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The use of fuzzy models for the analysis of the behavior of large-scale systems

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ABSTRACT
The paper discusses the use of fuzzy models for the analysis of stability and adaptability of large-scale systems. We understand large-scale systems as multiply connected systems. These include economic, social, ecological, linguistic and other systems, the behavior (functioning) of which is associated with decision-making. The stability of the state and disturbance in such systems depends not only on external factors, but also on the decisions made by the system itself. It is difficult to obtain adequate formal models for these systems. The paper gives a generalization of two approaches based on the presentation of the initial data in the form of fuzzy gradations proposed by the author: the matrix method and method of expansion in series. The method based on fuzzy matrices is used to analyze the propagation of disturbances in the system. It allows us to perform a qualitative analysis of changes in the stability and adaptability of the system depending on the level of disturbances. The method based on the representation of the function of the system's behavior in the form of a series is used to determine the area of stability of the system depending on the influence of external factors. These approaches allow us to understand how small changes in the system lead to abrupt changes and instability. Calculations are performed for various cases of external factors and control parameters.
Keywords and phrases: fuzzy arithmetic, large-scale systems, stability of systems, matrix method, series expansion method.

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I. Introduction

The concept of stability has a different meaning. It can relate to both the structure and the functioning of the system. Lagrange first formulated the concept of stability in relation to the motion of planetary masses (the famous three-body problem). Lagrange stability means that the motion of a system of bodies is enclosed within certain limits, and that a system will pass an infinite number of times arbitrarily close to its initial position. In this case, there is complete stability. Poisson developed this concept. Poisson stability means that the system will always pass again an infinite number of times arbitrarily close to the initial position, but it cannot be argued that the system does not significantly move away from it. Poincare and Lyapunov further developed the concept of stability.

The concept of stability is associated with the concept of adaptability, which is understood as the ability of a system to maintain behavior (functioning) under the action of external perturbations. The requirement of adaptability is stronger than stability, so the adaptive system is stable, but the opposite is not always true.

In classical works, the main attention is given to the analysis of perturbations of the system in the initial state; in current works, the emphasis is shifted to the analysis of perturbations in the structure of the system [1 – 3, 5, 7, 9, 10, 12, 13]. The problem is to find out how the behavior of the system changes under external influences, and to what extent the influences can change so that the system remains in a stable state [8, 14, 30]. One of the problems of the modern theory of stability is the study of how the stability region changes depending on external parameters, the so-called control parameters. Another problem is of interest: if a behavior function of system is given, then what will look like a family containing functions close to it. These problems are solved in bifurcation theory and catastrophe theory, where the subject of study is the analysis of the behavior of families of trajectories that arise when considering many close systems. The results obtained in this field relate to the so-called gradient dynamical systems [11, 16]. For most real systems of practical interest, the assumptions underlying the catastrophe theory are not satisfied. In the general case, the control parameters depend on time and state variables fluctuate. However, in this case, the catastrophe theory is a useful tool for a qualitative study of stability.

Methods of stability analysis of systems depend on the type of model used to describe them [8]. The most studied are systems that are described by differential equations [6, 15, 17 – 19, 28 – 31]. In the case of an internal description, algebraic methods are used [4]. For systems with an external description, topological methods can be used [22]. To analyze perturbations in systems, matrix methods are used [8, 22].

The purpose of this paper is to study and test models based on the use of fuzzy arithmetic for analysis of behavior and assessing the stability of large-scale systems. The paper gives a generalization of two approaches based on the presentation of the initial data in the form of fuzzy gradations proposed by the author: the matrix method and method of expansion in series.

It is customary to understand large-scale systems as multiply connected systems. These include economic, social, environmental, linguistic, etc. systems, the behavior (functioning) of which is associated with decision-making. Stability in state and perturbation in such systems depends not only on external systems, but also on decisions made by the system itself. For these systems, it is difficult to obtain adequate formal models. Therefore, qualitative methods for analysis of behavior, stability and other system properties are important. In many tasks related to the analysis of interconnections and mutual influences of parameters in the large-scale systems, such as decision-making, control and evaluation, the specific numerical content of the quantities does not matter, at least at the stage of solution search, but only the order relation between them. Therefore, it becomes necessary to operate on quantities without being tied to a numerical context. The author suggests an approach based on the use of fuzzy gradations, in which the numbers are replaced by quantities. In [21 – 27], the advantages of this approach in solving various problems were shown. It is closest to the theory of interval estimation and models of the linguistic description of the subject domain. The main advantage of this approach is that it allows us to perform arithmetic operations directly without using the membership function and its application does not require the procedure of rationing weights. The advantage of this approach is also the quick analysis of many alternatives in multi-parameter tasks using all the useful information.

