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Stresses in Granular Material

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Abstract:
Jute bags are generally used for carrying various granular materials. In this case, the development of pressure in the bags containing these materials seems to be closely related to the container containing granular materials. This paper deals with the development of stresses in granular materials alongside on its container of simple cylindrical geometry.
Keywords: granular material, bulk density, yield locus, Coulomb's powder, Mohr's Circle, flow value

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The Figure 1. shows a bin containing granular material and the different forces acting on a small slice of granular material.

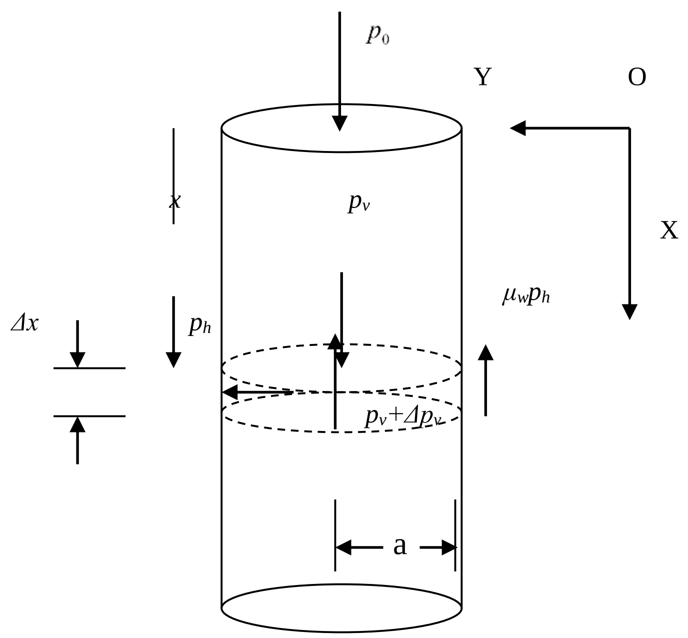

Figure 1. Forces acting on the slice of the granular material

Janssen [1] derived the following expression for the vertical pressure of granular material at a depth *x* from the top

$$p_v = \frac{\rho_b a}{\mu_w k}\left[1 - \exp\left(\frac{-\mu_w kx}{a}\right)\right] + p_0 \exp\left(\frac{-\mu_w kx}{a}\right) \tag{1}$$

Where, a is the radius of the container, the coefficient of friction between the granular material and the wall of the bin is μ_w, ρ_b is the bulk density of the granular material, p_0 is the external pressure acting on top of the granular material, p_h and p_v are the horizontal and vertical pressures of the granular material respectively and $k = p_h / p_v$; *k* is always less than unity in case of granular materials. Janssen assumed that the vertical pressure p_v is constant over horizontal plane and the bulk density of the granular material ρ_b is independent of depth. The horizontal pressure is given by

$$p_h = k\, p_v = \frac{\rho_b a}{\mu_w}\left[1 - \exp\left(\frac{-\mu_w kx}{a}\right)\right] + p_0\, k \exp\left(\frac{-\mu_w kx}{a}\right) \tag{2}$$

In the case of $p_0 = 0$, i.e., without external pressure

$$p_v = \frac{\rho_b a}{\mu_w k}\left[1 - \exp\left(\frac{-\mu_w kx}{a}\right)\right] \tag{3}$$

and

$$p_h = \frac{\rho_b a}{\mu_w}\left[1 - \exp\left(\frac{-\mu_w kx}{a}\right)\right] \tag{4}$$

also,

$$\exp(-\beta x) = (1 - \beta x + \cdots)$$

for small values of *x*, $1 - \exp(-\beta x) = \beta x$

in this case $\beta = \frac{\mu_w kx}{a}$

hence

$$p_v = \rho_b x \tag{5}$$

$$p_h = k \rho_b x \tag{6}$$

The equations (5) and (6) have some similarity with the liquid pressure, which is also the product of density and depth. But liquid exerts equal pressures in all directions at any point inside it, which does not happen in case of granular material. This difference is due to the fact that liquids cannot sustain shear forces but the bulk solids can. But at large *x*, when the exponential term is very small

$$p_v = \frac{a \rho_b}{\mu_w k} \tag{7}$$

$$p_h = \frac{a \rho_b}{\mu_w} \tag{8}$$

and the pressures become independent of depth, *x*, but depends upon the radius of the container which is an interesting characteristics of granular material.

