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RESEARCH ARTICLE OPEN ACCESS

IIR filter design using wavelets

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ABSTRACT
Digital signal processing offers a wide range of opportunity for research with the development of technology. Wavelets are the popular topic in the recent days as they are finding their applications in digital signal processing. They are developed as a breakthrough of Fourier transforms and may be used in designing the digital filters with the cut-off frequency of π/2. In this work, we explore the application of digital frequency band transformation to obtain the digital low pass filter with cut-off frequency other than π/2.
Keywords – Digital filters, digital signal processing, frequency transformation, low pass filter, wavelets

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I. INTRODUCTION

Digital filters most often find their applications in digital signal processing. Popularly worked on digital filters are the Finite Impulse response (FIR) filters and Infinite impulse response (IIR) filters. The FIR filters are commonly designed through the Window method and Frequency sampling method [1]. In the design of IIR filter, the analog filter is first obtained and later transformed to the digital domain. This digitizing of the transfer function is done through four methods namely: i) mapping of differentials, ii) impulse invariant transformation, iii) bilinear transformation and iv)matched z- transform technique [2].

Wavelets are used for all engineering domains. They are mainly used for de-noising and data compression, while their use in filter is not emphasized. Wavelets coefficients may be used for digital FIR filter design [3]. In this work, we explore their use further with the application of frequency transformation technique. This section is followed by brief discussion on wavelets in section 2 and frequency transformation in section 3. In section 4 we discuss the application of frequency transformation for the first order and third order FIR filter design followed by simulation study in section 5 and lastly, the conclusion.

II. Wavelets

Wavelets are the mathematical tools used in digital signal processing to obtain the information from the signal [3-4]. A wavelet is a small duration waveform with a zero average value [5]. The wavelet function breakup the signal into its frequency components and each component is analyzed giving rise to the frequency information at the instant of time.

A discrete signal is divided into two sub-signals. Each contains half the number of samples present in the original sequence. They are time scaled and time shifted in discrete steps. Each wavelet is characterized by its scaling function, represented as φ(t) and wavelet function ,η(t). The scaling function coefficients are used to derive the wavelet coefficient functions. Haar wavelet or Daubechies 1 (dB1) wavelet is the simplest wavelet. D2 to D20 (db1 to db10 respectively) are the popularly used Daubechies wavelets. The Haar wavelet coefficients (dB1) are given by equation 1 and the Daubechies 2 (dB2) wavelets coefficients are given by equation 2[3]:

Haar wavelet (dB1) wavelet:
Scaling coefficients, a_k = [0.707 0.707]
Wavelet coefficients, b_k = [0.707 -0.707] (1)

Daubechies 2 (dB2) wavelet:

Scaling coefficients, a_k = [0.4830 0.8365 0.2241 -0.1294]
Wavelet coefficients, b_k = [-0.1294 -0.2241 0.8365 -0.4830] (2)

The wavelet basis functions are summed and weighted using wavelet transform coefficients to construct any signal. The wavelet basis functions may be obtained from the scaling and wavelet coefficients iteratively, as explained by the following pseudo code [3].

Scaling function:
1: scal = a_k
2: scal_up=up sample (scal)

3: scal=convolve (scal_up, a_k)
4: repeat 2 and 3 until proper scaling function is obtained.
Wavelet function:
1: wav = b_k
2: wav_up=up sample (wav)
3: wav=convolve (wav_up, a_k)
4: repeat 2 and 3 until proper wavelet function is obtained.

Here, we consider the db1 and db2 wavelets for our study. The db1 (Haar) wavelets are shown in Fig. 1 and the db2 (Daubechies 2) wavelets are shown in Fig. 2.

2.1 Filter using wavelet coefficients

We have the scaling function, φ(t) as given in equation 3 and wavelet function, η(t) as in equation 4 [3]:

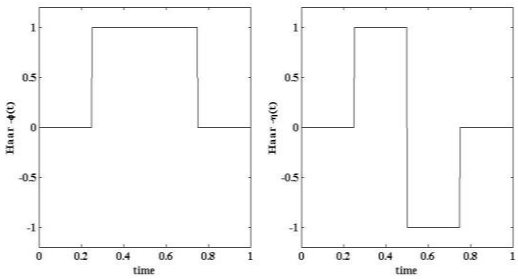


Fig. 1. Haar wavelets: Scaling function, φ(t) and wavelet function, η(t)

