

Multipurpose Multi Objective River Basin Optimization Using Hydraulic Jump Model

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ABSTRACT

This research is aimed at optimizing multipurpose multi objective river basin projects using hydraulic jump model with objectives of developing a hypothetical model, applying hydraulic jump model in six different water resources projects and developing optimum strategies for cost minimization and profit maximization of multipurpose multi objective river basin projects. River basin in Nigeria has being abandoned leading to destruction of aquatic lives, ecological imbalance and poverty. Game theory was used to optimize the accrued benefit obtained by application of the hydraulic jump simulated model in six different water resources management projects which includes hydroelectric power, irrigation, navigation, fishing, water supply and recreation. A hypothetical model was developed using Civil 3D, AutoCAD and Archicad. Digital elevation model of the hypothetical multi-purpose project was developed using Arc view GIS and Sufer 10. Results showed that the Maximin and Minimax values from the payoff matrix are 0.78 and 0.78 respectively. Therefore, player A maximizes his profit with an optimal strategy of $Z_{\max}=2.35x_1+0.78x_2+2.38x_3+1.30x_4+0.98x_5+1.73x_5$ to obtain a total score of ₦69,441,000,000 while player B minimizes his losses with an optimum strategy of $Z_{\min}=0.78y_1+0.40y_2+0.47y_3+0.71y_4+0.53y_5+0.22y_5$ to obtain a score of ₦20,208,000,00. Federal government should implement policies for the adoption of the developed optimization models in optimally allocating resources to various multi-purpose multi-objective river basin projects which include hydropower, irrigated agriculture, and navigation tourism and in fulfillment of the requirement of ISO 9001:2015.

KEYWORD: Hydraulic Jump, River Basin, Optimization Models, Froude Number, Game Theory.

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I. INTRODUCTION

The world's economy was shut down as a result of COVID 19 brouhaha. Virtually all aspects of economic activities were shut down unexpectedly sequel to the compulsory lockdown policy of industries. There is need to speedily recover from the shocking economic meltdown. A lucrative area to explore which has being abandoned today and can generate enormous amount of money is river basin. River basin if properly managed is unimaginable assets that can help nations recover quickly from the mishaps and help increase her GDP. River basin optimization is the panacea to the current world's economic collapse and could help achieve sustainable development goals even in the mist the pandemic.

Cost allocation of multi-purpose multi objective reservoir development is a serious factor

to consider in river basin optimization project (Okada 2015). Appropriate decision making in allocating water resources will help in optimally maximizing benefits to the society (Ahmad, Zhang, Liu, Naveed, Zaman, Tayyab, Waseem and Umar 2018). Optimal benefits of resources allocation with uncertainties was achieved using Bayesian Game models (Lei Xue, Changyin Sun, Fang and Yu 2017). Multipurpose multi objective simulation of river basins can help maximize unimaginable sustained profit (Mousavi J., Nasrin Rafiee N., Asl-Rousta B. & Kim J., 2017). The work of Gupta and Hira (2014), showed how game theory can be used to develop optimal strategies for profit maximization and cost minimization in executing multipurpose multi objective river basin projects.

River basins have some hydraulic problems. Akpan and Ledogo (2015) worked on

some hydraulic jump problems. They developed models that can simulate hydraulic jump characteristics. Most Scholars have not properly considered exploring hydraulic jump problems in river basin optimization. This research is aimed at optimizing river basin using hydraulic jump model with objectives of developing a hypothetical model, applying hydraulic jump model in five different water resources projects and developing optimum strategies for cost minimization and profit maximization of multipurpose multi objective river basin projects.

II. MATERIALS AND METHODS

The materials used to carry out this research includes, simulated hydraulic jump model as per (Akpan and Ledogo, 2015), Visual Basic, Suffer 8, Archi CAD, Civil 3D, Digitizer and Data acquisition software. Cost benefit analysis of five water resources management projects was done first while Game theory was used to develop the optimization models.

Determination of Accrued Benefit in Multipurpose Multi Objective Project

Assuming the dam can impound water with available head of 18m when it flows from upstream without obstruction at 20m/s. The head of water in the river after the jump is 30.36m when its flow with an initial velocity of 20m/s.

Hydroelectric power, irrigation, navigation, fishing, water supply and recreation were the multi-purpose river basin development projects considered in the analysis to determine the efficacy of the hydraulic jump simulated model.