It can be said that there is a critical depth $x_c = \dfrac{a}{\mu_w k}$ up to which the nature of the pressure development in bulk solid is similar to that of liquid and below this critical depth the pressure in bulk solid becomes independent of depth. Higher the radius of the cylinder higher is the critical length. This is shown in Figure 2.

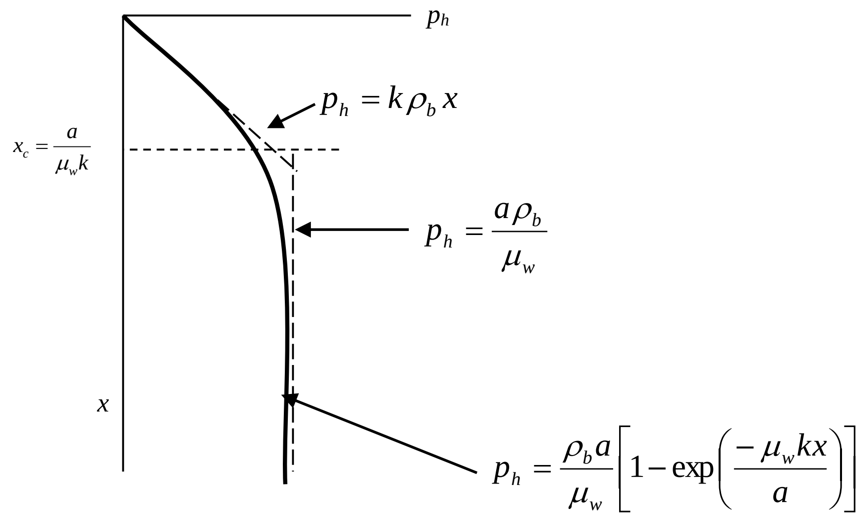

Figure2.Variation in horizontal pressure of granular material without external load

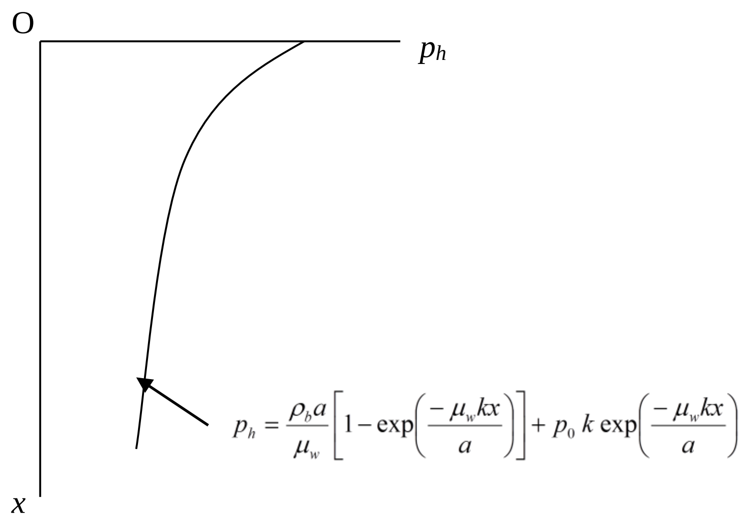

Figure 3. Variation in horizontal pressure of granular material under external load

Figure 3. shows that the effect of external load is prominent in the upper layers of the granular material and becomes insignificant at the bottom portion. In the expressions for horizontal pressure of granular material [Equation(2)], the bulk density ρ_b and the coefficient of friction between the granular material and the wall of the bin μ_w can be determined experimentally and are discussed in the next chapter but the value of $k = p_h / p_v$ is difficult to determine experimentally and so estimated from the following theoretical considerations.

