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The use of fuzzy arithmetic in problems of correlation analysis

Vadim Nikolayevich Romanov
Doctor of Technical Sciences, Professor
St.-Petersburg, Russia; vromanvpi@mail.ru

ABSTRACT
The paper studies the use of fuzzy arithmetic for correlation analysis. The implementation of the principal component method with fuzzy and incomplete data is considered. The author offers a new approach based on fuzzy gradations, which allows us to perform arithmetic operations directly. The advantages of the proposed approach are analyzed. This approach allows us to generalize the analysis scheme in case of uncertainty, to process together quantitative and qualitative data and smooth errors associated with data inconsistency; it significantly reduces the complexity of the analysis and provides an objective criterion for completing the procedure.

Article Highlights
Detection of the relationships between quantities and identification of their causes in case of fuzzy, inconsistent initial information.
Making reliable decisions by a few (2 or 3) of the most important criteria.
The proposed approach makes it possible to reduce the complexity and time of analysis, calculation and conclusions.

KEYWORDS AND PHRASES: fuzzy arithmetic, correlation analysis, method of principal components

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I. INTRODUCTION

The main applications of correlation analysis are the establishment of interconnections and the mutual influence of parameters in systems, the determination of significant factors in regression models. One of the important applications of correlation analysis is the detection of “hidden” parameters from directly observable quantities. The most advanced is statistical correlation analysis. The theoretical foundations of statistical correlation analysis and its applications are given in works [4, 10]. The use of serial correlation in the analysis of time series is considered in [1]. The method of principal components and methods of factor analysis were studied in [3]. The use of statistical correlation analysis is associated with a number of rather strong restrictions. Quantitative and qualitative data are processed separately, after which they are compared. The result strongly depends on the accuracy of the experimental data. For example, to determine the Spearman's rank correlation coefficient used in processing qualitative data, with an error of 10%, the number of objects must be at least 100; for calculation of typical numerical correlation coefficient, the amount of data is even greater. In real situations, we have to deal with a smaller amount of data, which increases the error in determining the correlation coefficient. When calculating the correlation coefficient, the quantities should have a normal distribution or close to normal. The use of nonparametric methods requires a large number of measurements; otherwise, it leads to inaccurate conclusions [10, 17]. The correlation coefficient gives a linear approximation and takes into account only the linear relationship of quantities. In a real situation, with a large number of variables, the relationships are more complex and can include nonlinear terms and interactions, which affect the value of the correlation coefficient and increase the error. In many cases, information about objects and criteria is incomplete, heterogeneous, and there is uncertainty in the evaluation of objects by criteria. Therefore, accurate numerical calculations do not make sense. In this case, fuzzy correlation analysis based on fuzzy set theory is used. The theoretical foundations of fuzzy correlation analysis are given in [6, 7]. The calculations use the Zadeh generalization principle and the approximation of the membership function by various distributions or its discrete representation [2, 5, 8, 9, 11, 12]. This approach does not allow us to take full advantage of fuzzy data representation. With a large number of variables, the calculations are cumbersome; the results are not very clear and difficult to interpret. In the problems of correlation analysis, associated with the choice of essential factors and the analysis of their influence on the result, the specific numerical content of the quantities often does not matter, but only the order relation between them. The article uses the approach proposed by the author, based on fuzzy gradations, which allows us to perform calculations without being tied to a numeric context. The purpose of the article is to develop a methodology for correlation analysis using fuzzy arithmetic, in particular, when detecting hidden parameters. The proposed approach uses the representation of data in the form of fuzzy gradations, which makes it possible to generalize the analysis scheme in case of uncertainty, significantly reduces the complexity of the analysis and provides an objective criterion for the completion of the analysis procedure. The article is based on the author's previous results related to the implementation of the rules of fuzzy arithmetic [14 – 16].