Hydroelectric power

Assuming the dam can impound water with available head of 20m when it flows from upstream without obstruction at 18m/s. The head of water in the river after the jump is 30.36m when its flow with an initial velocity of 18m/s. let efficiency of penstock, turbine and generator be 94%, 80%, 84% respectively. The discharge of water expected from the reservoir is $50\text{m}^3/\text{s}$,

$$\text{Then } P_{18} = \frac{nP \cdot nt \cdot ng \cdot w \cdot Q \cdot H}{0.735}$$

$$= 0.94 \times 0.8 \times 0.84 \times 9.81 \times 50 \times 18 / 0.735$$

$$\text{Power for head of 18} = \left(\frac{5577.1}{0.735} \right) \text{kw} = 7587.9\text{HP}$$

$$\text{Power for head of 30.36}$$

$$P_{30.36} = \frac{0.94 \times 0.8 \times 0.84 \times 9.81 \times 50 \times 30.36}{0.735}$$

$$P_{30.36} = \left(\frac{9406.7}{0.735} \right) \text{kw}$$

$$\text{Hence } P_{18} = 7587.9\text{HP and } P_{30.36} = 12798.2\text{HP}$$

Therefore the stimulated model has vehemently improved the power of efficiency from 7587.9HP to 12798.2HP. Where P = Power developed, n_p = Penstock efficiency, n_t = Turbine efficiency, n_g = Generator efficiency, w = Specific weight of water, Q = Quality of water available per year, H = Head of water.

Irrigation

Assume the entire area of the reservoir is 500m^2 for depth of 18m, volume is $10,000\text{m}^3$, and case A for a depth of 30.36, volume is $15,180\text{m}^3$, case B. The following data of a soil were to be used for irrigation purpose, Depth of root zone = 160cm = 1.6m, Existing water content = 11.0%, Dry weight of soil = $16.5\text{km}/\text{m}^2$, Area of land for irrigation = $90,000\text{m}^2$, Volume of water applied for case A & B = $10,000\text{m}^3$ and $15,180\text{m}^3$, Water loss due to evaporation and seepage etc = 18%, Water retained in the soil = 92%, Calculate for field capacity

Case A (Q=10,000m³)

$$\text{Water retained in the soil} = 92\% \text{ of water applied} = 0.92 \times 10,000\text{m}^3 = 9200\text{m}^3$$

$$\text{Water retained per unit area} = \frac{9200}{90,000} = 1.102\text{m}^3$$

$$\text{Weight of water retained in soil} = 0.102 \times 10.0 = 1.022\text{KN}$$

$$\text{Dry weight of soil per unit area} = 16.5 \times 1.6 = 26.40\text{KN}$$

$$\text{Percentage of water retained} = \frac{1.022}{26.4} = 0.00387 = 3.87\%$$

$$\text{But field capacity of soil} = 5.88\% + 11.0\% = 16.88\%$$

Case B where Q = 15.180m³

$$\text{Water retained in the soil} = 92\% \text{ of water applied} = 0.92 \times 15,180\text{m}^3 = 13,965.6\text{m}^3$$

$$\text{Water retained per unit area} = \frac{13,965.6}{90,000} = 0.1552$$

$$\text{Dry weight of soil per unit area} = 16.5 \times 1.6 = 26.40\text{KN}$$

$$\text{Weight of water retained in soil} = 0.1552 \times 10 = 1.5517\text{KN}$$

$$\text{Percentage of water retained} = \frac{1.5517}{26.4} = 0.00387 = 3.88\%$$

$$\text{But existing water content} = 11.0\%$$

$$\text{Hence, for head of 20m, field capacity} = 3.88\% + 11.0\% = 14.88\%$$

Therefore the simulated model has vehemently improved the field capacity from 14.88 to 16.88%.

Water supply

Case A: Head of water before the jump = 18m

Case B: Head of water after the jump = 30.36m

Assuming the detention time is same for both cases (3hrs). Water is to be discharged from the reservoir

to town A, B and C. if all losses are same. Area of reservoir is same = $500m^2$.

