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Cordial Decomposition in Various Graphs

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ABSTRACT:
The main goal of this paper is to introduce and investigate the results of cordial decomposition and cordial decomposition number $\pi_C(G)$ of a graphs. Also investigate some bounds of $\pi_C(G)$ in product graphs like Cartesian product, composition etc.

Keywords: Decomposition, labeling, cordial graphs, cordial decomposition and prime decomposition number.

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I. INTRODUCTION

In this Chapter, we define cordial decomposition and cordial decomposition number $\pi_C(G)$ of a graphs. Also investigate some bounds of $\pi_C(G)$ in product graphs like Cartesian product, composition etc.

A bijection $f: V(G) \to \{0,1\}$ is called binary vertex labeling of $G(V,E)$ and $f(v)$ is called the label of the vertex $v \in G$ under f. For an edge $e = uv$, the induced edge labeling $f^*: E(G) \to \{0,1\}$ is given by $f^*(e) = |f(u) - f(v)|$. Let $v_f(0), v_f(1)$ be the number of vertices of G having labels 0 and 1 respectively under f and $e_f(0), e_f(1)$ be the number of edges having labels 0 and 1 respectively under f^*. A graph $G(V,E)$ is cordial if it admits cordial labelling. In this paper, we investigate the cordial decomposition for join and composition of some graphs.

II. RESULTS ON CORDIAL DECOMPOSITION

In this work, we investigate the cordial labeling for join and composition of some graphs.

Definition 2.1: Let $G(V,E)$ be a graph. A mapping $f: V(G) \to \{0,1\}$ is called binary vertex labeling of $G(V,E)$ and $f(v)$ is called the label of the vertex $v \in G$ under f.

For an edge $e = uv$, the induced edge labeling $f^*: E(G) \to \{0,1\}$ is given by $f^*(e) = |f(u) - f(v)|$. Let $v_f(0), v_f(1)$ be the number of vertices of G having labels 0 and 1 respectively under f and $e_f(0), e_f(1)$ be the number of edges having labels 0 and 1 respectively under f^*. A graph $G(V,E)$ is cordial if it admits cordial labelling.

Definition 2.2: A decomposition of G is a collection $\psi_C = \{H_1, H_2, ...H_r\}$ such that H_i are edge disjoint and every edges in H_i belongs to G. If each H_i is a cordial graphs, then ψ_C is called a cordial decomposition of G. The minimum cardinality of a cordial decomposition of G is called the cordial decomposition number of G and it is denoted by $\pi_C(G)$.

Theorem 2.1. The upper bounds of cordial decomposition number of the complete bipartite graph $K_{m,n}$ is $\pi_C(K_{m,n}) \le (mn)$.

Proof: The complete bipartite graphs $K_{m,n}$ having the set of vertices $V = \{\{u_i | 1 \le i \le m\} \cup \{v_j | 1 \le j \le n\}\}$. Note that there is $(m+n)$ vertices in $K_{m,n}$. The edge set edges set $E(K_{m,n})=\{u_i v_j \mid 1\le i\le m, 1\le i\le n\}$. Therefore number of P_2 in $K_{m,n}$ is mn. Hence $\pi_C(K_{m,n})\le(mn)$.