II. Arithmetic operations on fuzzy gradations

The rules of fuzzy arithmetic and the evaluation of the reliability of the results are considered in the previous works of the author, so we give them here in abbreviated form to make the results of calculations clear. To describe the object area we use the fuzzy gradations in the range VL…VH. The range comprising gradations VL, L, M, H, VH, where VL – very low value, L – low, M – middle, H – high, VH – very high value, we call the basic scale and with the adding of the intermediate gradations (see below) – extended scale. Introduce also two limit gradations out of range: VVL (lowest value) and VVH (highest value). Depending on condition of the task, we can interpret the gradation VVL as zero, lower bound, exact lower bound, etc. and the gradation VVH as unit, infinity, upper bound, exact upper bound, etc. On the set of fuzzy gradations, a relation of a non-strict order (preference relation) is established, which has the properties of reflexivity, antisymmetry and transitivity. Fuzzy gradations form an ordinal scale VVL < VL < …< VH < VVH, in which an admissible transformation is arbitrary monotone function that does not change the order of gradations. In particular, all gradations can be simultaneously multiplied or divided, as well as increased or decreased by the same number, so that the values do not go beyond the range 0 ... 1. This is another advantage of using fuzzy gradations to represent the initial data.

There is the one-to-one correspondence between each fuzzy gradation of the scale and the corresponding numerical interval. It is assumed that the value of the gradation is concentrated in the center of the interval, so modal values of fuzzy gradations VL, L, M, H, VH correspond with the values 0.1; 0.3; 0.5; 0.7; 0.9 respectively. Modal values of intermediate gradations VL-L (between very low and low values), L-M (between low and middle values), M-H (between middle and high values), H-VH (between high and very high values) correspond to values 0.2; 0.4; 0.6; 0.8 respectively. The value VVL means that the result is outside the left boundary of the range (this value corresponds to 0); the value VVH means that the result is outside the right boundary of the range (this value corresponds to 1).

The initial quantitative and qualitative information about objects and criteria, obtained using measurements and expert methods, is transformed into fuzzy gradations as follows. Each named variable is assigned a standardized (normalized) variable, varying in the interval [0, 1]. Then a fuzzy gradation is assigned to the standardized variable. In this case, the value 0 corresponds to the gradation VVL (the lowest value), the value 1 corresponds to the gradation VVH (the highest value), etc. (see above). The transition from physical to standardized variable is determined by the ratio $x = (z - z_{\min})/(z_{\max} - z_{\min}) \pm 0.1$, where the plus sign corresponds to the value of $z_{\min}$, and the minus sign to the value of $z_{\max}$. Here x is a standardized variable from the interval (0, 1); z is a "physical" variable, determined by measurement or expert method, which takes values in the interval $[z_{\min}, z_{\max}]$. Values of z can be represented by named numbers or dimensionless estimates.

The summation and multiplication operations on fuzzy gradations are performed in the same way as in ordinary arithmetic. For instance, for summation we have VL + VL = (VL-L), (VL-L) + VL = L, etc., (H-VH) + VL = VH, VH + VL = VVH. Similarly, summation is performed for other fuzzy gradations. For multiplication operation we have VL * VL = VVL, VL * (VL-L) = VVL, etc., VL * M = VL, etc., VL * VH = VL. Similarly, multiplication is performed for other fuzzy gradations. Calculations on fuzzy gradations are greatly simplified if all gradations are expressed in terms of the smallest gradation (VL), namely, (VL-L) = 2VL, L = 3VL, etc., VH = 9VL, VVH = 10VL. This representation makes it possible to extend calculations formally outside the range 0 ... 1. We use this technique in subsequent calculations.

