$
x_5	$\{x_7, x_9, x_{11}, x_{12}\}$	$\{x_6, x_8, x_{10}, x_{14}, x_{15}\}$
x_6	$\{x_8, x_{12}\}$	$\{x_7, x_9, x_{10}, x_{11}, x_{14}, x_{15}\}$
x_7	$\{x_8, x_9, x_{10}, x_{12}, x_{14}\}$	$\{x_{11}, x_{15}\}$
x_8	$\{x_9, x_{11}, x_{12}, x_{14}, x_{15}\}$	$\{x_{10}\}$

x_9	$\{x_{10}, x_{11}, x_{12}, x_{14}, x_{15}\}$	-
x_{10}	$\{x_{12}\}$	$\{x_{11}, x_{14}, x_{15}\}$
x_{11}	$\{x_{12}, x_{15}\}$	$\{x_{14}\}$
x_{12}	{all x}	$\varnothing$
x_{14}	-	$\{x_{15}\}$
x_{15}	-	-

From the table 3 it follows that the element x_1 is consistent with six elements; elements x_3, x_4, x_5, x_{11}, x_{14} are consistent with five elements (taking into account the symmetry of α); elements x_6, x_{10} – with four elements, x_{15} – with three, x_7 – with eight, x_8 – with nine, x_9 – with ten, x_{12} is consistent with all other elements. It follows from the conditions of minimal sufficiency and strongest combination that chains should be composed of elements, for which the degree of consistency is minimal, i.e. element x_i should be combined first of all with elements from $\overline{X}_i$. The smaller the set $\overline{X}_i$, the corresponding element x_i is more universal. In our case, the most universal element is x_{12}, provided that gradations H and VH are permissible. The best solution when combining elements is determined by the expression

$$x^* = \max_{l,m,\ldots} \min_j K_j(x_l, x_m, \ldots). \quad (5)$$

When combining elements in a chain according to (5), gradations M are replaced with the gradations H or VH, and graduations H – with gradations VH, which leads to the best result. It should be borne in mind that the result, obtained from the relation (5), depends on the purposes and constraints given by external systems. If we want to define a chain where there is a certain number of gradations VH without additional conditions, then the main elements are those that contain this gradation, and other elements have a supporting role, so duplication of gradations H or M can take place. If the purpose is the synthesis of the chain with maximum number of gradations VH in the absence of duplication of gradations VH and H, then in our case, the solution has the form $< x_1, x_4, x_6, x_{15} >$ and contains four graduations VH. In the general case, to obtain the full set of admissible solutions (combinations) that satisfy special requirements, for example, a given number of gradations VH or H by specific criteria, it is needed redundancy of the initial data according to the number of elements. For obvious reasons it is impossible in the framework of the example to provide a large redundancy of the initial data.

Now we consider the application of genetic algorithm operations to change or improve the solutions of elementary level. The applied method must satisfy two conditions: to give meaningful solutions and not to worsen the final solution. To change the solutions of an elementary level according to individual criteria, it is expedient to use the inversion operation. Table 1 shows that the elements x_1, x_{14} are equivalent by the number of criteria with the same gradations and overlap in the gradation VH. Therefore, without worsening the final solution, we can change the position of the gradation VH in one of the equivalent elements. Suppose we need to get $K_3 = \mathrm{VH}$ for x_{14}. Apply the inverse operation to the element x_{14}. We take in the line x_{14} of the table 1 two points: one before the first position, and the second between the third and fourth position. Then the line x_{14} in the table 1 takes the form $< \mathrm{H, M, VH, M, M, ..., M} >$, which gives the required result. To make this transformation really feasible, we need to examine the structure of the element x_{14} and see if we can change it to get the desired result. To improve solutions at an elementary level, it is advisable to use the crossover (crossing-over) operation. We use this operation to get an element with a given number of gradations H or VH. For example, we need to get all gradations VH in line x_1 of table 1. This is achieved through the following transformations. Apply the crossover operation to x_1 and x_5, choosing the break point between the first and second positions. We get $x_{1,5}$ = <VH, VH, M, M, M, M, M, H>. Hereinafter, we show only one of the two formed structures. Apply the crossover operation to $x_{1,5}$ and x_2, choosing a break point between the second and third positions. We obtain $x_{1,5,2}$ = <VH, VH, VH, M, H, L, M, M>. Apply the crossover operation to $x_{1,5,2}$ and x_4, choosing the break point between the third and fourth positions.