Example 2.1. The cordial decomposition of the graph $K_{3,4}$

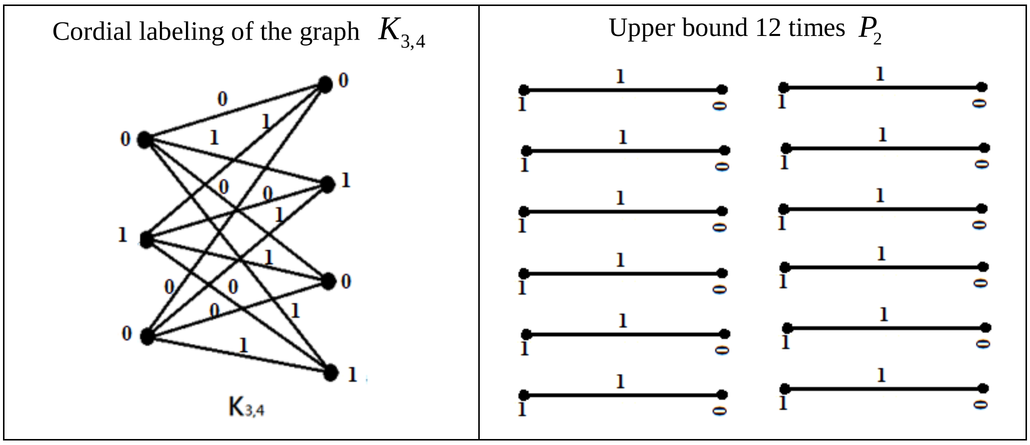


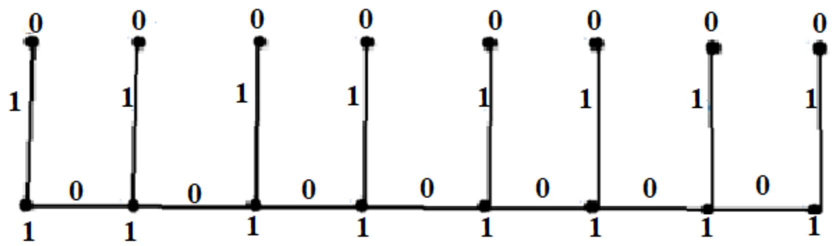
Figure 2.1. Thecordial decomposition of the graph $K_{3,4}$

Theorem 2.2: The bounds of cordial decomposition number of the Brush graph B_n is $(n+1)\le\pi_C(B_n)\le(2n-1)$.

Proof: The Brush graph B_n constructed by the path P_n and n number of P_2 paths. Therefore Brush graph B_n having the set of vertices $V=\{u_i, v_i, 1\le i\le n\}$. Note that P_n and P_2 be two cordialgraphs. This implies $\psi_S \supseteq \{P_n \cup P_2 \cup P_2 ... \cup n \ times\}$ and $|\psi_C| \ge |\{P_n \cup P_2 \cup P_2 ... \cup n \, times\}|$.Hence we get $\pi_C(B_n)\ge(n+1)$.

The edge set is $E = P_n \cup P_2 \cup ... n \, times \, P_2$,. Note that P_2 a cordialgraphs. Therefore number of P_2 in B_n is $S(P_n)+nS(P_2)$. This implies $|\psi_S| \le |S(P_n)+nS(P_2)|$. Hence $\pi_C(P_m+P_n)\le(2n-1)$.

Example 2.2: The cordial decomposition of Brush graph B_8



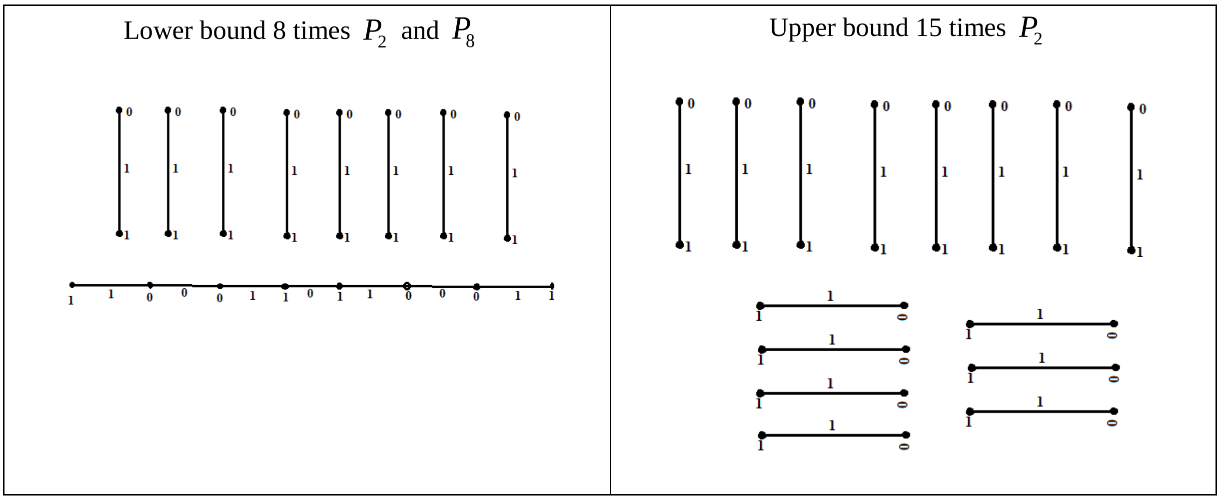


Figure 2.2: Cordial decomposition of Brush graph B_8

Theorem 2.3. The bounds of cordial decomposition number of the ladder graph L_n is $(n+2) \le \pi_C(L_n) \le (2n-1)+n.$