III. The use of fuzzy matrices for the analysis of system stability

In this section, we study the possibility of using fuzzy matrices to represent and evaluate the interconnections (interactions, mutual influences) of system components. This approach is a generalization of the graph and matrix method for analyzing disturbances in systems [4, 8]. The problem is formulated as follows. There is a system on elements of which a fuzzy binary relation $R \subset X \times Y$ is given; the sets X and Y can coincide or be different. It is required to evaluate the stability, adaptability and other properties of the system under the action of perturbations. We represent the relation in the form of a matrix S consisting of fuzzy gradations. Typically, the values of the matrix are interpreted as the degree of confidence in the fulfillment of the relationship. For this case, we interpret the matrix as a scheme of interconnections (interactions, mutual influences) between system components. Depending on the level of analysis, the elements of the matrix can be objects, their states, or state characteristics. Suppose that the values can vary in the interval [VL, VH], which determines the stability region of the system. The general form of the matrix S for coincident sets X and Y is given in table 1.

Table 1
Matrix of interconnections (mutual influence) of system components

Components	x_1	x_2	…	x_n
x_1	s_{11}	s_{12}	…	s_{1n}
x_2	s_{21}	s_{22}	…	s_{2n}
…	…	…	…	…
x_n	s_{n1}	s_{n2}		s_{nn}

Note. Elements in matrix cells are represented by fuzzy gradations.

The value of fuzzy gradation in the cell (i, k) of the table 3 shows the strength of the interconnection (influence) of the x_i component on the x_k component; the sign of gradation determines the direction of influence: the "+" sign means amplification, and the "–" sign means weakening. For example, if $s_{12} = \mathrm{M}$, then component x_1 enhances component x_2 to an average degree; if $s_{21} = \mathrm{H}^{-}$, then x_2 weakens x_1 to a high degree, etc. The matrix of interconnections is symmetric; the mutual influence matrix can be asymmetric. Some of the components can be associated with system inputs or outputs.

The matrix is analyzed at three levels VL ∨ L, M and H ∨ VH. A component of the system with a level of interconnection VL ∨ L has a low sensitivity to external disturbances, a component with a level M has a middle sensitivity, and a component with a level H ∨ VH has a high sensitivity to external disturbances (hereinafter, ∨ is a the logical connective "or"). The adaptability of the system is due to the influence (limitation) of external systems; therefore, the adaptability for components with a level of interconnection VL ∨ L is higher than for components with a level M and, especially, with a level H ∨ VH. If the sets *X* and *Y* are different, then the elements of *Y* are considered as functions of the elements of *X*, namely, the matrix $Y \times Y$ is analyzed for the element x_1, then for the element x_2, etc. The analysis is performed at the three levels of gradations indicated above. Similarly, the elements of the set *X* are considered as functions of the elements of the set *Y*, namely, for each element of the set *Y*, the corresponding matrices $X \times X$ are analyzed. If the power of a set with a low level of interaction of elements increases, then horizontal isolation develops in the system, and it becomes more stable. If the power of a set with a high level of interaction increases, then vertical integrity develops in the system, and the system becomes less stable. If the power of a set with an average level of interaction increases, then the system is in a balanced state with respect to both processes and stability is retained. The ratio of the change in the power of a set with a given level of interaction to the power of the whole set shows the degree of change in a property (for example, stability, adaptability, isolation, integrity, etc.).

Consider the process of perturbation propagation in a system with one input and one output. Suppose that a perturbation β acts on the component x_i of the system at time t_0. Then the perturbation of the component x_j at time t caused by the component x_i can be represented in the form

$$\Delta x_j(t) = \Delta x_j(t_0) + \beta(I + S + S^2 + ... + S^k)_{ij}, \qquad (1)$$

where $t = t_0 + k$, $\Delta x_j(t_0)$ is the initial perturbation of the component x_j, I is the identity matrix, i is the row number, j is the column number of the corresponding matrix *S*. For arbitrary *j*, *t*he condition of system stability by state in the time interval $t - t_0 = k$ have the form

$$\left|\Delta x_j(t)\right| \le \text{VH}. \qquad (2)$$

The condition for the stability of the system by perturbation for arbitrary *i* and *j* has the form