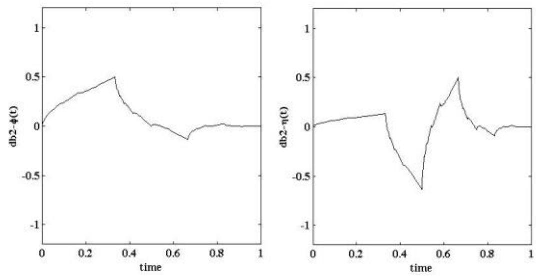


Fig. 2. Daubechies 2 wavelets: Scaling function, φ(t) and wavelet function, η(t)

$$\varphi(t) = \sum_{k=0}^{2N-1} a_k \varphi(2t - k) \qquad (3)$$

$$\eta(t) = \sum_{k=0}^{2N-1} b_k \varphi(2t - k) \qquad (4)$$

Where, the filter order, N is even always. The coefficients, a_k and b_k represent that of the filter coefficients. Equation 3 and equation 4 show the relation of wavelets to filters. Scaling coefficients show the characteristics of that of a low pass filter while the wavelet coefficients display that of the high pass filter [3]. Scaling function can be thought of as a signal with a spectrum of low-pass filter and wavelet functions as the signal with high-pass spectrum. As a result, the low pass digital FIR filter can be obtained with the coefficients of the scaling function of the wavelets.

The frequency responses of filters using the wavelets have the cut off frequency of π/2 always. Attempt is made here to design the filter with the different cut-off frequency using frequency transformation technique. The next section describes the digital frequency transformation that can be applied for the digital filter design.

III. FREQUENCY TRANSFORMATION

Frequency transformation is the technique to design filters like the high pass filter or the band pass filter or the band stop filter from the low pass filter. A low pass filter with the different cut off frequency can also be designed from the low pass filter using frequency transformations. Two cases of frequency transformations are possible [2][6]:

Continuous frequency band transformation: Here, analog low pass filter is transformed to different analog filter that is digitized to obtain the desired digital filter.

Digital frequency band transformation: where, analog low pass filter is digitized followed by the application of digital frequency transformation to get the digital filter.

Many design formulas are available for the continuous–time filter design. They are all related to the low pass filters [6]. Continuous Frequency transformation in filter design preserves the equiripple characteristics of the prototype filter [2]. Earlier, the frequency transformations in digital filters were done through continuous filters. With the development of digital filters, the frequency transformation on z-plane found its importance [7]. In Digital transformation, z^{-1} in the transfer function of the low pass filter is replaced by the all pass function to obtain the desired new transfer function. This all pass function may be first order or second order [6]. Frequency transformation technique in the s-domain and bilinear z-transformations is being used in a lot of research regarding the design of digital filter. These methods were stable in the mapping of poles and zeros from one domain to the other [8].

One of the ways in which digital FIR filters can be designed is using the wavelet coefficients. FIR filters are preferred because of their stability and linearity in phase. In the following section, we discuss the application of frequency transformation for the FIR filter design using Daubechies wavelet coefficients.

IV. FREQUENCY TRANSFORMATION ON WAVELET COEFFICIENTS

Frequency transformation can be done on a digital low pass filter by replacing the variable z^{-1} as given by equation 5. A suitable set of simple transformations is available to convert the digital low pass filter [2]. Here we consider the conversion from the low pass filter with cut-off frequency, $\omega_c = \pi/2$ to low pass filter of different cut-off frequency. From the set of transformations, we have equation 5 and equation 6 [2]:

$$z^{-1} \rightarrow \frac{z^{-1} - \alpha}{1 - \alpha z^{-1}} \qquad (5)$$

Where,

$$\alpha = \frac{\sin[(\omega_c - \omega_n)/2]}{\sin[(\omega_c + \omega_n)/2]} \qquad (6)$$

Here, ω_n is the cut-off frequency of the new filter. All-pass type of transformation where the unit circle is mapped into itself more than once is used, maintaining the magnitude response of the low pass filter. The transformed filter is obtained by transforming the prototype low pass filter into a suitable low pass design and applying the first or second order substitution for z^{-1} [2]

We start with the low pass filter design using scaling coefficients of the wavelets. The cut-off frequency of this low pass filter is always $\pi/2$. In order to transform it to low pass filter with new cut-off frequency, let the desired filter have the cut-off frequency ω_n, which is other than $\pi/2$. Using equation 6, the value of α is determined. The frequency transformation is then applied to get the new filter coefficients. These new coefficients are then considered to compute the frequency response of the transformed filter. In this process FIR filter becomes IIR filter. We consider the first order and third order systems for our work.