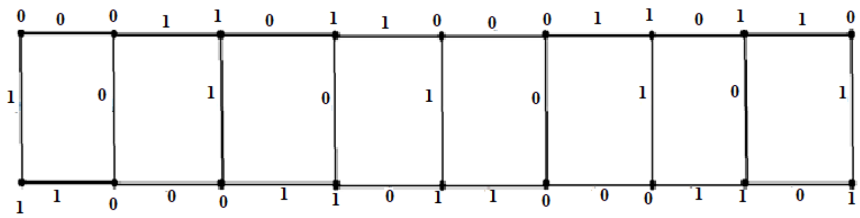
Proof: The ladder graph L_n, constructed by the graphs P_2 *and* P_n. Therefore the graph L_n having the set of vertices $V = \{u_1 v_i, 1 \le i \le n\} \cup \{u_2 v_i, 1 \le i \le n\}$. Note that there is $(2n)$ vertices in L_n. The graph L_n contains the n number of graphs P_2 and 2 number of path P_n. Note that P_m and P_n be two cordialgraphs. This implies $\psi_C \supseteq \{P_n \cup P_n \cup (n\, times\ P_2)\}$ and $|\psi_C| \ge |\{P_n \cup P_n \cup (n\, times\ P_2)\}|$.

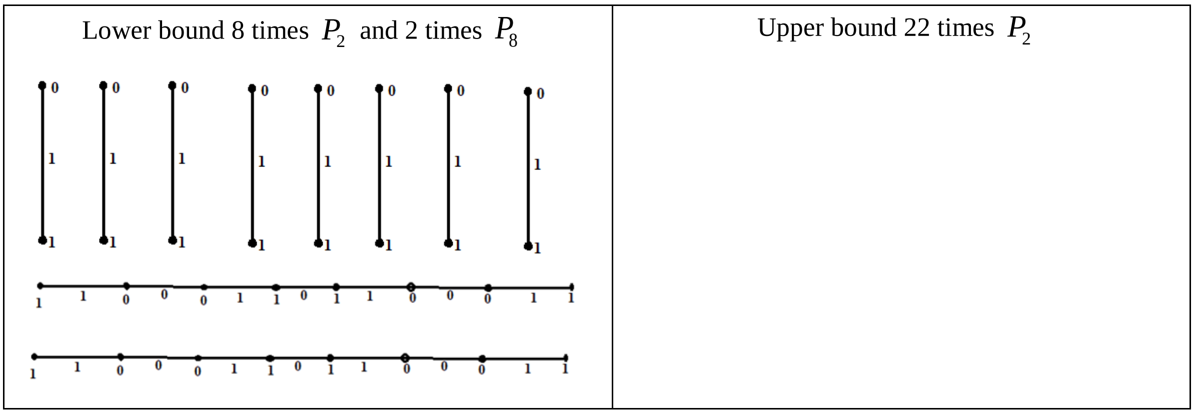
Hence $\pi_C(L_n) \ge (n+2)$.

The edge set is $E = S(P_n) \cup S(P_n) \cup n\, times\ S(P_2)$, Note that P_2 a cordial graphs. Therefore number of P_2 in (L_n) is $S(P_n) + S(P_n) + n\, times\ S(P_2)$. This implies $|\psi_C| \le |S(P_n) + S(P_n) + n\, times\ S(P_2)|$.

Hence $\pi_C(L_n) \le (2(n-1)+n)$.

Example .2.3: The cordial decomposition of the graph (L_9)





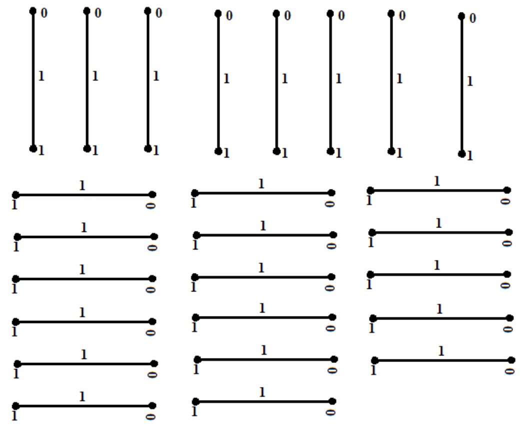


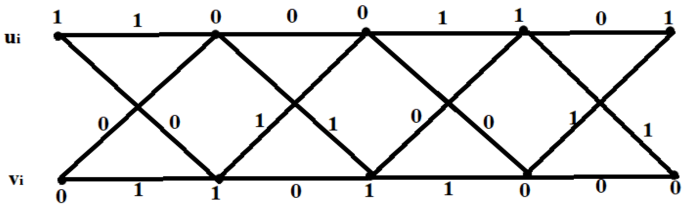
Figure 2.3: Cordial decomposition of the graph (L_9)

Theorem 2.4: The bounds of cordial decomposition number of the shadow graph $(D_2(P_n)$ is $(4) \le \pi_C(L_n) \le 4(n-1)$.