$$\left|S_{ij}^k(t)\right| \le \text{VH}. \qquad (3)$$

Now let the perturbation at the moment t_0 affect all components of the system, and the perturbation β_i corresponds to the component x_i. Then the perturbation of the component x_j caused by the component x_i is determined by the expression

$$\Delta x_{j(i)}(t) = \Delta x_j(t_0) + \beta_i(I + S + S^2 + ... + S^k)_{ij}, \qquad (4)$$

where $t = t_0 + k$, $\Delta x_j(t_0) = \beta_{j0}$, indices *i, j* independently run through the values 1, 2, ..., *n*. The conditions of system instability by state (by value) have the form

$$\max_{i,j}\left|\Delta x_{j(i)}(t)\right| > \text{VH}. \qquad (5)$$

The conditions of instability by perturbation are determined by the inequality

$$\max_{i,j}\left|S_{ij}^k(t)\right| > \text{VH}. \qquad (6)$$

For definiteness, let x_1 is the input and x_n is the output of the system. Then the instability by the state (value) at the output due to disturbance (perturbation) at the input is determined by the relation

$$\left|\Delta x_{n(1)}(t)\right| > \text{VH}. \qquad (7)$$

The instability by perturbation is given by the relation

$$\left|S_{n(1)}^k(t)\right| > \text{VH}. \qquad (8)$$

Consider an example from the field of economics. There are several regions. It is required to analyze their stability. Let the set *X* consist of elements: productive forces, employment rate, free capital, free land. The set *Y* includes the following elements: rent, interest (income from investments), wages. Elements can take on different values in different regions. Let's make a matrix *S*. The ratios between the elements of the matrix depend on the level of development of society, regulatory mechanisms, legislation, economic policy of the state and local authorities, centralization, protectionism, etc. These factors in this case are hidden parameters. The study of relationships between the elements of the influence matrices allows us to determine the hidden parameters that have the greatest impact on the behavior of the system [27]. For the convenience of analysis, by rearranging the rows and columns, we present the matrix in a block form. Assume that the initial matrix has the form of table 2.

Table 2
The matrix of interconnections (mutual influence) for example

VL ∨ L (I)	M (II)
M (III)	H ∨ VH (IV)

Note. Here, as above, VL is a very low value, L is low, M is middle, H is high, and VH is a very high value. Signs of interconnections (mutual influence) of elements inside blocks can be both positive and negative.

Calculations using relations (1), (4) show that if the fourth block has the value H, then the system is stable by perturbation for arbitrary input signal that varies in the permissible range [VL, VH], but unstable by state. If the value VH is used in the fourth block, then the system as a whole is unstable both by perturbation and by state. These conclusions are valid if there are both positive and negative connections between system components.

Obviously, if the initial perturbation of a system in a certain state is significant (gradations M, H or VH), then under the action of a perturbation, it is likely to be unstable by state, which follows from relations (1), (4), (5). If the initial perturbation is small (gradations VL or L), then the system will be more stable by state. We considered single perturbations. With multiple perturbations, the stability of the system can be violated even under the action of small perturbations, which depends on the frequency of perturbation and the relaxation time (inertia) of the system. The results obtained are applicable in the general case to systems with a different type of structure, for example, to systems consisting only of components with weak, middle, or hard connections (bonds). In this case, the adaptability of the system is more associated with stability by perturbation, and stability of the system with stability by state. The consideration above allows us to understand how small and minor changes in the system, accumulating, over time lead to unpredictable consequences and loss of stability.

IV. The use of fuzzy series for the analysis of the behavior of the system

We consider an approach to the analysis of behavior and determination of stability and other properties of a system under conditions of uncertainty using expansion in series. This approach allows us to understand how small changes in parameters (factors) lead to a sharp change in the behavior of the system and loss of stability. Suppose that the function *y* characterizing the behavior of the system can be represented with sufficient accuracy in the form of a series. If *y* satisfies a linear differential equation, then the analysis of the convergence of the series can be performed using the Poincare method of decomposition in a small parameter [20]. We will consider the case where the equation is unknown. The values of the function *y* and parameters (factors) are found from experimental data or using expert estimates. The dependence of *y* on factors is approximated by a series. The approximation error is determined, for example, by the least squares method based on the minimum residual variance. Hereinafter, we assume that the error in determining the factors and parameters is about one gradation.