Free flowing granular materials are characterized by a yield locus, also called the line of rupture, which is a plot of shear strength (S_f) under different normal stresses (N). When the applied shear stress equals the shear strength of the granular material, sliding of the granular material occurs. The relation between shear strength, S_f, (shear resistance per unit area), and the normal stress, N, (Normal load per unit area) is represented by an empirical equation:

$$S_f = \mu N + C \qquad (9)$$

There is a linear relation between shear strength and normal stress. This equation is represented graphically in Figure 4.

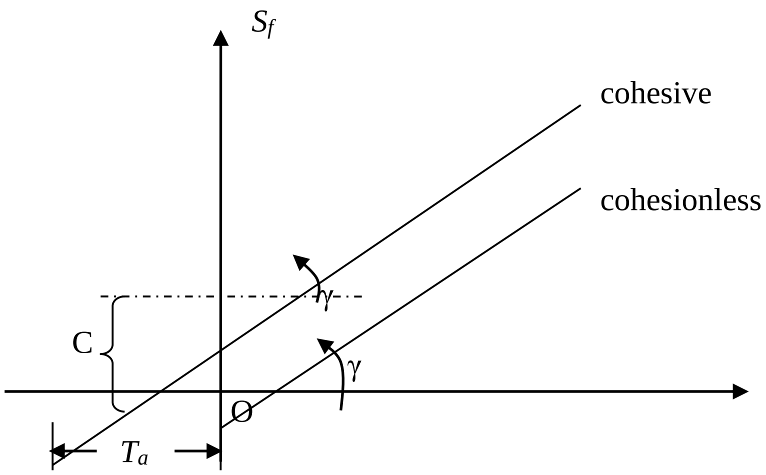

Figure 4. Yield loci of two types of granular materials

μ in equation (9) is the coefficient of friction between the granules and is equal to tan γ (Figure 4), γ is called the angle of internal friction. C represents the cohesion of the granular materials and is the shear strength at zero normal stress.

For cohesionless granular material (such as dry sand) $C = 0$ and equation (3.58) becomes

$$S_f = \mu N \qquad (10)$$

The equation (10) is known as Coulomb's equation and granular materials having linear yield locus are called Coulomb's powder [2].

The state of stress in every point inside the granular materials can be represented graphically by Mohr's circle which is shown in Figure 5.

The coordinates of the point D in the Mohr's Circle represent the value of σ and τ for a plane defined by the angle λ. called the aspect of the plane. For each different aspects of the plane in Figure 5 defined by the angle λ there is a corresponding point on the circle, the coordinate of which represent the normal and shear stresses on that plane. For granular material the normal stress is compressive in nature.

For example, when λ = 0, the plane coincides with the major principal plane I-I, and corresponds to the point A on the circle, where $\sigma = \sigma_2 = \sigma_v$ and $\tau = 0$, σ_v is the vertical compressive stress.

Similarly, for $\lambda = 45^0$, the point D coincides with the point F on the circle at which

$\sigma = \frac{1}{2}(\sigma_1 + \sigma_2)$ and $\tau = \tau_{max} = \frac{1}{2}(\sigma_2 - \sigma_1)$. The absolute value of τ is maximum for $\lambda = 45^0$.