Proof: The shadow graph of path P_n $(D_2(P_n))$ contains 2n vertices and 4(n-1) edges. Let $v_1, v_2, v_3, \ldots v_n$ be the vertices of path P_n . In a P_n shadow graph there is an image of n vertices $u_1, u_2, u_3, \ldots u_n$. Therefore shadow $(D_2(P_n))$ contains 2n vertices. There is a different 4 types of P_n in the shadow graph $(D_2(P_n))$. $\psi_C \supseteq \{P_n \cup P_n \cup (4\, times\; P_2)\}$ and $|\psi_C| \ge |\{P_n \cup P_n \cup (4\, times\; P_2)\}|$. Hence $\pi_C(L_n) \ge (4)$.

The edge set of $(D_2(P_n))$ is $E = \{(u_i u_{i+1} | 1 \le i \le n)\} \cup \{(v_i v_{i+1} | 1 \le i \le n)\} \cup \{(u_i v_{i+1} | 1 \le i \le n)\} \cup \{(v_i u_{i+1} | 1 \le i \le n)\}$ Note that P_2 a cordial graphs. Therefore number of P_2 in $(D_2(P_n))$ is $S(P_n) + S(P_n) + S(P_n) + S(P_n)$. This implies $|\psi_C| \le |S(P_n) + S(P_n) + S(P_n) + S(P_n)|$. Hence $\pi_C(L_n) \le (4(n-1))$.