Recall that $Q = \frac{V}{t} = \text{Area} \times \text{Head of water}$

$$\text{Discharge } Q_{18} = \frac{500 \times 20}{3 \times 360} = \left(\frac{3333.33}{3600}\right) m^3/s = 0.93m^3/s$$

$$\text{Discharge } Q_{30.36} = \frac{500 \times 30.36}{3 \times 360} = \frac{(15180.0)}{10.800} = 1.40m^3/s$$

Therefore, the simulated model has vehemently improve the discharge from $0.93m^3/s$ to $1.40m^3/s$

Recreation

Case A: Head of water before the jump = 18

Case B: Head of water after the jump =

30.36m

$$\text{Volume of water stored in clear reservoir } (Q_{18}) = \frac{500 \times 20}{1} = 10,000m^3/hr$$

$$\text{Volume of water stored in clear reservoir } (Q_{30.36}) = \frac{500 \times 30.36}{1} = 1,5180m^3/hr$$

Assumption

- Water pumped from clear water reservoir to the three towns will take 16hrs.
- Detention period of 8 hours is provided in clear water reservoir to ensure 24hrs supply capacity of water.

$$\text{Capacity of clear water reservoir for case A} = \frac{10,000 \frac{m^3}{hr} \times 8 \text{ hrs}}{24 \text{ hrs}} = 3,333m^3$$

For assumed L:B ratio of 2.5:1 adopt $20.4 \times 8.16 \times 20$ Assuming free board of 0.5 is to be provided, Then use reservoir capacity of $20.4 \times 8.16 \times 20.5$.

$$\text{Capacity of clear water reservoir for case B (head of } 30.36m) = \frac{15,180m^3 \times 8 \text{ hrs}}{24 \text{ hrs}} = 5,060m^3$$

Adopt $20.4 \times 8.16 \times 20$

Assuming free board of 0.5 is to be provided, then provide reservoir capacity of $20.4 \times 8.16 \times 20.5$

Therefore, the simulated model has vehemently improved the capacity of reservoir from $3,333m^3$ to $5,060m^3$

Fish pond

Assumption:

Since water will come into the storage tank that distributes water to the fish pond in 16hrs/day pumping time. The water is also expected to flow to the service reservoir (fish pond) by gravity flow in a time period of 18hrs. Storage of 2hrs is required on the storage tank. Hence a provision of 4 hours storage is to be used for the design so that the scheme can put 0hrs both as the water treatment, plant and the onward transmission of water to the service reservoirs. The two hours (2hrs) overlap of storage capacity is sufficient.

$$\text{Capacity of tank required for case A (at } 18m) = \frac{10,000 \times 4}{24} = 1,66.7 m^3.$$

$$\text{Capacity of tank required for case B (at } 30.36m) = \frac{15,180 \times 4}{24} = 2,530m^3$$

For a L× b ratio of 2.5b : b, Adopt = $14.4 \times 5.8 \times 30.3$

Assuming a freeboard of 0.5m is to be constructed, Adopt ($14.4 \times 5.8 \times 30.8$ freeboard) Therefore, the simulated model has greatly improved the capacity of storage reservoir for the fishpond from a storage capacity of $1,666.7m^3$ to $2,530m^3$.

Navigation

$$\text{Density} = \frac{\text{mass}}{\text{volume}} \text{ (Archimedes principle)}$$

Assume a ship has a total mass of 88,500KN, and area of $500m^3$.

$$\text{Then for case A where head (h) = 18, Density} = \frac{88,500KN}{500 \times 20} = \frac{88,500KN}{10,000m^3} = 8.85KN/m^3$$

For case B where head of water (H) = 30.36,

$$\text{Density of ship} = \frac{88,500KN}{500 \times 30.36} = \frac{88,500kn}{15,180m^3} = 5.83KN/m^3$$

The ship in case A (H = 18) has a density of $8.85KN/m^3$ while case B (H = 30.36) has a density of $5.83KN/m^3$. Hence bigger ship can safely navigate when the head of water measures from 18.0m to 30.36m, since the volume of water has increased from $10,000m^3$ to $15,180m^3$.

Note: The ship becomes less denser $8.85KN/m^3$ to $5.83KN/m^3$ when the head of water increases from 18 to 30.36. tabulated results of application before and after the jump x and y for case A and B are presented in table 4.1 below. The cost/benefit table of power efficiency, field capacity, volume, storage capacity, discharge and capacity of reservoir is presented in table 4.2 below

Optimization Modeling of the Hypothetical Multipurpose Multi Objective Projects

Game theory was used to develop the optimization model. Two strategies were employed to obtain optimal strategy. Strategy A is geared towards maximizing profit when investing in multipurpose multi objective projects while strategy B is geared towards minimizing losses when embarking on the multipurpose multi objective river basin project. To maximize profit, player A allocate 0.25%, 0.13%, 0.15%, 0.23%, 0.17% and 0.07% resources to A1, A2, A3, A4, A5 and A6 respectively. A1, A2, A3, A4, A5 and A6 represent hydroelectric power, irrigation, navigation, fishing, water resources and recreation respectively. Table 4.1 showed how the cost/benefit were obtained. The payoffs of accrued benefits are computed based on the percentages of resources allocation necessary to maximize profit by player A and minimize cost by Player B as shown in Table 3.1. below.