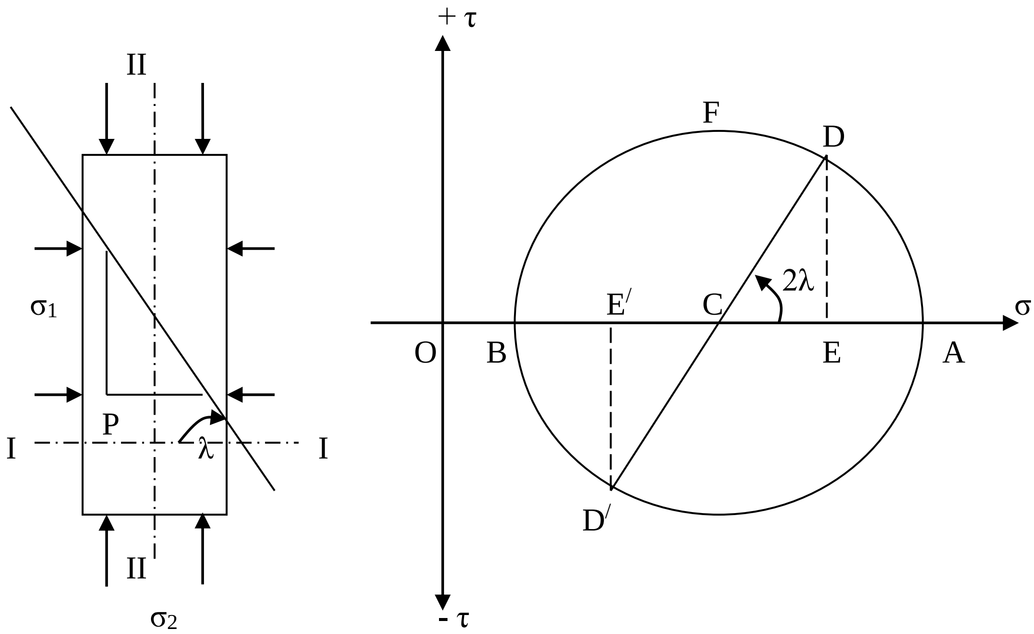

Figure 5 State of stress in granular material and Mohr's circle

(a) (b)

For $\lambda = 90^0$, the plane coincides with the minor principal plane II – II, and the point D coincides with the point B on the circle, where $\sigma = \sigma_1 = \sigma_h$ and $\tau = 0$, σ_h is the horizontal compressive stress.

Figure 6. illustrates the conditions of rest or sliding of the granular material. M_0M is the yield locus of the granular material. If the Mohr's circle does not touch the yield locus then the granular material is said to be in a state of rest.

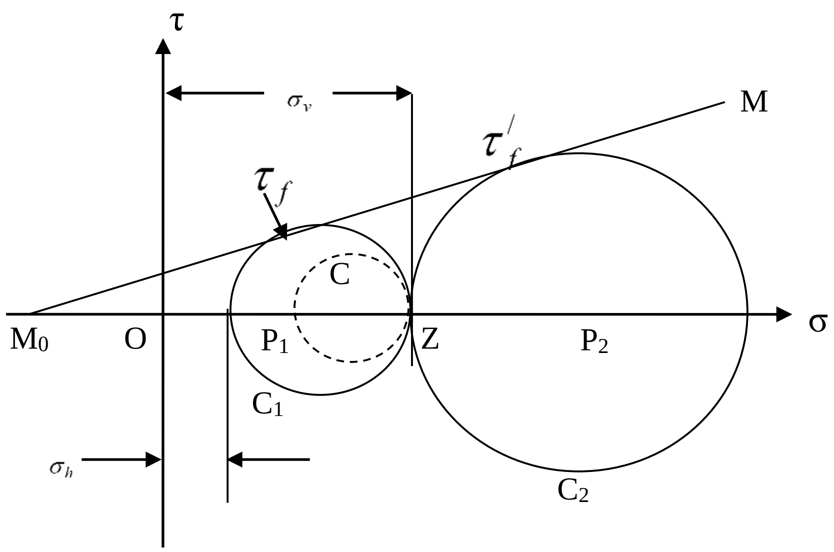

Figure 6. Mobilization of friction in Coulomb powder

On the other hand, if the Mohr's circles, like circles C_1 and C_2, touch the yield locus then the shear stress in the plane of the granular material corresponding to the point of tangency equals the shear strengths τ_f and $\tau_f^{/}$ and the granular material is in a point of incipient failure near the point P_1 and P_2 in that plane. In this case the frictional forces are fully mobilized. The granular material is then in a state of plastic equilibrium because once the sliding starts it will continue indefinitely until the state of stress changes to one similar to that shown by Mohr's circle C.