We get $x_{1,\,5,\,2,\,4}$ = <VH, VH, VH, VH, M, M, H, M>. Apply the crossover operation to $x_{1,\,5,\,2,\,4}$ and x_3, choosing the break point between the fourth and fifth positions. We obtain $x_{1,\,5,\,2,\,4,\,3}$ = <VH, VH, VH, VH, VH, H, M, M>. Apply the crossover operation to $x_{1,\,5,\,2,\,4,\,3}$ and x_6, choosing the break point between the fifth and sixth positions. We get $x_{1,\,5,\,2,\,4,\,3,\,6}$ = <VH, VH, VH, VH, VH, VH, M, M>. Apply the crossover operation to $x_{1,\,5,\,2,\,4,\,3,\,6}$ and x_{10}, choosing the break point between the sixth and seventh positions, we get $x_{1,\,5,\,2,\,4,\,3,\,6,\,10}$ = <VH, VH, VH, VH, VH, VH, VH, M>. Finally, apply the crossover operation to $x_{1,\,5,\,2,\,4,\,3,\,6,\,10}$ and x_{11}, choosing the break point between the seventh and eighth positions, we get the required result $x_{1,\,5,\,2,\,4,\,3,\,6,\,10,\,11}$ = <VH, VH, VH, VH, VH, VH, VH, VH>. Of course, the chosen sequence of transformations is not the only one. Other chains can be obtained in a similar way. To make these transformations really feasible, we need to examine the structure of the elements and see if we can change them to get the desired result.

III. Discussion of the results

We restricted our consideration to linear combinations of the elements, as any treatment method is fulfilled consistently, and it makes no sense to complicate the structure. Of course, individual elements can include cycles and parallel components, but this task is more low-level analysis. The order of the elements in the chain is determined by external priorities, for example, by class of method. In our case, gentle methods are considered. We can also accept that the order of the elements in the subsystem is regulated by an auxiliary criterion.

When a set of chains of elements is obtained, we can move on to the next level of analysis and consider the relationship of sub-systems (treatments) with the methods in general (as a whole). If we design chains of elements as y_1, y_2, etc. and introduce generalized criteria, as above, we reduce the task at the level of subsystems to the problem has already been considered at the level of elements. Now the initial data consist of chains, estimated by criteria, and we are looking for combinations of chains to obtain the best methodology. The execution order of treatments is determined by external priorities and may be regulated by an auxiliary criterion. The choice of the best combination of treatments in the framework of methodology is determined by the expression similar to (5) with corresponding change of variables. We can also use the distance measure defined in table 2.We have for arbitrary treatment y_k

$$d(y_k, y_0) = \bar{\alpha}(y_k, y_0)$$

(6)

where y_0 is the ideal solution, determined by the results obtained on the elemental level, which correspond to the maximum values of all criteria. The best treatment is given by the expression similar to (4)

$$y* = \arg\min_k d(y_k, y_0)$$

(7)

To determine $d(y_k, y_0)$, we can use the Hamming measure or the choice by the greatest or smallest difference depending on external purposes. Calculations are performed as above according to the rules of fuzzy arithmetic. Otherwise, the problem of synthesis at this level has no peculiarities in comparison with the elementary level considered above.

The combined use of the genetic algorithm and the representation of initial data in the form of fuzzy gradations facilitates the solution of the synthesis problem in comparison with the traditional numerical methods. The results obtained in the article depend mainly on the structure of the initial data. The disadvantages of the traditional methods are the laboriousness of the calculations, the considerable time costs and the strong dependence of the result on the errors of the initial data.

IV. Conclusion

The proposed approach based on fuzzy gradations significantly facilitates informal procedures for analyzing the set of acceptable solutions and obtaining the best solution in the problem of synthesis of the soft systems. Comparison of the proposed algorithm with the traditional approach based on the generalized Zadeh principle shows its advantages. The application of this method does not depend on the specific numerical context since it uses an order scale. The proposed approach makes it possible to use all advantages of fuzzy data representation and, at the same time, preserve clarity and certainty in the interpretation of the results obtained. The proposed approach allows solving the synthesis problem with ill conditioned initial data.

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