Example 2.4: The cordial decomposition of shadow graph $D_2(P_5)$



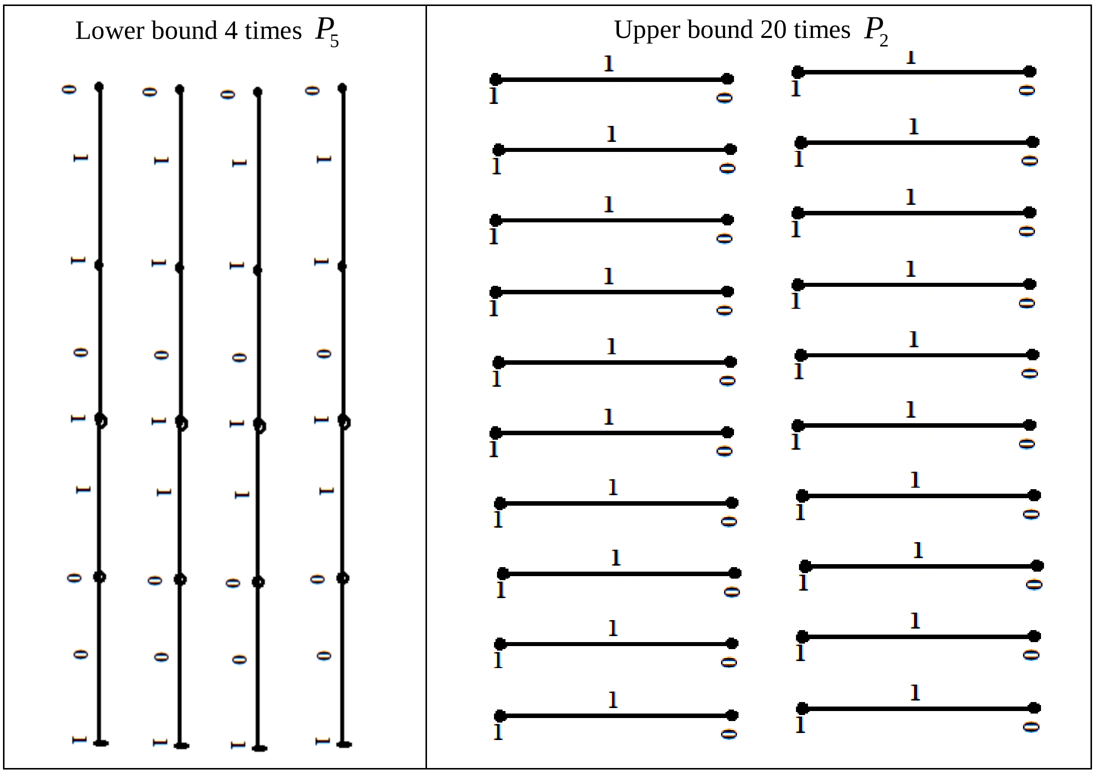


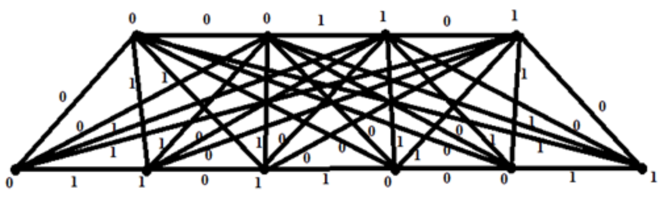
Figure 2.4: The cordial decomposition of shadow graph $D_2(P_5)$

Theorem 2.5: A graph $(P_m + P_n)$ is a join of two path cordial graphs with $(m < n)$ .The bounds of cordial decomposition number of the graph $(P_m + P_n)$ is, $3 \le \pi_C(P_m + P_n) \le (mn + m + n - 2)$..

Proof: Let P_m and P_n be two path cordial graphs of order m and n $(m > n)$ respectively and $(P_m + P_n)$ is a join of P_m and P_n with edge set E. The graph $(P_m + P_n)$ contains $(m + n)$ vertices In the graph $(P_m + P_n)$ there are graphs P_m , P_n and the complete bipartite graphs $K_{m,n}$ .Note that P_m and P_n be two cordial graphs and complete bipartite graphs $K_{m,n}$ cordial graph. This implies $\psi_S \supseteq \{P_m \cup P_n \cup K_{mn}\}$ and $|\psi_S| \ge |\{P_m \cup P_n \cup K_{mn}\}|$. Note that the graphs P_m , P_n and $K_{m,n}$ are subtract divisor cordial graphs. Hence $\pi_C(P_m + P_n) \ge (3)$.

The edge set is $E = E_1 \cup E_2 \cup S(K_{m,n})$, Here $S(K_{m,n})$ is a size of a complete bipartite graph $K_{m,n}$. Note that P_2 a cordialgraphs. Therefore number of P_2 in $(P_m + P_n)$ is $S(P_m) + S(P_n) + S(K_{m,n})$. This implies $|\psi_S| \le |\{S(P_m) + S(P_n) + S(K_{m,n})\}|$. Hence $\pi_C(P_m + P_n) \le (mn + m + n - 2)$.

Example 2.5. The cordial decomposition of the graph $(P_4 + P_6)$



$(P_4 + P_6)$

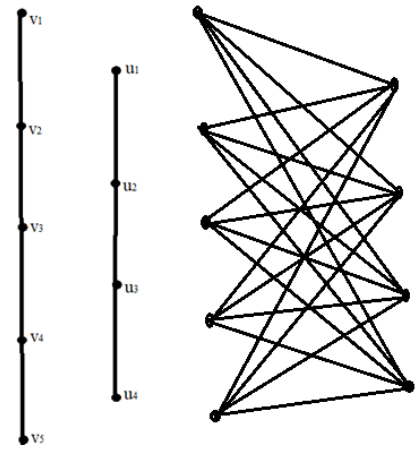
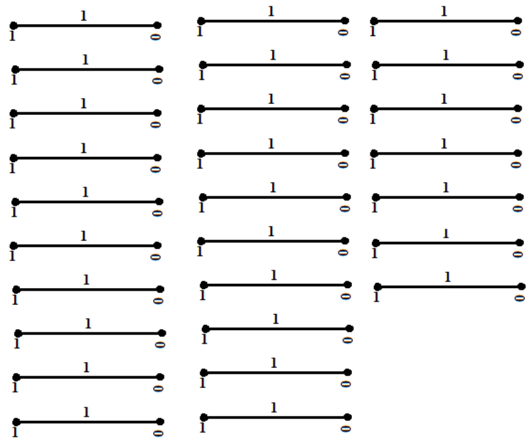
Lower bound of 3 cardial graphs, P_5, P_4 & $K_{5,4}$	Upper bound 27 times P_2
	

