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Development of Stresses in a Filled Jute Bag of Elliptic Cross-Section

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ABSTRACT:
The shape of a filled jute bag when laid flatduring stacking in storehouse is simplified into three different geometric shapes in order to calculate the development of stresses in different portions of the bag under uniform pressure. The purpose of such calculation is know the zone where stress development is maximum that may cause failure of the bag. The predicted zone that develops maximum stress matches the actual failure of jute bags reported by earlier research scientists..

Keywords: hemi-prolate spheroid, linear elastic membrane, elliptic cylinder,longitudinal stress, circumferential stress.

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I. Introduction

Jute bags are used to pack various kinds of material, like food grains, sand, sugar, etc. The filled bags are stacked vertically up to a height of 30 to 40 feet in the storehouse. Hence the bag at the lower portion of the stack are under severe pressure. A paper [1] has been published by the present authors where the necessity for calculating the stresses in the different portions of the bag have been discussed; an idealized structure of the bag has been considered where the bag is divided into three sections, central part is right circular cylinder with stitched flat top, bottom is hemispherical or hemi oblate spheroid and two right circular cones at the bottom corners. But the elliptic cross-section of the central cylindrical part closely resembles the geometry of a filled jute bag when the bag is laid flat or in case of stacking in a storehouse. Parsons [2] also considered the central part as an elliptic cylinder. In this case the spheroid at the bottom would be a hemi-prolate one, which is obtained by rotating an ellipse about its major diameter [Figure 1]. The cones at the corners will become oblique cones. The bag material is assumed to be continuous, homogeneous, isotropic, linear elastic membrane.

In this geometry only the hemi-prolate spheroid is a surface of revolution, the other two segments, the elliptic cylinder and oblique cone, are not surfaces of revolution. The analysis of stresses in membrane shells that are not surfaces of revolution is more complex than that in case of membrane shell of revolution. The complexity further increases in case of oblique cone since it is not a symmetrical structure. The development of stresses in the elliptic cylinder and hemi-prolate spheroid are discussed below. The stress development in oblique cone is not considered because it has been found [1] in the theoretical analyses that maximum stress always develops in the circumferential direction of the cylindrical segment and development of stress in the cone is minimum, which also corroborates the observation of Banerjee and others [3] - [5].

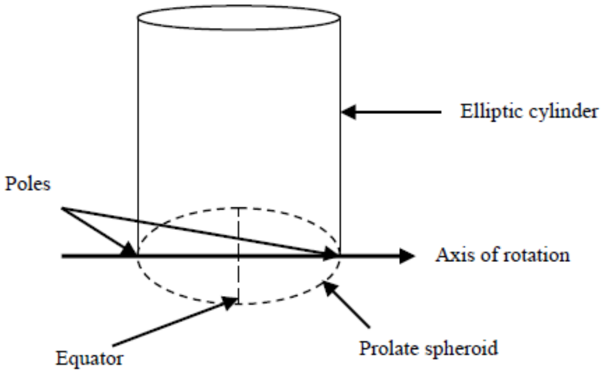

Figure 1 Elliptic cylinder with hemi-prolate spheroid at one end

The cylindrical segment is therefore appears to be the most important segment from stress development point of view in the bag. The position of a point on the surface of the elliptic cylinder is given by the coordinates *x* and *s*, where *x* is the distance from one end of the cylinder and *s* is the length of the arc in the circumferential direction measured from a definite generator shown in Figure 3. For elliptic cylinder, which is not a surface of revolution, the stresses in the corresponding directions are called longitudinal stress N_x and circumferential stress N_s.

For elliptic cylinder, Figure 2, $r_\varphi = \infty$ and $r_\theta = \dfrac{b^2/a}{\left(1-e^2\cos^2\varphi\right)^{\frac{3}{2}}}$

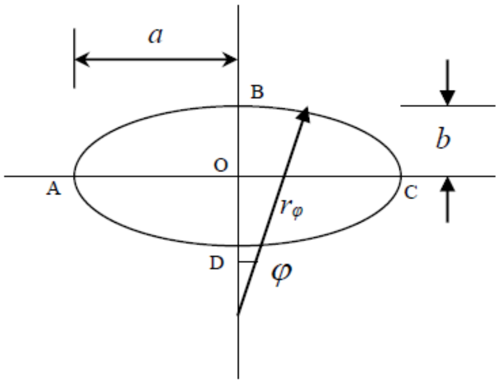

Figure 2.Geometry of elliptic cross-section

a, *b* are the semi-major and semi-minor diameters and *e* is the eccentricity of the ellipse;

$e = \sqrt{1-\dfrac{b^2}{a^2}}$; φ is the angle between the normal to the ellipse and the minor axis.