There are only two circles C_1 and C_2 through the point Z which satisfy the condition for plastic equilibrium. One of these circles, i.e., the circle C_1, is located on the left hand side and the other, C_2, on the right hand side of the point Z. In the first case, the major principal stress is in the vertical direction, $\sigma_v > \sigma_h$ and this state is called Active Rankine state [3] which occurs in the granular materials in bags. The second case is called Passive Rankine state where the major principal stress is in the horizontal direction, and $\sigma_v < \sigma_h$.

The principal stresses σ_1 and σ_2 for a mass of granular material at plastic equilibrium in the Active Rankine state at the point P can be obtained from the Figure 7:

Radius PA = PB = $\left(\sigma_2 - \sigma_1\right)/2 = \dfrac{(\sigma_2 - \sigma_a) - (\sigma_1 - \sigma_a)}{2}$

OP = $\dfrac{(\sigma_2 + \sigma_1)}{2}$ OM$_0$ = $-\sigma_a$ (apparent tensile strength)

M$_0$P = OP + OM = $\dfrac{(\sigma_2 + \sigma_1)}{2} - \sigma_a = \dfrac{(\sigma_2 - \sigma_a) + (\sigma_1 - \sigma_a)}{2}$

Also PA = M$_0$P sin γ

Therefore, $\sin\gamma = \dfrac{\mathrm{PA}}{\mathrm{M_0P}} = \dfrac{(\sigma_2 - \sigma_a) - (\sigma_1 - \sigma_a)}{(\sigma_2 - \sigma_a) + (\sigma_1 - \sigma_a)}$, $\dfrac{1 - \sin\gamma}{1 + \sin\gamma} = \dfrac{\sigma_1 - \sigma_a}{\sigma_2 - \sigma_a}$

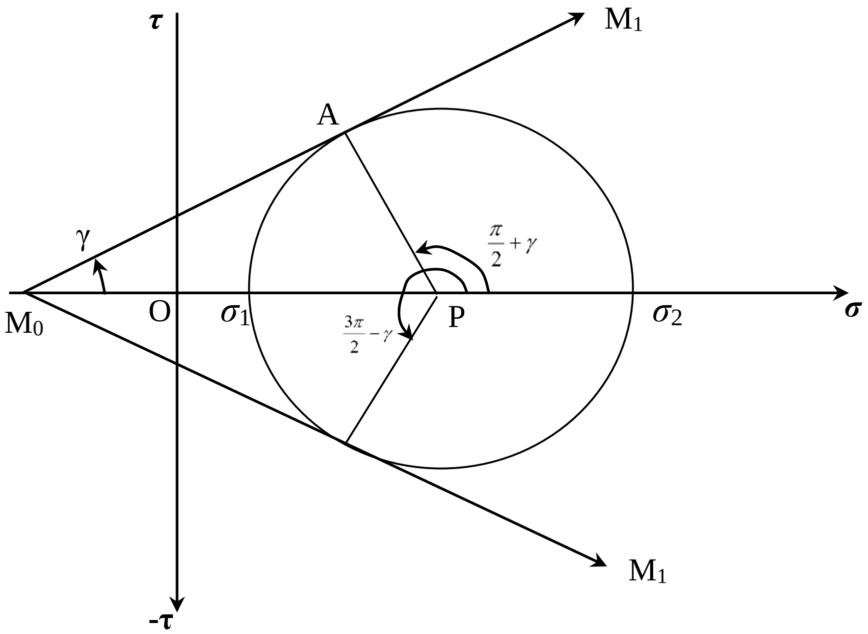

Figure 7. Direction of sliding of granular material in plastic equilibrium

If σ_a is small,

$$\frac{\sigma_1}{\sigma_2} \approx \frac{1-\sin\gamma}{1+\sin\gamma}$$

For cohesionless bulk solid, $\sigma_a = 0$, and the equation is exact.