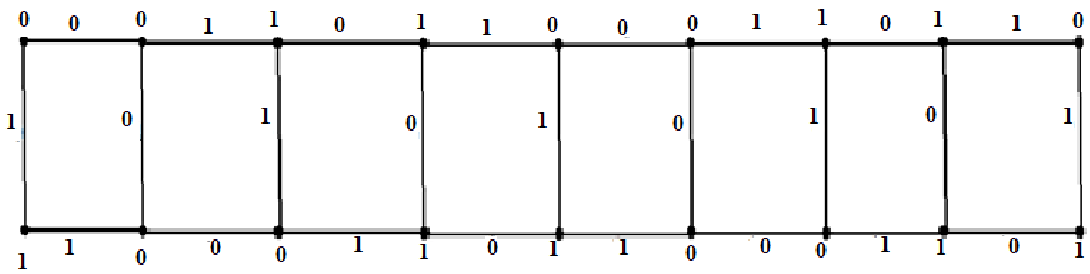
Figure 2.5: The cordial decomposition of shadow graph $(P_4 + P_6)$

Theorem 2.6. The bounds of cordial decomposition number of the composition of the graphs P_2 *and* P_n is $(n+2) \le \pi_C(P_2 \circ P_n) \le (2n-1)+n.$

Proof: The composition of the graphs P_2 *and* P_n, constructed by the graphs P_2 *and* P_n. Therefore the graph $P_2 \circ P_n$ having the set of vertices $V = \{u_1 v_i, 1 \le i \le n\} \cup \{u_2 v_i, 1 \le i \le n\}$. Note that there is $(2n)$ vertices in $P_2 \circ P_n$. The graph $P_2 \circ P_n$ contains the n number of graphs P_2 and 2 number of path P_n. Note that P_m and P_n be two cordialgraphs. This implies $\psi_C \supseteq \{P_n \cup P_n \cup (n\,times\ P_2)\}$ and $|\psi_C| \ge |\{P_n \cup P_n \cup (n\,times\ P_2)\}|$. Hence $\pi_C(P_2 \circ P_n) \ge (n+2)$.

The edge set is $E = S(P_n) \cup S(P_n) \cup n\,times\ S(P_2)$, Note that P_2 a cordialgraphs. Therefore number of P_2 in $(P_2 \circ P_n)$ is $S(P_n) + S(P_n) + n\,times\ S(P_2)$. Thisimplies $|\psi_C| \le |S(P_n) + S(P_n) + n\,times\ S(P_2)|$. Hence $\pi_C(P_2 \circ P_n) \le (2(n-1)+n)$.