The article is organized as follows. First, we briefly consider the rules of fuzzy arithmetic and the evaluation of the reliability (certainty) of the results. Then the correlation analysis algorithm is generalized for the case of data representation in the form of fuzzy gradations. Further, using an example, we consider the methodology of applying correlation analysis in the method of principal components. Finally, we discuss the obtained results and give recommendations for their application.

II. ARITHMETIC OPERATIONS ON FUZZY GRADATIONS AND RELIABILITY EVALUATION

To describe the object area we use the fuzzy gradations in the range VL…VH. The range includes gradations VL – very low value, (VL-L) – between very low and low, L – low, (L-M) – between low and middle, M – middle (medium), (M-H) – between middle and high, H – high, (H-VH) – between high and very high, VH – very high. Boundary gradations out of range are also known, namely VVL (lowest value) and VVH (highest value). Depending on condition of the task gradation VVL can be interpreted as zero, lower bound, exact lower bound etc. and gradation VVH – as unit, infinity, upper bound, exact upper bound, etc. Fuzzy gradations form an ordinal scale in which an admissible transformation is arbitrary monotone function that does not change the order of gradations. This is another advantage of using fuzzy gradations. In particular, all gradations can be simultaneously multiplied or divided, as well as increased or decreased by the same number, so that the values do not go beyond the range 0 ... 1. The rules of fuzzy arithmetic and the evaluation of the reliability of the results are considered in the previous works of the author, so we give them here in abbreviated form to make the results of calculations clear. The summation and multiplication operations on fuzzy gradations are performed in the same way as in ordinary arithmetic. For instance, for summation we have VL + VL = (VL-L), (VL-L) + VL = L, etc., (H-VH) + VL = VH, VH + VL = VVH. Similarly, summation is performed for other fuzzy gradations. For multiplication operation we have VL * VL = VVL, VL * (VL-L) = VVL, etc., VL * M = VL, etc., VL * VH = VL. Similarly, multiplication is performed for other fuzzy gradations. In calculations, it makes no sense to introduce small gradation shares, and rounding should be used towards the nearest gradation, since this does not affect the accuracy of the final results. When the number of factors (summands) is more than two, the result is determined similarly. The process quickly converges as the number of components (factors or summands) increases; so for three to four components, the extreme limits of the range are reached. We can also determine the results for inverse operations (subtraction and division). When performing calculations, multiplication or division by an integer or rational number are defined using the summation operation. Subtraction and division operations are defined through the operations of addition and multiplication, respectively. The exponentiation and root extraction operations are defined through the multiplication operation. Calculations can be performed directly in fuzzy gradations or using modal values corresponding to fuzzy gradations, with the subsequent representation of the numerical results in the form of fuzzy gradations. Calculations on fuzzy gradations are greatly simplified if all gradations are expressed in terms of the smallest gradation (VL), namely, (VL-L) = 2VL, L = 3VL, etc., VH = 9VL, VVH = 10VL. This representation makes it possible to extend calculations formally outside the range 0 ... 1. We use this technique in subsequent calculations. We now estimate the reliability of the results of calculations on fuzzy gradations. Designate x, y, etc. – the values represented as fuzzy gradations; ν_x, ν_y, etc. − the indexes of fuzziness corresponding to values x, y, etc., respectively, are also represented in the form of fuzzy gradations. Each gradation is a function value of which is concentrated in the center of the corresponding interval, and the value of the membership function is 1. In this sense, the accuracy of determining the gradation is equal to 1(the index of fuzziness is 0), if it is not stipulated special conditions. We consider the distribution on the gradations. The accuracy (certainty) of the result is defined as for fuzzy set elements of which are individual gradations. For the index of fuzziness, we use two expressions. The first of these has the form

$$\nu_x = 2\min(x, \bar{x}), \quad (1)$$

where $\bar{x}$ – the opposite value to x; for example, if x = VL, then $\bar{x}$ = VH etc. Multiplication by the number 2 is understood as the summation of two equal values represented as fuzzy gradations. The expression (1) corresponds to the strong condition β > ν, or