$$\frac{\sigma_1}{\sigma_2} = \frac{1-\sin\gamma}{1+\sin\gamma}$$

$$= \frac{\sin^2\frac{\gamma}{2} + \cos^2\frac{\gamma}{2} - 2\sin\frac{\gamma}{2}\cos\frac{\gamma}{2}}{\sin^2\frac{\gamma}{2} + \cos^2\frac{\gamma}{2} + 2\sin\frac{\gamma}{2}\cos\frac{\gamma}{2}}$$

$$= \frac{\left(\cos\frac{\gamma}{2} - \sin\frac{\gamma}{2}\right)^2}{\left(\cos\frac{\gamma}{2} + \sin\frac{\gamma}{2}\right)^2} = \frac{\left(1 - \tan\frac{\gamma}{2}\right)^2}{\left(1 + \tan\frac{\gamma}{2}\right)^2}$$

$$= \frac{\left(\tan\frac{\pi}{4} - \tan\frac{\gamma}{2}\right)^2}{\left(\tan\frac{\pi}{4} + \tan\frac{\gamma}{2}\right)^2} = \tan^2\left(\frac{\pi}{4} - \frac{\gamma}{2}\right) = k \qquad (11)$$

Since the ratio $\frac{\sigma_1}{\sigma_2} = \frac{\sigma_h}{\sigma_v}$ and is similar to the ratio $\frac{p_h}{p_v} = k$ used in Janssen's equation [Equation (1)].

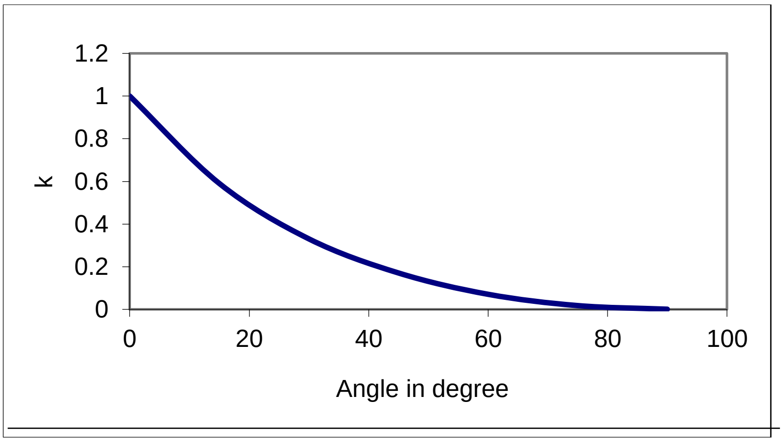

Figure 8. Variation in k for different values of γ

In soil mechanics [3] k is called the flow value. For materials without shear strength, like liquids at rest, $\gamma = 0$, $k = 1$ and $\sigma_1 = \sigma_2$. For $\gamma = \pi/2$, $k = 0$ i.e., material is having maximum shear strength, without any transverse deformation, $\sigma_2 = 0$.

The Yield loci, μ and μ_w are measured for three types of granular materials, rice, sand and flour [4], from which γ and k are calculated and shown in Table 1.

Table 1. Values of Angle of Internal Friction (γ) and Flow Value (k)

Material	μ_w (Table 4.12)	μ (Table 4.10)	$\gamma = \tan^{-1} \mu$	$k = \tan^2\left(\frac{\pi}{4} - \frac{\gamma}{2}\right)$	$\mu_w k$
Rice	0.403	0.554	28.98	0.35	0.14
Sand	0.566	0.576	29.94	0.33	0.19
Flour	0.717	0.777	37.84	0.24	0.17

Richard and Brown [1] also found that the product $\mu_w k$ for different types of granular materials to vary from 0.1 to 0.22 which is in agreement with the present findings.

Conclusions

Following the Janssen's equation, the horizontal component of pressure in the cylindrical portion of the bag containing granular material, builds up exponentially with depth. This pressure depend on the bulk density of the granular material, friction between the granular material and the bag wall, flow value of the granular material and radius of the cylindrical segment. There is a critical depth up to which the grain pressure is approximately similar to that of liquid beyond which it becomes independent of depth. The critical depth is found to be higher, in all the cases, than the height of the bag considered in the present study. .Effect of external pressure is severe in the upper layers of the column of the granular material and become insignificant in the bottom layers of a tall column.

References

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