Table 3.1. Payoff of the accrued benefit to maximize profit

		Player B							
		B1	B2	B3	B4	B5	B6		
Player A	A1	0.25	9.4	2.35	0.78	2.38	1.30	0.93	1.73
	A2	0.13	3.1	1.22	0.40	1.24	0.68	0.48	0.90
	A3	0.15	9.5	1.41	0.47	1.43	0.78	0.56	1.04
	A4	0.23	5.2	2.16	0.71	2.19	1.20	0.85	1.59
	A5	0.17	3.7	1.60	0.53	1.62	0.88	0.63	1.17
	A6	0.07	6.9	0.66	0.22	0.67	0.36	0.26	0.48
				9.4	3.1	9.5	5.2	3.7	6.9

Table 3.1 above is used to determine the payoff matrix as shown in table 4.3 below. Analyzing for existence of saddle point, from table 4.2 the Maximin and Minimax values are 0.78 and 0.78 respectively. Hence the game is strictly determinable with a game value of 0.78. Therefore player A maximizes his profit with an optimal

strategy of
 $Z_{max}=2.35x_1+0.78x_2+2.38x_3+1.30x_4+0.98x_5+1.73x_6$
 to obtain total score of ₦69,441,000,000 while player B minimizes his losses with an optimum strategy of
 $Z_{min}=0.78y_1+0.40y_2+0.47y_3+0.71y_4+0.53y_5+0.22y_6$
 to obtain total score of ₦20,208,000,000.

III. RESULTS AND DISCUSSION

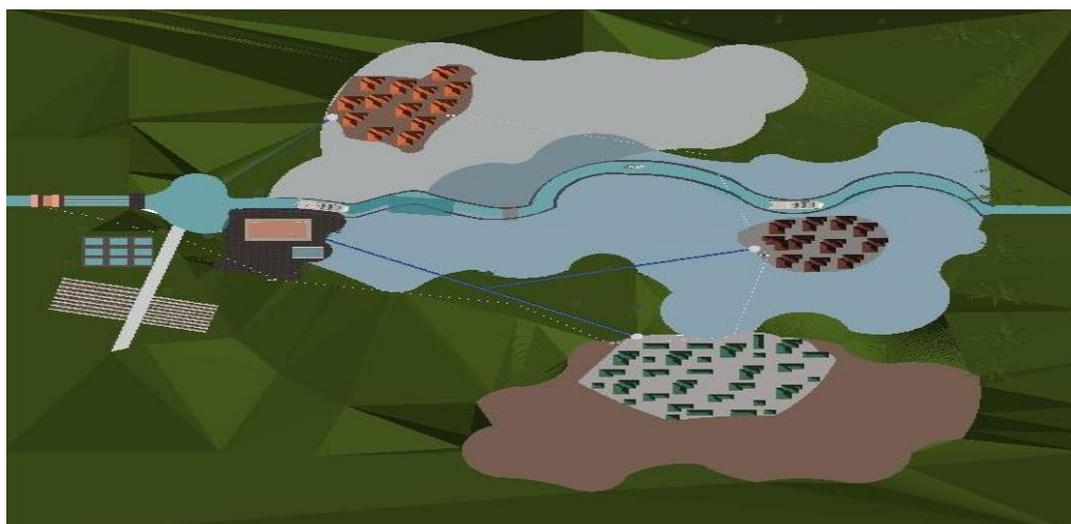


Figure 4.1. Top view showing the entire river Basin.

Discussion of results in figure 4.1 above

i. The simulated model in this research is applicable in river basin optimization. A hypothetical multipurpose project development in a river basin area was modeled to effectively

demonstrate the application of the new simulated model in river basin optimization.

ii. The model was developed using Civil 3D, AutoCAD and Archicad.