The transformed filter coefficients may be obtained as explained below

4.1 First order systems:

For digital low pass FIR filter of length N=2, considering the scaling coefficients, a_k, we have H(z) given by equation 7:

$$H(z) = h_0 + h_1 z^{-1} \qquad (7)$$

Using equation 5 in equation 7 we have the new transfer function as in equation 8:

$$H_{new}(z) = h_0 + h_1\left(\frac{z^{-1} - \alpha}{1 - \alpha z^{-1}}\right) \qquad (8)$$

From equation 8 we have

$$N_1 = h_0\left(1 - \alpha z^{-1}\right)$$
$$N_2 = h_1\left(-\alpha + z^{-1}\right) \qquad (9)$$

Collecting the respective terms from equation 9, we have the new filter coefficients given by equation 10:

$$H_0 = h_0 - \alpha h_1$$
$$H_1 = \left(-\alpha h_0 + h_1\right) z^{-1} \qquad (10)$$

The new transfer function is now as in equation 11:

$$H_{new}(z) = \frac{H_0 + H_1 z^{-1}}{1 - \alpha z^{-1}} \qquad (11)$$

That now resembles an IIR filter.

4.2 Third order systems:

Considering the scaling coefficients, a_k for the third order systems with length, N=4, the transfer function of the low pass digital FIR filter is given by equation 12:

$$H(z) = h_0 + h_1 z^{-1} + h_2 z^{-2} + h_3 z^{-3} \qquad (12)$$

Using equation 5 in equation 12 we have the new transfer function given by equation 13:

$$H_{new}(z) = h_0 + h_1\left(\frac{z^{-1} - \alpha}{1 - \alpha z^{-1}}\right) + h_2\left(\frac{z^{-1} - \alpha}{1 - \alpha z^{-1}}\right)^2 + h_3\left(\frac{z^{-1} - \alpha}{1 - \alpha z^{-1}}\right)^3 \qquad (13)$$

From equation 13, we have equation 14:

$$N_1 = h_0\left(1 - \alpha z^{-1}\right)^3$$
$$= h_0\left(1 - 3\alpha z^{-1} + 3\alpha^2 z^{-2} - \alpha^3 z^{-3}\right)$$
$$N_2 = h_1\left(z^{-1} - \alpha\right)\left(1 - \alpha z^{-1}\right)^2$$
$$= h_1\left(-\alpha + \left(1 + 2\alpha^2\right) z^{-1} - \left(2\alpha + \alpha^3\right) z^{-2} + \alpha^2 z^{-3}\right)$$
$$N_3 = h_2\left(z^{-1} - \alpha\right)^2\left(1 - \alpha z^{-1}\right)$$
$$= h_2\left(\alpha^2 - \left(2\alpha + \alpha^3\right) z^{-1} + \left(1 + 2\alpha^2\right) z^{-2} - \alpha z^{-3}\right)$$
$$N_4 = h_3\left(z^{-1} - \alpha\right)^3$$
$$= h_3\left(-\alpha^3 + 3\alpha^2 z^{-1} - 3\alpha z^{-2} + z^{-3}\right) \qquad (14)$$

Collecting the respective terms from equation 14, we have the new filter coefficients given by equation 15:

$$H_0 = h_0 - \alpha h_1 + \alpha^2 h_2 - \alpha^3 h_3$$
$$H_1 = \left(-3\alpha h_0 + \left(1+2\alpha^2\right) h_1 - \left(2\alpha + \alpha^3\right) h_2 + 3\alpha^2 h_3\right) z^{-1}$$
$$H_2 = \left(3\alpha^2 h_0 - \left(2\alpha + \alpha^3\right) h_1 + \left(1+2\alpha^2\right) h_2 - 3\alpha h_3\right) z^{-2}$$
$$H_3 = \left(-\alpha^3 h_1 + \alpha^2 h_2 - \alpha h_3 + h_4\right) z^{-3} \quad (15)$$

The transfer function is now said to be given by equation 16 representing the IIR filter:

$$H_{new}(z) = \frac{H_0 + H_1 z^{-1} + H_2 z^{-2} + H_3 z^{-3}}{1 - 3\alpha z^{-1} + 3\alpha^2 z^{-2} - \alpha^3 z^{-3}} \quad (16)$$

The frequency response of the new filter is computed.

Design procedure is summarized as below

1. Consider the low pass filter with the $\omega_c = \pi/2$, designed using wavelet coefficients
2. For the new cut-off frequency, ω_n other than $\pi/2$, find the value of α.
3. Obtain the value of the new filter coefficients
4. Compute the frequency response of the filter thus designed.

