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Estimation of Non-Stationary in Time Stochastic Signals

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ABSTRACT: The method of construction of the modified filter of Kalman and his application is considered to the tasks of filtration of non-stationary in time casual signals.
KEYWORDS: Stochastic Signals, Filtration, Casual Process, Estimation

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I. INTRODUCTION

For the increase of exactness of stochastic signals estimation it is necessary to take into account greater totally of factors, influencing on the results. In this case tstimated signal must be examine not as determined, but as casual process and his estimation can be received with the use of stochastic methods of treatment by the synthesis of algorithms of filtration, adequate the probed classes of signals.

For application of theory of filtration to the technical systems development of mathematical theory is needed, and also methods of design of the systems, synthesis of filters and their practical realization. For the linear systems with the linear measurings at Gausse noises the theory of filtration is well developed and there are exact algorithms of optimum decision. However, for engineering practice the reliable methods of decision of design tasks, planning and realization of filters, adequate the probed signals, are needed, in addition. In practice, as a rule, in place of determination «optimum» apply the term of «suboptimum». Because optimization of process of realization of filter usually plugs in itself so a lot of factors, that it strongly hampers or does impossible mathematical description, an exact optimum decision is usually used only for the estimation of descriptions of the real filter at the known mathematical model of the probed signal.

[1,2].There are a few basic methods of filtration of descriptions of signals. A starting point for creation of theory of filters were works of Kolmogorov A.N. and Viner N., executed in 1940-th for stationary ergodic processes.

However much the Viner filter got practical application not immediately. It is explained difficulties, at first, receipts of exact decisions of equalizations of filtration, secondly, idenntifications of the required spectral distributing of signals and noises and, thirdly, designs of the physically realized systems. Many systems, presenting practical interest, simply fallen short of suppositions, to accepted in the theory of Viner filter.

II. MATERIALS AND METHODS

Use of estimations of kind

$$X = \sum S(n)$$

requires the set of statistics on the great number of realization, while we, frequently, dispose only one. Application of autoregressive methods assumes preliminary researches of processes with the purpose of construction of adequate stochastic models for the probed temporal rows and identification of signals characteristics. Thus it is also necessary to have a representative sampling of realization of the probed processes in the order.

[3, 4]. Development of Kalman-Bucy filter in 1960th removed the necessity of supposition about the stationarity of the system and about the presence of information on an endless time domain. In addition, a decision, got as a recurrent computational algorithm, does possible the direct synthesis of evaluation chart by computer and allows to get the real-time estimations of signal.

Receiving an evaluation of the current value in the form of dependence from previous values allows easy replacement of the n-th assessments on n+k make the forecast process in k steps forward.

For the case of the discrete measurings of signal *Sn*, will present him as additive mixture

$$S_n = X_n + Q_n \ , \qquad (1)$$

where X_n - useful signal; Q_n ‾ additive noise with the universal mean $M\left[Q_n\right] = 0$; R it is dispersion of noise, determined as

$$R = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2; \quad \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \ .$$

For the model of signal (1) Kalman filter equations can be written as

$$\begin{aligned} \hat{X}_n &= F_n \hat{X}_{n-1} + K_n\left[S_n - H_n F_n X_{n-1}\right] \\ K_n &= A_n H_n^T\left[H_n A_n H_n^T + R\right]^{-1} \\ A_n &= F_n P_{n-1} F_n^T \\ P_n &= A_n - K_n H_n A_n . \end{aligned} \qquad (2)$$

Where:

$\hat{X}_n$: is the estimated state vector;

Fn: is the matrix of transition from the state $n-1$ to n;

K_n : is the Kalman filter gain;

S_n : is the measured value of signal;

P_n: is the estimated state covariance matrix;

H_n : is the matrix of measurement conditions;

the index "T" means the transposition of the matrix.

Equations (2) can be applied for the receipt of estimation directly, if the matrix of transition F_n is known, that essentially, analytical type of the processed dependence. In practice, as a rule, analytical expression for the estimated signal is unknown, and determination of coefficients of make sense according to the first and second derivatives.

Known from the theory of filters causes of divergence (bias estimations), associated in this case, mainly with the ultimate value of m, there is a need for more control divergence. For these aims procedure which is taken to the count of statistics of kind is offered

$$B_M = \sum_{l=1}^{M} b_l , \quad B_0 = 0, \quad l = 1,2,..., \ M \qquad (6)$$

matrix is the difficult and labour intensive process of identification.