Example 2.6: The cordial decomposition of the graph $(P_2 \circ P_n)$



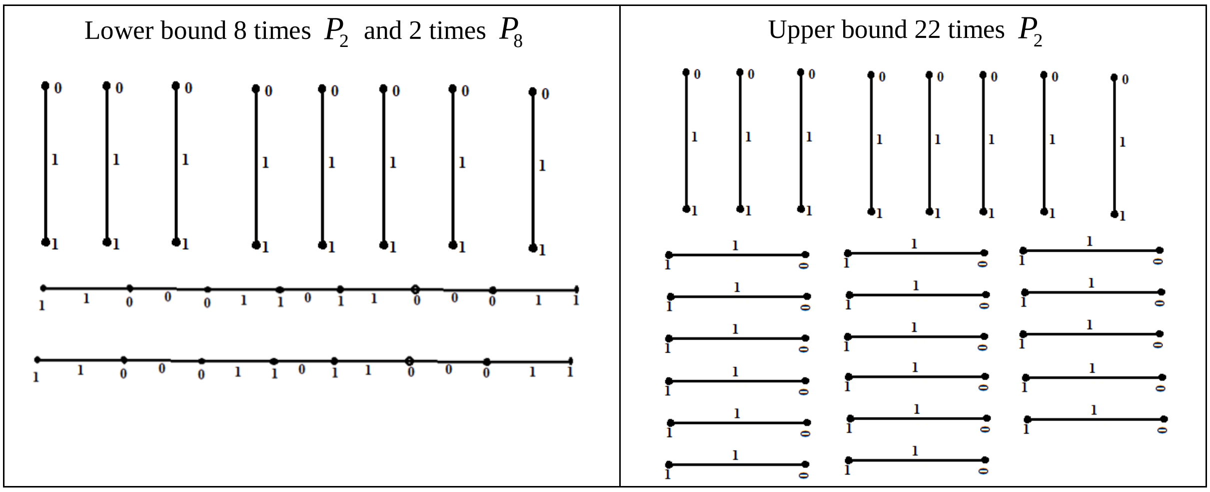


Figure 2.6: Cordial decomposition of the graph $(P_2 \circ P_n)$

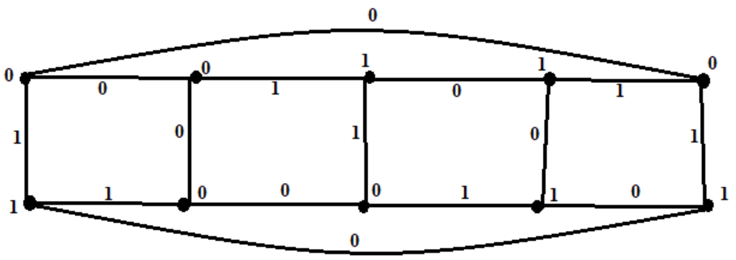
Theorem 2.7: The bounds of cordial decomposition number of the composition of the graphs P_2 *and* C_n is $(n+1) \le \pi_C(P_2 \circ C_n) \le (3n)$.

Proof: The composition of the graphs P_2 *and* C_n, constructed by the graphs P_2 *and* C_n . Therefore the graph $(P_2 \circ C_n)$ having the set of vertices $V = \{u_1 v_i, 1 \le i \le n\} \cup \{u_2 v_i, 1 \le i \le n\}$. Note that there is $(2n)$ vertices in $(P_2 \circ C_n)$. The graph $(P_2 \circ C_n)$ contains the graphs 2 times C_n and n times P_2 . Note that P_2 and C_n be two cordialgraphs.

This implies $\psi_C \supseteq \{C_n \cup C_n \cup (n\, times\, P_2)\}$ and $|\psi_C| \ge |\{C_n \cup C_n \cup (n\, times\, P_2)\}|$. Hence $\pi_C(P_2 \circ C_n) \ge (n+2)$.

The edge set is $E = S(C_n) \cup S(C_n) \cup n\, times\, S(P_2)$, Note that P_2 a cordialgraphs. Therefore number of P_2 in $(P_2 \circ C_n)$ is $S(C_n) + n\, times\, S(P_2)$. This implies $|\psi_C| \le |S(C_n) + n\, times\, S(C_2)|$. Hence $\pi_C(P_2 \circ C_n) \le (3n)$.