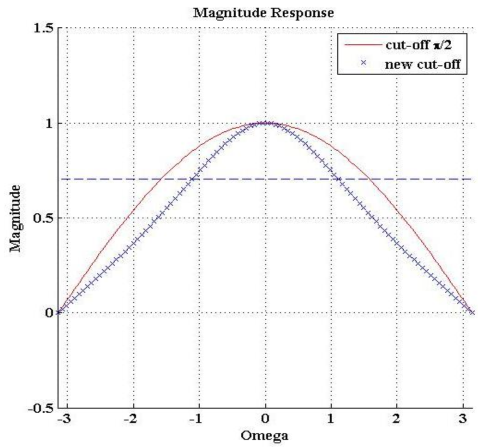
V. SIMULATION

Here, we discuss the design of the low pass digital filter using wavelet coefficients through frequency transformation. In this work, we have considered the Daubechies1 and Daubechies2 wavelet. Scaling coefficients show the characteristics of the low pass filter with $\omega_c = 0.5\pi$ always, irrespective of the order of the filter. Hence, attempt is made to transform the low pass filter designed using wavelet coefficients to the low pass filter of different cut-off frequency using the digital frequency transformation.

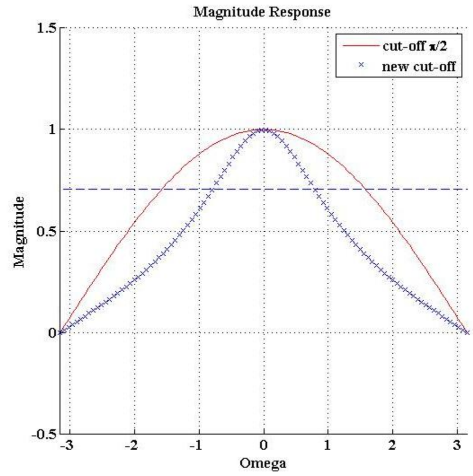
Consider the Daubechies1 wavelet with the new cut-off frequency, ω_n=0.35π, and the scaling coefficients, a_k=[1 1]. From equation 6, we have equation 17:

$$\alpha = \frac{\sin[(0.5\pi - 0.35\pi)/2]}{\sin[(0.5\pi + 0.35\pi)/2]} \quad (17)$$

The frequency transformation is applied using the value of α as in equation 17, to obtain the new filter coefficients as given by equation 10 leading to the new transfer function given by equation 11. The simulation result shows that the cut-off frequency is shifted to new value of 0.35π, from the cut-off of 0.5π as shown in Fig.3a. It is observed that the magnitude of the response remains unchanged as desired. Similar observation is noticed for Harr wavelet with cut-off frequency at 0.25 π, 0.65π and 0.75 π as shown in Fig. 3b, Fig. 3c and Fig. 3d respectively.



a)



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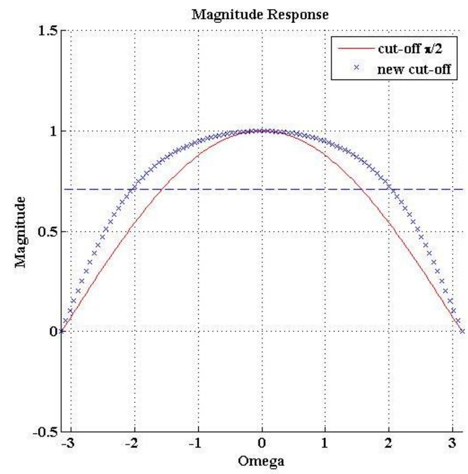

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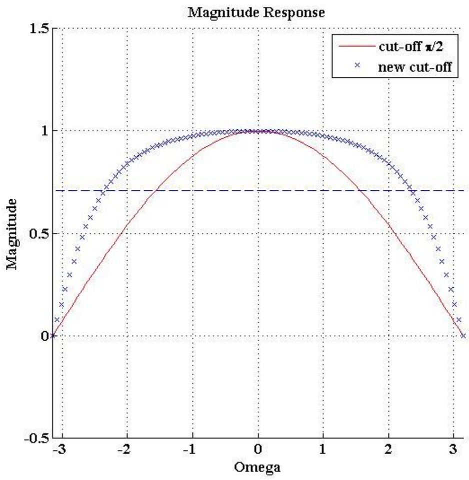
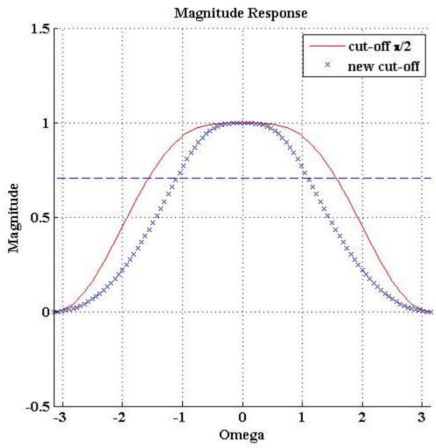


d)