The proposed modification of Kalman filter (2), consists in the following.

It is known that the signal of free-form can be presented as decomposition in a row, for example, Teylor row. Limited to the members not higher than *m* order, for every element of *F* matrix can write down

$$F_{ij} = \begin{cases} \dfrac{j!}{i!\left(j-i\right)!} - \left(j-i\right), 0 \le i \le m, i \le j \le m, \\ 0, \quad j < i. \end{cases} \qquad (3)$$

Putting for definiteness $m = 2$, will get

$$F = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{vmatrix} . \qquad (4)$$

As measured only the signal itself, and not its derivatives, the matrix of measurement conditions $H = [\ 1 \ \ 0 \ \ 0\ ]$. Then substitution (4) in (2) gives procedure of estimation of signal $\hat{X}_n$ for the accepted approximation in a scalar form, as

$$\begin{aligned} \hat{x}_n &= \hat{x}_{n-1} + \hat{y}_{n-1} + \hat{z}_{n-1} + \alpha_n C_n \\ \hat{y}_n &= \hat{y}_{n-1} + 2\hat{z}_{n-1} + \beta_n C_n \\ \hat{z}_n &= \hat{z}_{n-1} + \gamma_n C_n \\ C_n &= S_n - \hat{x}_{n-1} - \hat{y}_{n-1} - \hat{z}_{n-1} , \end{aligned} \qquad (5)$$

where $\alpha_n = P_n^{11} R^{-1}$, $\beta_n = P_n^{21} R^{-1}$, $\gamma_n = P_n^{31} R^{-1}$ is elements of matrix Kalman filter gain $K_n = P_n \ H^T_n \ R^{-1}$, determined through the elements P_{ij} of the matrix P_n (2), and y_n and z_n

$$b_l = \mathrm{sgn}(\ S_l - \hat{X}_l\ ) = \begin{cases} +1, & S_l - \hat{X}_l \ge 0 \\ -1, & S_l - X_l < 0 \end{cases}$$

on an interval $(n - M\ , \ \ n)$. Defined on the interval values (B_M - $minB_M$) and ($maxB_M$ - B_M) compared to the threshold of *h*. When exceeding the values *h* one of the values of the decision on the divergence, the filter parameters are assigned initial values, and filtering lasts from the moment of (n - M). Particular interest are issues of forecasting future values of a time series. Such task can be decided on the base of the offered modified method of Kalman filtration. For this purpose sufficiently in (2) to put $H_n = [000\ \ ]$, and to present the index of n as $n = n + k$, where k *is* number of the steps set in advance.

With respect to the proposed modification (equation (5) and (6)), consider a block diagram of the software implementation suboptimal filter is presented in Fig. 1.

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The program's algorithm estimation of the measured signal is the following. First produced in the initial installation. As matrix R, as it is known from the theory of filtration, accepted the diagonal matrix of size 3 x 3 (in obedience to the accepted approximation). Moreover, the elements of the main diagonal are assigned the values of a priori found dispersion of the investigated process. The required number of measurings is set. As the mathematical expectation for non-stationary processes, it is expedient to choose the measurement value S_1.