Example 2.7: The cordial decomposition of the graph $(P_2 \circ C_5)$



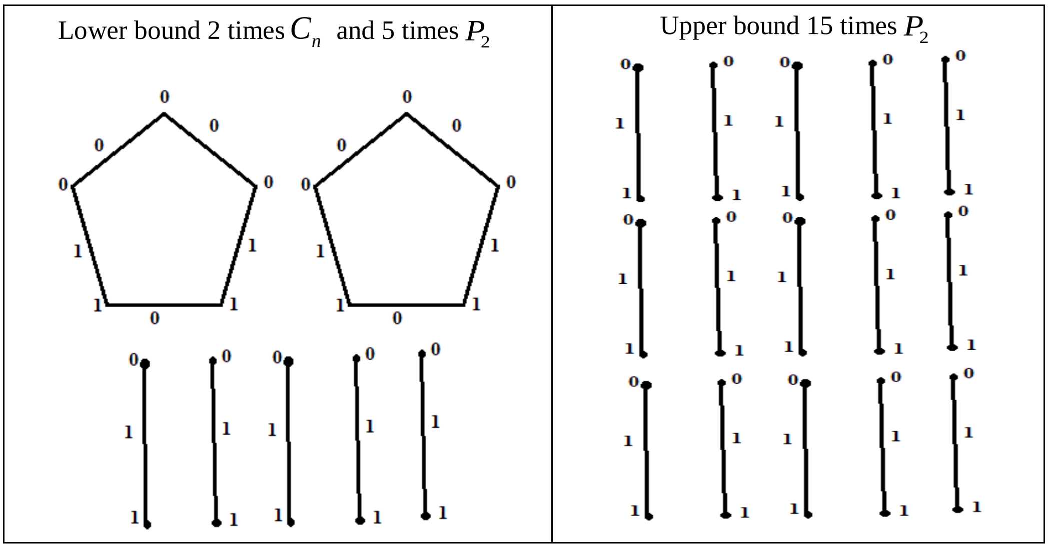


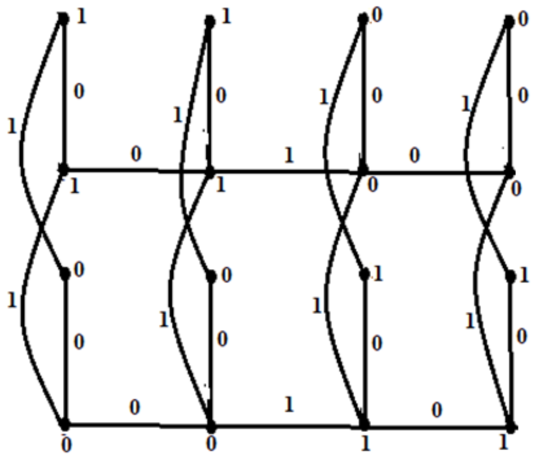
Figure2.7: Cordial decomposition of the graph $(P_2 \circ C_5)$

Theorem 2.8: The bounds of cordial decomposition number of the composition of the graphs P_2 *and* B_n is $(n+1) \le \pi_C(P_2 \circ B_n) \le (3n)$.

Proof: The composition of the graphs P_2 *and* B_n, constructed by the two graphs P_2 *and* B_n. Therefore the graph $(P_2 \circ B_n)$ having the set of vertices $V = \{u_1v_i, 1 \le i \le n\} \cup \{u_2v_i, 1 \le i \le n\} \cup \{u_1w_i, 1 \le j \le n\} \cup \{u_2w_i, 1 \le j \le n\}$. Note that there is $(4n)$ vertices in $(P_2 \circ B_n)$. Note that P_2 and B_n be two cordialgraphs. This implies $\psi_C \supseteq \{B_n \cup B_n \cup (2n\, times\, P_2)\}$ and $|\psi_C| \ge |\{B_n \cup B_n \cup (2n\, times\, P_2)\}|$. Hence $\pi_C(P_2 \circ B_n) \ge 2(n+1)$.

The edge set is $E = S(B_n) \cup S(B_n) \cup 2n\, times\, S(P_2)$, Note that P_2 a cordialgraphs. Therefore number of P_2 in $(P_2 \circ B_n)$ is $S(B_n) + S(B_n) + n\, times\, S(P_2)$. This implies $|\psi_C| \le |S(B_n) + S(B_n) + 2n\, times\, S(P_2)|$. Hence $\pi_C(P_2 \circ B_n) \le (6n-2)$.

Example 2.8: The cordial decomposition of the graph $(P_2 \circ B_4)$



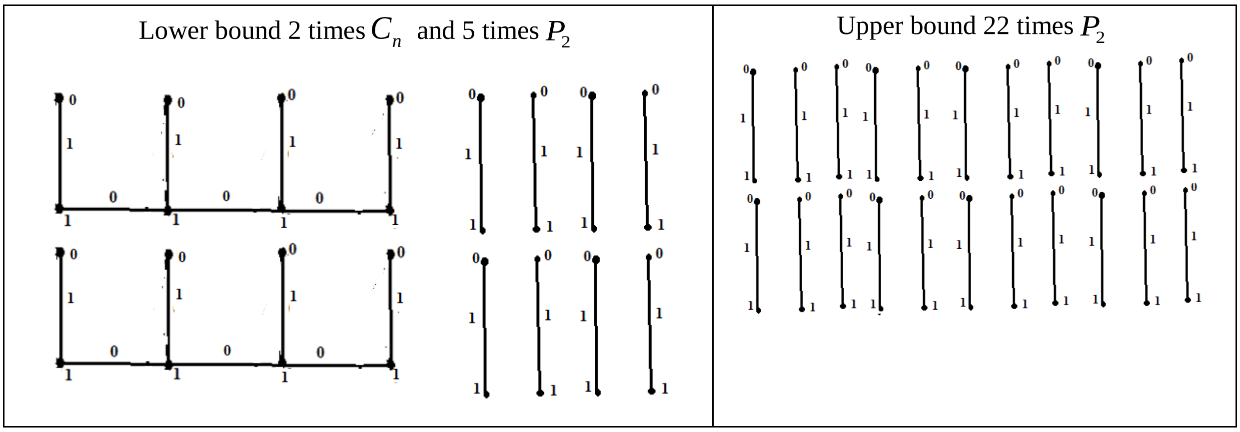


Figure 2.8: Cordial decomposition of the graph $(P_2 \circ B_4)$

III. CONCLUSION

In this paper we define cordial decomposition and cordial number $\pi_C(G)$ of graphs. Also investigate some bounds of $\pi_C(G)$ in product graphs like Cartesian product, composition etc. In future we will investigate the decomposition number in various labeling in graphs.

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