To determine the behavior of the system, consider a linear model taking into account interactions of different orders

$$y = \sum_i a_i f(x_i) + \sum_{\substack{i,j \\ i \neq j}} a_{ij} f(x_i, x_j) + \dots , \quad (9)$$

where *y* determines the deviation of the system from the initial (equilibrium) state, the value of which characterizes the instability of the system (a constant value y_0 is chosen for the zero level); x_i – sources of influence (factors) with acceptable thresholds (limits) L_{x_i}; f –approximating functions, the form of which depends on a priori information about the system and is not predetermined; a_i – parameters that depend on the thresholds L_{x_i} and can change abruptly (stepwise) for some values of factors. The specific nature of such sharp changes is not considered. It is assumed that the thresholds L_{x_i} are relative, and we are in the admissible range of values of factors, far from the absolute limits, when the system is destroyed, and irreversible changes occur in it. Factors x_i can be internal and external. For example, for a manufacturing firm, internal factors include functional and economic factors. The functional factors include the quality and reliability of products, their competitiveness; economic factors include productivity, direct and indirect costs. External factors include the pace of industry development, market share compared to leading competitors, rent, bank interest (investment income), economic legislation, government policy in the economic sphere, etc.

In (9), the first sum is linear with respect to factors. It represents the contribution of individual factors and consists of *n* terms. We denote this contribution as y_1. The second sum is the contribution from first-order interactions, or pair interactions of factors, and consists of $C_n^2 = n(n-1)/2$ terms. Denote this contribution as y_2. Subsequent sums give the contributions of the second, third, etc. orders from the interaction of three, four, etc. factors, respectively. The third sum consists of C_n^3 terms of the form $a_{ijk}x_ix_jx_k$, where the indices *i*, *j*, *k* are pairwise different. Denote this contribution as y_3. The fourth sum consists of C_n^4 terms of the form $a_{ijkl}x_ix_jx_kx_l$, where the indices *i*, *j*, *k*, *l* are pairwise different. Denote this contribution as y_4. Thus, in the general case, $y = y_1 + y_2 + y_3 + y_4 + \ldots$.

For a generality of analysis, factors and functions are described by fuzzy gradations in the interval [VL, VH] and take, as above, the values VL (very low), L (low), M (middle), H (high), VH (very high). Gradations that are intermediate between the basic ones are also known, namely VL-L (between very low and low values), etc. It is assumed that the functions $f(x_i)$ are periodic in the interval [VL, VH], in the sense that after the factors reach the limits L_{x_i}, the functions change in the same interval, which, generally speaking, can differ from the previous one in numerical value. Since we do not specify the form of the function, these assumptions are not a limitation of generality, and we make them for the convenience of calculations. The system has stable behavior (the system is stable) if $y <$ VH, and unstable behavior (the system is unstable) otherwise.

Since x_i and $f(x_i)$ have the same range of variation in a fuzzy scale, it is sufficient to investigate the change in the factors (variables) x_i and parameters a_i. First, we will consider a linear model without taking into account the interaction of factors. We start with the simple case of a one-factor model, which has the form

$$y = a_1 f(x_1). \tag{10}$$

If the value of factor x_1 is less than the threshold, i.e. $x_1 < L_{x_1}$, then we take fuzzy gradation L (low value) as the initial values for the factor and parameter. The upper value for the factor is VH, and the parameter has a constant value $a_1 = \mathrm{L}$. When the factor reaches the threshold value, the parameter undergoes an abrupt change by one gradation and becomes equal M (middle value), and the factor again changes in the interval [VL, VH]. Hereinafter, we perform calculations for the boundary values of the factors $x =$ VL and $x =$ VH, since this is sufficient to obtain correct results.