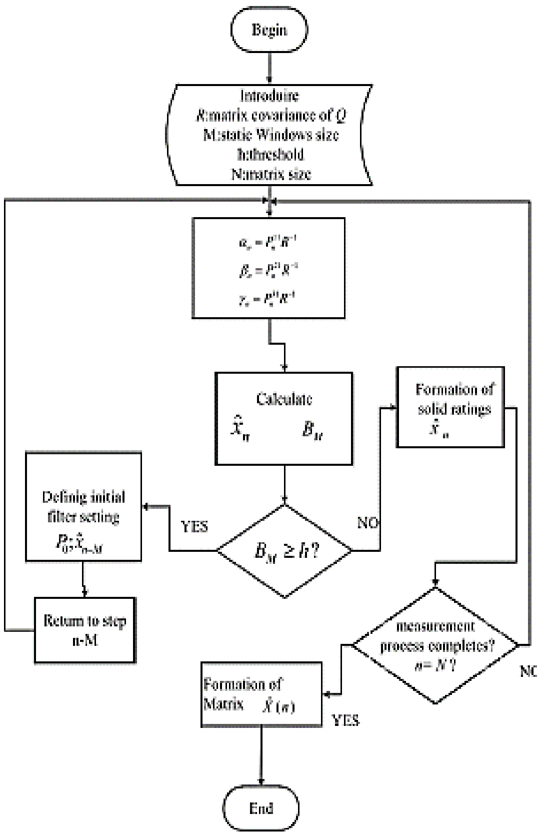

Fig. 1. Block diagram of algorithm

After starting the program calculates the gain of the filter and the current value $\hat{X}_n$ of estimation of the object state. Executes control the bias estimations and, if necessary, ajustments to the initial values of the filter. Forming of array estimations of the state object and calculation of his prognosis values is further made. All of process repeats oneself then. Under reaching the set number of measurings, the output of information is made.

III. EXPERIMENTATION

In connection with the fact that the gain Kalman filter presents an iterative ratio and, therefore, it cannot be expressed as a universal stationary solutions, efficiency of application of the modified algorithm will demonstrate a model example.

With this purpose, as the useful signal X_n (1) take a Gaussian signal type

$$X_n = e^{-(n-n_0)^2/2}, \qquad (7)$$

for which a transition matrix is determined as

$$F_n = \frac{X_n}{X_{n-1}} = \frac{e^{-(n-n_0)^2/2}}{e^{-(n-1-n_0)^2/2}} = e^{-(2n-2n_0-3)}. \quad (8)$$

As H=1, the system of equalizations (2) can be writtenin in a kind

$$P_n = \frac{F_n^2 P_{n-1} R}{F_n^2 P_{n-1} + R}, \qquad (9)$$

$$\hat{X}_n = F_n \hat{X}_{n-1} + P_n R^{-1} S_n - P_n R^{-1} F_n \hat{X}_{n-1}. \quad (10)$$

Thus as an estimation of error of process of filtration for the known matrix of transition P_n is accepted from (9), and for the modified algorithm of element P_{11} matrices of *Pn* from (2).

On a fig. 2 the result of the work of the modified algorithm of filter of Kalman performance is presented. On a fig. 3 a change the rationed error of estimation is resulted for the known and modified matrices of transition.

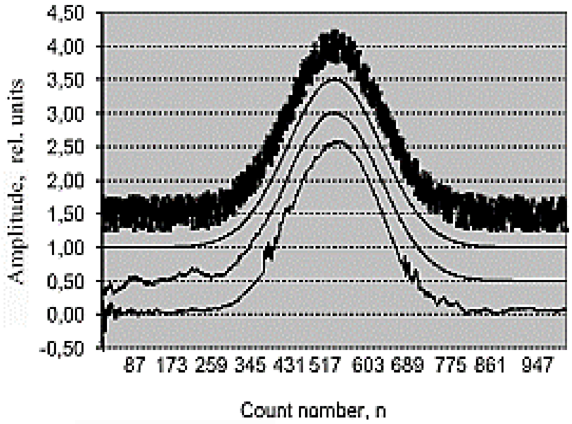

Fig. 2. Result of the work of the modified algorithm of Kalman filter performance, where from top to bottom measureable signal of S_n, useful signal of X_n, estimations of signal of $X_n(1)$ with the known matrix of transition and $X_n(2)$ with the modified matrix of transition, accordingly.

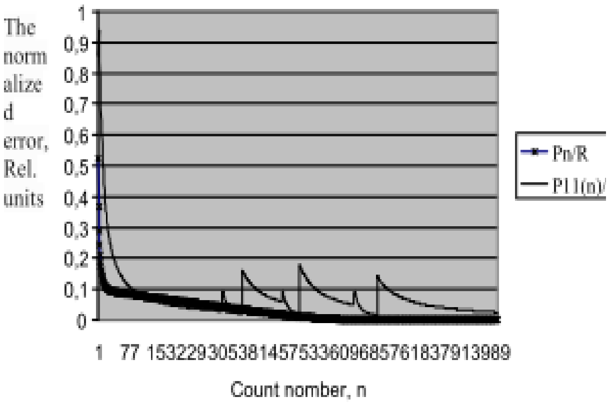

Fig. 3 Change the rationed error of estimation for the known and modified matrices of transition.

IV. CONCLUSION

The developed method of filtration, based on modification of filter of Kalman, allows to get suboptimum in sense of minimum dispersions of estimation of signals, characterized a substantial unstationarity in time and not having exact analytical description (the mathematical model).

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