The circumferential stress is [6]

$$N_s = p\,\frac{b^2/a}{\left(1-e^2\cos^2\varphi\right)^{\frac{3}{2}}} \qquad (1)$$

N_s is a function of φ in case of elliptic cylinder.

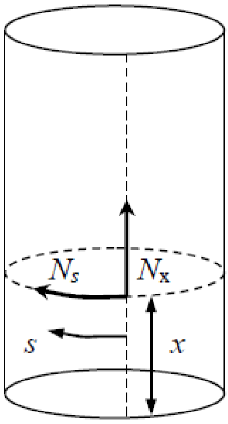

Figure 3. Stresses in elliptic cylinder

Morley [7] presented a simplified formula for finding longitudinal stress, N_x, which is as follows:

$$N_x = p\frac{(\text{area of the elliptic section})}{(\text{perimeter of the ellipse})} \qquad (2)$$

The circumferential stress, N_s , reaches the maximum value at the points B and D, shown in Figure 2, where $\varphi = 0$

$$(N_s)_{\max} = \frac{pa^2}{b} \qquad (3)$$

It reaches the minimum value at the points A and C, shown in Figure 2, where $\varphi = \pi/2$

$$(N_s)_{\min} = \frac{pb^2}{a} \qquad (4)$$

The minimum value of N_s at the points A and C is advantageous since the seams lie at these points and seam strength is lower than the fabric strength [30] and chances of failure are reduced with elliptic cross-section of the filled bag. The ratio of these two stresses is

$$\frac{(N_s)_{\min}}{(N_s)_{\max}} = \left(\frac{b}{a}\right)^3 = (1-e^2)^{\frac{3}{2}} \quad (5)$$

As the eccentricity increases the value of this ratio decreases which means the minimum value of N_s also decreases.
The perimeter of the ellipse is given by

$$4a\int_0^{\frac{\pi}{2}} \sqrt{1-e^2\sin^2\alpha}\, d\alpha = 2W$$

The size of a 50 kg B. Twill jute bag
Parsons [2] had given an equation that is used to determine the value of e from the ratio (R) of flat width (W) and flat length (L) of the bag. For 50 kg capacity B.Twill bags for packing food grain (IS:12650; 2003) is $L = 94$ cm, $W = 57$ cm,$R = 57 / 94 = 0.606$ for which $e = 0.35$ and the value of the elliptic integral is 1.5238 [8]. Therefore,

$$a = \frac{2\times 57}{4\times 1.5238} = 18.7 \text{ cm}$$

$$b = 17.49 \text{ cm}$$

Assuming an internal pressure p = 1 kg/cm^2

$$(N_s)_{\max} = \frac{pa^2}{b} = 19.99 \text{ kg/cm}$$

$$(N_s)_{\min} = \frac{pb^2}{a} = 16.36 \text{ kg/cm}$$

$$N_\varphi = \frac{p(\pi ab)}{2W} = 9.01 \text{ kg/cm}$$

Stresses are also calculated with internal pressures 0.5 and 2.0 kg/cm^2. These three stresses are calculated for other two dimensions *L/W* are 33 cm / 20 cm and 165 cm / 100 cm, keeping the same value of R = 0.606,with internal pressures 0.5, 1.0, 2.0 kg/cm^2 [9]. The values are shown in Table 1.

In the case of the hemi-prolate spheroid the semi-major and the semi-minor diameters are equal to that of the elliptic cylinder since it is joined at the end of the elliptic cylinder (Figure 2)

The principal radii of curvature of the hemi-prolate spheroid are

$$r_\varphi = \frac{b^2/a}{\left(1 - e^2 \sin^2\varphi\right)^{\frac{3}{2}}} \quad (6)$$

$$r_\theta = \frac{b^2/a}{\left(1 - e^2 \sin^2\varphi\right)^{\frac{1}{2}}} \quad (7)$$

So from equations (3) and (4)

$$N_\varphi = \frac{pr_\theta}{2} = \frac{pb^2}{2a} \frac{1}{\left(1 - e^2 \sin^2\varphi\right)^{\frac{1}{2}}} \quad (8)$$

$$N_\theta = pr_\theta \left(1 - \frac{r_\theta}{2r_\varphi}\right)$$

$$= \frac{pb^2}{a\left(1 - e^2 \sin^2\varphi\right)^{\frac{1}{2}}} \left[1 - \frac{\left(1 - e^2 \sin^2\varphi\right)}{2}\right] \quad (9)$$

At pole, φ = 0,

$$N_\varphi = N_\theta = \frac{pb^2}{2a} \quad (10)$$

At equator, φ = π/2,

$$N_\varphi = \frac{pb}{2} \text{ and } N_\theta = pb\left(1 - \frac{b^2}{2a^2}\right) \quad (11)$$