Hereinafter, we have excluded the value $a =$ VL, since in this case the change in the behavior of the system *y* is equal to zero or does not exceed VL, which corresponds (is typical for) a closed system. In this case, the value of *y* may exceed VH, i.e. the system loses stability, only under the joint action of a large number of external factors $n > 10$, each of which takes the maximum permissible value VH. For the same reason, the value $x =$ VL is excluded, since in this case the change in the behavior *y* is equal to zero or does not exceed VL. In this case, the value of *y* may exceed VH, i.e. the system loses stability, only under the joint action of a large number of external factors $n > 20$, each of which takes the maximum value VH, and besides $a =$ M.

For model (10), calculations using the rules of fuzzy arithmetic give

1). $x_1 < L_{x_1}$; $a_1 =$ L; $x_1 =$ L $\rightarrow y = a_1x_1 =$ VL;

2). $x_1 < L_{x_1}$; $a_1 =$ L; $x_1 =$ VH $\rightarrow y = a_1x_1 =$ L;

3). $x_1 \geq L_{x_1}$; $a_1 =$ M; $x_1 =$ L $\rightarrow y = a_1x_1 =$ VL-L;

4). $x_1 \geq L_{x_1}$; $a_1 =$ M; $x_1 =$ VH $\rightarrow y = a_1x_1 =$ M.

Thus, there are four different combinations of factors, which correspond to four different values of *y*, or four variants of behavior. The system is stable, since $y <$ VH.

It is not difficult to obtain general relations for a linear model with n factors. If for all x, $x < L_x$ or $x \geq L_x$, then the number of different combinations of factors is equal to $n + 1$. If only for one x, $x < L_x$ or $x \geq L_x$, then the number of different combinations of factors is $2n$. If n_1 factors x_i have values less than the threshold, i.e. $x_i < L_{x_i}$, where $i = 1, 2$, etc., n_1, and n_2 factors x_k have values not less than the threshold, i.e. $x_k \geq L_{x_k}$, where $k = 1, 2$, etc., n_2, so that $n_1 + n_2 = n$, then the number of different combinations is $(n_1 + 1)(n_2 + 1)$. The general ratio for the number of different combinations of factors has the form

$$N_y = (n+1) + 2n + 3(n-1) + 4(n-2) + ... + 2n + (n+1). \qquad (11)$$

In (11), the terms equidistant from the ends are the same. The first term in (11) gives the number of combinations when no x exceeds the threshold; the second term gives the number of combinations when only one x exceeds the threshold, etc.; the last term gives the number of combinations when all x are above the thresholds. In general, for some different combinations, the values of y may be the same. In particular, for a two-factor model without taking into account interactions, i.e. for $n = 2$, we find that there are 10 combinations. The stability of the system is violated in one case with the following values of parameters and factors:

1). $x_1 \geq L_{x_1}$, $x_2 \geq L_{x_2}$; $a_1 = a_2 =$ M; $x_1 = x_2 =$ VH $\rightarrow a_1x_1 = a_2x_2 \approx$ M, $y =$ VVH.

For other combinations, the system is stable. For the three-factor model without interaction, there are 20 combinations. The system is unstable in six cases:

1). $x_1 \geq L_{x_1}$, $x_2 < L_{x_2}$, $x_3 < L_{x_3}$; $a_1 =$ M, $a_2 = a_3 =$ L; $x_1 = x_2 = x_3 =$VH $\rightarrow a_1x_1 =$ M, $a_2x_2 = a_3x_3 =$ L, $y =$ VVH;

2). $x_1 \geq L_{x_1}$, $x_2 \geq L_{x_2}$, $x_3 < L_{x_3}$; $a_1 = a_2 =$ M, $a_3 =$ L; $x_1 =$ L, $x_2 = x_3 =$VH $\rightarrow a_1x_1 =$ VL-L, $a_2x_2 =$ M, $a_3x_3 =$ L, $y =$ VVH;

3). $x_1 \geq L_{x_1}$, $x_2 \geq L_{x_2}$, $x_3 < L_{x_3}$; $a_1 = a_2 =$ M, $a_3 =$ L; $x_1 = x_2 = x_3 =$VH $\rightarrow a_1x_1 = a_2x_2 =$ M, $a_3x_3 =$ L, $y =$ VVH;