β > H (0.7), (2)

where β is an estimate in the form of fuzzy gradation for the result r or $\bar{r}$. The result is value of x or its reliability. Here and below in parentheses the numerical value corresponding to the maximum (modal value) of the gradation is indicated. The second expression for the fuzziness index has the form

$$\nu = \min(x, \bar{x}), \quad (3)$$

which corresponds to a softer condition β > ν/2 or

β > M (0.5). (4)

III. ALGORITHM OF CORRELATION ANALYSIS

We formulate the problem as follows. Given the set of objects $X = \{x_1,\dots, x_m\}$, which are evaluated according to the set of criteria (features, properties) $K_1,\dots, K_n$, represented by fuzzy gradations in the range [VL, VH]. It is required to determine the correlation (interdependence) between the criteria and its possible causes. The solution algorithm includes the following steps:

1. The initial quantitative and qualitative information about objects and criteria, obtained using measurements and expert methods, is transformed into fuzzy gradations as follows. Each named variable is assigned a standardized (normalized) variable, varying in the interval [0, 1]. Then a fuzzy gradation is assigned to the standardized variable. In this case, the value 0 corresponds to the gradation VVL (the lowest value), and the value 1 corresponds to the gradation VVH (the highest value). A value of 0.1 corresponds to the modal value of the VL gradation (very low value); similarly a value of 0.3 – gradation of L (low value); a value of 0.5 – gradation M (middle value); the value of 0.7 – gradation H (high value), the value of 0.9 – gradation VH (very high value). The transition from physical to standardized variable is determined by the ratio $x = (z - z_{\min})/(z_{\max} - z_{\min}) \pm 0.1$, where the plus sign corresponds to the value of $z_{\min}$, and the minus sign to the value of $z_{\max}$. Here x is a standardized variable from the interval (0, 1); z is a "physical" variable, determined by measurement or expert method, which takes values in the interval $[z_{\min}, z_{\max}]$. Its values are represented by named numbers or dimensionless estimates.
2. The correlation coefficient is determined for each pair of criteria from a known ratio modified for our case with the only difference that all values are represented in the form of fuzzy gradations:

$$\rho_{lm} = \frac{\sum_{i=1}^{n}(K_i^l - \bar{K}^l)(K_i^m - \bar{K}^m)}{\left[\sum_{i=1}^{n}(K_i^l - \bar{K}^l)^2 \sum_{i=1}^{n}(K_i^m - \bar{K}^m)^2\right]^{1/2}}$$

(5)

The subscript in (5) corresponds to the object, and the superscript to the criterion. The line at the top indicates the mean value. Calculations are performed using the fuzzy arithmetic rules (see above).

3. Objects are distributed in groups in accordance with the values of the correlation coefficient.
4. To identify "hidden" parameters, the principal component method is used.
5. Based on the results of the analysis, a conclusion is made on the possible causes of the observed relationships.

IV. EXAMPLE OF STUDY

Consider an example. Let the initial data are given in table 1. The specific interpretation of quantities (objects and criteria) depends on the subject area and does not matter for subsequent calculations. It is assumed that the error of estimates in the table 1 is about one gradation (does not exceed one gradation). Based on the initial data, the correlation coefficient was calculated using the relation (5). The results of the calculations are given in table 2. We will give examples of calculating the values from tables 1 and 2. Calculate the mean values $\bar{K}$ in table 1. For the value of $\bar{K}^1$ we get $\bar{K}^1$ = [2(L-M) + 6M + (VL-L) + 3H + 4(M-H) + (H-VH) + VH + 2VL]/20 = 104VL/20 = 5VL = M. For $\bar{K}^2$ we have $\bar{K}^2$ = [5M + (L-M) + 4L + 3(VL-L) + H + 3VL + (M-H) + (H-VH) + VH]/20 =
= 80VL/20 = 4VL = L-M. The remaining mean values are calculated similarly. Calculate values of ρ_{lm} in table 2. From (5) it follows that on the main diagonal of table 2 all values ρ_{ll} = 1 = VVH. Calculate ρ_{12} based on the data from the first and second columns of table 1. We write ρ_{12} in the form $\rho_{12} = B_{12}/(B_1 * B_2)^{1/2}$. From (5) we obtain B_{12} = ((L-M) – M)*(M – (L-M)) + … + (H – M)*((VH – (L-M)) = (– VL) * VL +…+ 2VL * 5VL= $34(VL)^2$; B_1=((L-M) – M)^2 + … + (H – M)^2 =(VL)^2 + … + (2VL)^2 = 83(VL)^2; B_2 = (M–(L-M))^2 + … + (VH – (L-M))^2 = (VL)^2 + … + (5VL)^2 = 104(VL)^2. For the coefficient ρ_{12} we obtain $\rho_{12} = \rho_{21}$ =