Table 4.1 . Cost/Benefit table of powere efficiency, Field capacity, Volume, Storage capacity, Discharge and Capacity of reservoir

X	Y	Benefit	Cost	Cost (Billion)
12798	7585	5213	9,400,000,000	9.4
17	15	2	3,100,000,000	3.1
15180	10000	5180	9,500,000,000	9.5
2530	1667	863	5,200,000,000	5.2
1.4	0.93	0.47	3,700,000,000	3.7
5060	3333	1727	6,900,000,000	6.9

Discussion of results in table 4.1

i. The cost of executing the projects are assumed to be 9.4,3.1, 9.5, 5.2, 3.7 and 6.9 billion naira for hydroelectric power, irrigation, navigation, fishing, water supply and recreation respectively.

ii. The benefit accrued from application of the hydraulic jump model in Case A and B are 5213hp, 2%, 5180m³, 863m³, 0.47m³/s and 1727m³ respectively.

Table4.2 . Summary Table of Application of the Simulated Models in River Basin Multi-purpose River Basin Projects.

S/N	H ₂ O Resources Mgt	Parameter	Results Case A (H = 18m)	Case B (H = 30.36m)
1.	Hydroelectric power	Power efficiency	7,588HP	12,798HP
2.	Irrigation	Field capacity	14.88%	16.88%
3.	Navigation	Volume	10,000m ³	15,180m ³
4.	Fishing	Storage capacity	1,667m ³	2,530m ³
5.	Water supply	Discharge	0.93m ³ /s	1.40m/s
6.	Recreation	Capacity of reservoir	3333m ³	5060m ³

Discussion of results in table 4.2

i. Six component of water resources management was considered which include hydroelectric power, irrigation, navigation, fishing, water supply, recreation
 ii. Assuming the dam can impound water with available head of 18m when it flows from upstream without obstruction at 20m/s. The head of water in the river after the jump is 30.36m when its flow with an initial velocity of 20m/s.

Discussion of results in table 4.3

i. From table 4.3 above the Maximin and Minimaz values are 0.78 and 0.78 respectively. Hence the game is strictly determinable with a game value of 0.78.
 ii. Therefore player A maximizes his profit with an optimal strategy of $Z_{max}=2.35x_1+0.78x_2+2.38x_3+1.30x_4+0.98x_5+1.73x_5$ to obtain a total score of ₦69,441,000 while player B minimizes his losses with an optimum strategy of $Z_{min}=0.78y_1+0.40y_2+0.47y_3+ 0.71y_4+0.53y_5+ 0.22y_5$ to obtain a score of ₦20,208,000.

Table 4.3. Payoff matrix for optimization of multipurpose multi-objective projects

0.78	2.38	1.3	0.93	1.73	0.78
0.4	1.24	0.68	0.48	0.9	0.4
0.47	1.43	0.78	0.56	1.04	0.47
0.71	2.19	1.2	0.85	1.59	0.71
0.53	1.62	0.88	0.63	1.17	0.53
0.22	0.67	0.36	0.26	0.48	0.22

0.78 2.38 1.3 0.93 1.73

IV. CONCLUSION

i. A hypothetical multipurpose river basin project development was modeled to effectively demonstrate the application of the new simulated model in river basin optimization.
 ii. The costs of executing the projects are assumed to be 9.4, 3.1, 9.5, 5.2, 3.7 and 6.9 billion naira for hydroelectric power, irrigation, navigation, fishing, water supply and recreation respectively.
 iii. The benefit accrued from application of the hydraulic jump model in Case A and B are 5213hp,

2%, 5180m³, 863m³, 0.47m³/s and 1727m³ respectively.

- iv. Player A maximizes his profit with an optimal strategy of $Z_{\max}=2.35x_1+0.78x_2+2.38x_3+1.30x_4+0.98x_5+1.73x_5$ to obtain a total score of ₦69,441,000 while player B minimizes his losses with an optimum strategy of $Z_{\min}=0.78y_1+0.40y_2+0.47y_3+0.71y_4+0.53y_5+0.22y_5$ to obtain a score of ₦20,208,000.

V. RECOMMENDATION

- i. Federal government should implement policies for the adoption of the developed optimization models in optimal allocation of resources to various multi-purpose multi-objective river basin projects which include hydropower, irrigated agriculture, and navigation tourism as it agrees with the requirement of quality management system (ISO 9001:2015).
- ii. Ministry of power, agriculture, tourism, and water resources should adopt the simulated hydraulic jump model sequel to the fact that the head of flow required to produce the desired jump for hydroelectric power, irrigation, navigation, fishing, water supply, and recreation can be obtained to optimally accrue mouth watering benefit.

CONTRIBUTION TO KNOWLEDGE

This thought will help researchers expand their knowledge in application of hydraulic jump problems in river basin optimization.

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