The longitudinal and circumferential stresses at pole and equator of the hemi-prolate spheroid are calculated for the bag dimensions given in Table 3.1 with internal pressures 0.5, 1.0 and 2 kg/cm^2 and the results are given in Table 1.(a), (b), (c):

Table 1 Stresses in Different Segments of Bag with Elliptic Cross-section under Different Internal Pressures

(a) Bag size: 33 cm / 20 cm

Pressure: 0.5 kg/cm²					
Stress (kg/cm)	Elliptic cylinder			Hemi-prolate spheroid	
	Eqn. (2)	Maximum Eqn. (3)	Minimum Eqn. (4)	Pole Eqn. (10)	Equator Eqn. (11)
Longitudinal	1.58	- (*)	-	1.43	1.53
Circumferential		3.50	2.87	1.43	1.72
Pressure: 1.0 kg/cm²					
Longitudinal	3.16	-	-	2.87	3.07
Circumferential		7.01	5.75	2.87	3.44
Pressure: 2.0 kg/cm²					
Longitudinal	6.32	-	-	5.74	6.14
Circumferential		14.02	11.5	5.74	6.88

(b) Bag size: 94 cm / 57 cm

Pressure: 0.5 kg/cm²					
Stress (kg/cm)	Elliptic cylinder			Hemi-prolate spheroid	
	Eqn. (2)	Maximum Eqn. (3)	Minimum Eqn. (4)	Pole Eqn. (10)	Equator Eqn. (11)
Longitudinal	4.51	-	-	4.09	4.37
Circumferential		9.99	8.18	4.09	4.91
Pressure: 1.0 kg/cm²					
Longitudinal	9.01	-	-	8.19	8.74
Circumferential		19.99	16.36	8.19	9.82
Pressure: 2.0 kg/cm²					
Longitudinal	18.02	-	-	16.38	17.48
Circumferential		39.98	32.72	16.38	19.64

(c) Bag size: 165 cm / 100 cm

Pressure: 0.5 kg/cm²					
Stress (kg/cm)	Elliptic cylinder			Hemi-prolate spheroid	
	Eqn. (2)	Maximum Eqn. (3)	Minimum Eqn. (4)	Pole Eqn. (10)	Equator Eqn. (11)
Longitudinal	7.91	-	-	7.18	7.67
Circumferential		17.52	14.37	7.18	3.44
Pressure: 1.0 kg/cm²					
Longitudinal	15.82	-	-	14.37	15.35
Circumferential		35.05	28.74	14.37	17.19
Pressure: 2.0 kg/cm²					
Longitudinal	31.64	-	-	28.74	30.70
Circumferential		70.10	57.48	28.74	34.38

[*Maximum and minimum values of longitudinal stress are not applicable since it is constant across the perimeter of the elliptic cross-section]

The Table 3.4 shows that the longitudinal and circumferential stresses increase in both the segments with the increase in the size of the bag and the internal pressure. For a given bag size and pressure stress in the circumferential direction of the elliptic cylinder is higher than the other stresses. The circumferential stress in the elliptic cylinder is maximum at the end of the minor diameter of the elliptic cylinder, points B and D in Figure 3.21. The circumferential stresses at the end of major diameter where the stitched portion of the bag lies, points A and C in Figure 3.21, is 82 % of the maximum circumferential stress for R = 0.606. Since the stitched portion of the bag is weaker than the corresponding fabric, so the elliptic cross-section is expected to give better performance of the bag in the stack. Equation (3.45) shows that the ratio of the minimum and maximum circumferential stresses decreases with the increase in eccentricity of the elliptic cross-section. The eccentricity increases with the increase in *R* [22], i.e., with smaller width and larger length of the bag. The eccentricity decreases as the degree of filling is increased and the cross-section of the bag approaches towards more circular one. Parsons [22] mentioned that to reduce chances of rupture bags should not be filled too tightly. Van der Feen et al [25] also observed that the maximum permissible load that could be borne by a filled sack decreased as the degree of filling was increased. The practical observations are in agreement with the theoretical analysis. The A. Morley, *Strength of Materials*, Longmans, Green and Co. Eleventh edition, 1954, Page 347.

II. Conclusions

With the increase in the dimensions of the bag the stresses the three segments increase. Stronger bag material is needed to manufacture bags of larger dimensions since there is an increase in stresses with the increase in the dimensions of the bag. For elliptic cross-section of the cylinder, the circumferential stress is lower at the ends of the major axis than that at the end of the minor axis. The ratio of these two increases with the increase in eccentricity (*e*). The eccentricity of the cross-section depends on the degree of filling and the width/length ratio of the bag. If the bag is partially filled it will give a flatter structure, means higher eccentricity, when the bag is laid down on its side (long dimension) during stacking which corroborates the findings of Persons [2]. Since the sewn portions also lie at the end of the major axis so the stress on it will be lessened as the degree of filling is diminished.

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