4). $x_1 \geq L_{x_1}$, $x_2 \geq L_{x_2}$, $x_3 < L_{x_3}$; $a_1 = a_2 =$ M, $a_3 =$ L; $x_1 = x_2 =$ VH, $x_3 =$ L $\rightarrow a_1x_1 = a_2x_2 =$ M, $a_3x_3 =$ VL, $y =$ VVH;

5). $x_1 \geq L_{x_1}$, $x_2 \geq L_{x_2}$, $x_3 \geq L_{x_3}$; $a_1 = a_2 = a_3 =$ M; $x_1 =$ L, $x_2 = x_3 =$ VH, $\rightarrow a_1x_1 =$ VL-L, $a_2x_2 = a_3x_3 =$ M, $y =$ VVH;

6). $x_1 \geq L_{x_1}$, $x_2 \geq L_{x_2}$, $x_3 \geq L_{x_3}$; $a_1 = a_2 = a_3 =$ M; $x_1 = x_2 = x_3 =$ VH, $\rightarrow a_1x_1 = a_2x_2 = a_3x_3 =$ M, $y =$ VVH;

In the general case, for a linear model with n factors, we have the following results. If all $a_i =$ L and all $x_i =$ L, then for y we obtain with rounding $y = y_1 = n$VL. The system will be unstable for $n \geq 10$. If, for the same values of a_i, all $x_i =$ VH, then we obtain with rounding $y = y_1 = n$L. The system will be unstable for $n \geq 4$. If all $a_i =$ M and all $x_i =$ L, then for y we obtain with rounding $y = y_1 = n$(VL-L). The system is unstable for $n \geq 5$. If, for the same values of a_i, i.e. for $a_i =$ M, all $x_i =$ VH, then we obtain with rounding $y = y_1 = n$M. The system will be unstable for $n \geq 2$.

Now we take into account the interaction of factors. Consider model (9), which takes into account only pair interactions. As above, we perform calculations for the boundary values $x_i =$ L, and $x_i =$ VH. We assume that when only one of the factors reaches the threshold, the interaction parameter a_{ij} increases by one gradation, and when two or more factors reach the threshold, we consider two cases: a_{ij} increases by one or two gradations. The number of possible combinations is determined by (11), in which all terms, starting from the third, should be multiplied by 2, since when two or more x exceed the corresponding threshold values, then, according to our assumption, a_{ij} can take two values M and H. In particular, for the two-factor model with pair interactions, the number of different combinations is 13.

To perform the analysis, we will assume that the function describing pair interaction is equal to the product of functions of individual factors, i.e. $f(x_i, x_j) = f(x_i) \cdot f(x_j)$, and similarly for functions of a higher order. This condition means that the terms describing interactions of different orders decrease with an increase in the interaction order and the finite segment of series (9) approximates function y quite accurately, although the series as a whole may diverge.

In the general case, for a model with n factors, which takes into account pair interactions, we have the following results. If all $a_i =$ L, $a_{ij} =$ L and all $x_i =$ L, then the contribution $y_2 \approx 0$. The relative error in determining the contribution y_2 is $n/3$, and already at $n = 3$ it is equal to 1. Therefore, we cannot reliably determine this contribution more than for two factors. If, for the same values of a_i = L and a_{ij} = L, all x_i = VH, then the contribution $y_2 = n(n-1)$(VL-L)/2, and it is comparable with the contribution y_1. The relative error in determining the contribution y_2 in this case is $n/9$, and for n = 9 it is 1. Therefore, we cannot reliably determine this contribution more than for eight factors. The sum of contributions $y = y_1 + y_2 = n$L + $n(n-1)$(VL-L)/2. Therefore, the system is unstable for $n \geq 3$. If all a_i = M, a_{ij} = M and all x_i = L, then the contribution $y_2 \approx n(n-1)$VL/2 (with rounding), and it is comparable with the contribution y_1. The sum of contributions $y = y_1 + y_2 = n$(VL-L) + $n(n-1)$VL/2. Therefore, the system is unstable for $n > 3$. If, for the same values of a_i = M, a_{ij} = M, all x_i = VH, then the contribution $y_2 = n(n-1)$(L-M)/2, and it is comparable with the contribution y_1. The sum of contributions $y = y_1 + y_2 = n$M + $n(n-1)$(L-M)/2. Therefore, the system is unstable for $n \geq 2$. If a_i = M, a_{ij} = H and all x_i = L, then the contribution $y_2 \approx n(n-1)$VL/2 (with rounding), and it is comparable with the contribution y_1. The sum of contributions $y = y_1 + y_2 = n$(VL-L) + $n(n-1)$VL/2. Therefore, the system is unstable for $n > 3$. If, for the same values of a_i = M, a_{ij} = H, all x_i = VH, then the contribution $y_2 \approx n(n-1)$(M-H)/2, and it is comparable with the contribution y_1. The sum of contributions $y = y_1 + y_2 = n$M + $n(n-1)$(M-H)/2. Therefore, the system is unstable for $n \geq 2$.