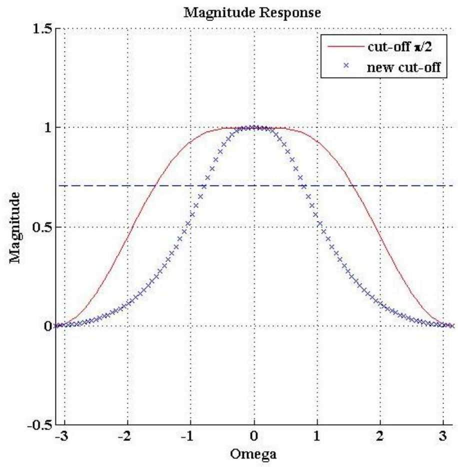
Fig. 3. Magnitude response of digital low pass filter with N=2, for the cut-off of a) 0.35π, b) 0.25π c) 0.65π, d) 0.75π

Considering the Daubechies 2 wavelet for the similar cases of cut-off frequency of 0.35π, the new filter coefficients as given by equation 15 giving rise to the new transfer function given by equation 16.

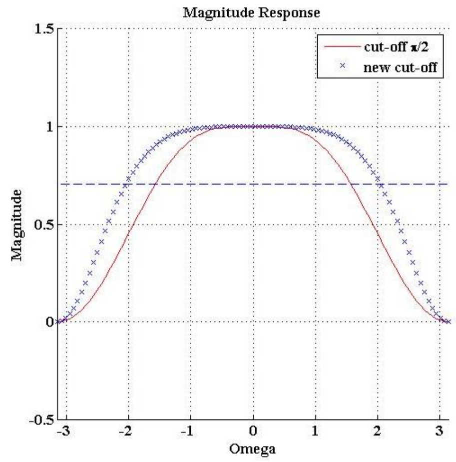
The simulation result shows that the cut-off frequency is shifted to new value of 0.35π, from the cut-off of 0.5π as shown in Fig. 4a.



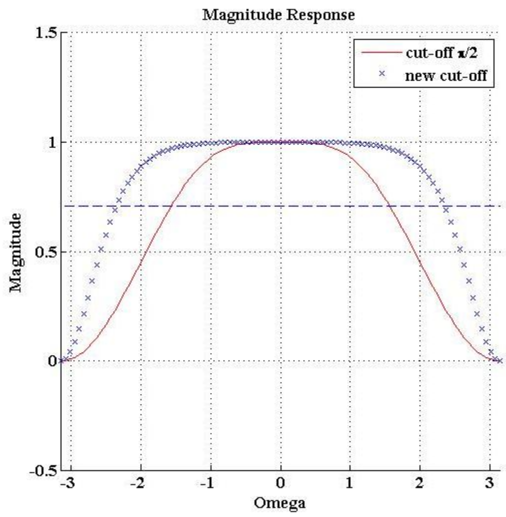
a)



b)



c)



d)

Fig. 4. Magnitude response of digital low pass filter with N=4, for the cut-off of a) 0.35π, b) 0.25π c) 0.65π, d) 0.75π

Similar observation is noticed for cut-off frequency at 0.25 π, 0.65π and 0.75 π as shown in Fig. 4b, Fig. 4c and Fig. 4d respectively. Observation shows that the magnitude of the response is maintained.

VI. CONCLUSION

Digital filter transformation applied to the filter design using wavelet coefficients is presented. Digital FIR filters are designed using different techniques. The filter designed using the wavelet coefficients have the cut-off of 0.5π always. Using the set of frequency transformations it is possible to design the low pass filter of any cut-off frequency from the low pass filter of 0.5π cut-off frequency. It is observed that the cut-off frequency of the filter is changed to new value, preserving the magnitude of the original filter response. This filter is now an IIR filter. This work may be applied to the higher order systems, with the length of the filter being even always. The work can be continued to compare the performance of the IIR filter with the IIR filter designed by transforming the analog filter.

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