In particular, for a two-factor model with pair interactions, that is, for n = 2, the system is unstable in three cases:

1). $x_1 \geq L_{x_1}$, $x_2 < L_{x_2}$; $a_1 = a_{12} = \mathrm{M}$, $a_2 = \mathrm{L}$, $x_1 = x_2 = \mathrm{VH} \rightarrow$ $a_1 x_1 = \mathrm{M}$, $a_2 x_2 = \mathrm{L}$, $a_{12} x_1 x_2 = \text{L-M}$, $y = \mathrm{VVH}$;

2). $x_1 \geq L_{x_1}$, $x_2 \geq L_{x_2}$; $a_1 = a_2 = a_{12} = \mathrm{M}$, $x_1 = x_2 = \mathrm{VH} \rightarrow a_1 x_1 = a_2 x_2 \approx \mathrm{M}$, $a_{12} x_1 x_2 = \text{L-M}$, $y = \mathrm{VVH}$;

3). $x_1 \geq L_{x_1}$, $x_2 \geq L_{x_2}$; $a_1 = a_2 = \mathrm{M}$, $a_{12} = \mathrm{H}$, $x_1 = x_2 = \mathrm{VH} \rightarrow a_1 x_1 = a_2 x_2 \approx \mathrm{M}$, $a_{12} x_1 x_2 = \text{M-H}$, $y = \mathrm{VVH}$;

The contributions from higher-order interactions can be considered in a similar way. It is clear that when we take into account the interactions of the second, third, and higher orders, the stability threshold of the system can only decrease, since additional terms appear that contribute to the value of y. For a given n, with an increase in the order of interaction, the contributions to y from the interaction decrease, although not very strongly, and, at the same time, the error in determining the contributions increases. The value of the gradation, starting from which the contributions become significant, increases with an increase in the order of interaction, although not very strongly. In addition, the value of n at which the interaction contributes to y increases with the order of the interaction. In particular, the second-order interaction is "switched on" starting with n = 3; the third-order interaction is switched on starting with n = 4, and so on. Therefore, in most cases, it is sufficient to take into account the contribution from the first-order interaction, i.e. from pair interactions.

V. Discussion of results

The results above show that a system represented by a linear two-factor model can be unstable when both factors take their maximum values VH. With an increase in the number of factors, the number of their combinations leading to system instability increases. Taking into account the interaction shifts the instability boundary towards lower values of the factors, and the number of combinations for which the system is unstable increases in comparison with the corresponding linear model. If all a_i = L, a_{ij} = L and all x_i = L, then the stability boundary is determined by the linear term in (9). If the values of the factors exceed L, then the contributions from the interaction in (9) can be comparable with the contributions from individual factors.

The obtained relations allow us to solve also the inverse problem, namely, to determine the permissible values of factors for which the change in stability of the system is within the given limits.

VI. Conclusion

Thus, the study shows the possibility of using the proposed approaches to assess the stability area of a system depending on the influence of many factors under uncertainty when factors and control parameters change, and it is difficult to give a formal description of the system. Application of classical approaches in this case is impossible or very difficult. The proposed approaches are useful for the systems of different types: social, economic, biological, environmental, etc., which affects only the interpretation of the elements. The representation of data as fuzzy gradations allows us to make estimates without being tied to a numerical context. The first proposed method is useful for qualitative analysis; the second method provides quantitative estimates.

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