$34(VL)^2/((83*104)^{1/2}(VL)^2) = 0,4$ = L-M (with rounding). The rest of the values in table 2 are calculated similarly.

Table 1
Initial data for an example

Objects	Criteria						
	K^1	K^2	K^3	K^4	K^5	K^6	K^7
x_1	L-M	M	M-H	L-M	VL-L	VL-L	L
x_2	M	M	L-M	L-M	H-VH	L-M	M-H
x_3	M	L-M	L	VL-L	M-H	VL-L	VL
x_4	M	L	M	VL-L	M	VL-L	VL
x_5	M	VL-L	L-M	L-M	L-M	L	VL-L
x_6	VL-L	VL-L	L	L	VL-L	L	VL
x_7	M	H	L	L-M	H-VH	L-M	VL
x_8	H	M	VH	H-VH	L-M	VH	H
x_9	M-H	M	M	VH	VH	VL-L	VL-L
x_{10}	H-VH	L	VL	VL-L	VH	L-M	VL
x_{11}	M-H	L	L-M	VL	VH	L-M	VL
x_{12}	L-M	L	L-M	L	M	VL	VL
x_{13}	H	VL	VL-L	L	H-VH	VL	VL
x_{14}	VH	M-H	M	L-M	L-M	M	M-H
x_{15}	M-H	H-VH	VL-L	L	VL	VL-L	VL
x_{16}	M-H	VL	H	VL-L	L-M	L-M	VL
x_{17}	VL	VL	VL	L	L	L	VL
x_{18}	VL	VL-L	M	VL-L	L	VL-L	VL-L
x_{19}	M	M	L	L	H	L	VL-L
x_{20}	H	VH	H	VL	M-H	VL-L	VL
$\bar{K}$	M	L-M	L-M	L	M	L	VL-L

Table 2
Correlation coefficients matrix

	K^1	K^2	K^3	K^4	K^5	K^6	K^7
K^1	VVH	L-M	VL-L	VL-L	L-M	L-M	L
K^2	L-M	VVH	L-M	VL-L	VL	VL	VL-L
K^3	VL-L	L-M	VVH	L-M	VL^-	M	M
K^4	VL-L	VL-L	L-M	VVH	VL	L-M	M-H
K^5	L-M	VL	VL^-	VL	VVH	VVL	VL^-
K^6	L-M	VL	M	L-M	VVL	VVH	H
K^7	L	VL-L	M	M-H	VL^-	H	VVH

Note. Hereinafter, the following notation is used: VVL is the value corresponding to 0, VVH is the value corresponding to 1; the minus sign at the top means that the gradation is located to the left of the VVL (has a negative value).

To determine the "hidden" parameters, we use the principal component method. It is required to bring the correlation matrix to a diagonal form by solving the equation $AX = \lambda X$, where A is the correlation matrix, λ – the proper numbers playing the role of hidden parameters, X – the eigenvectors. The solution is carried out iteratively, and all calculations are performed in fuzzy gradations. To calculate the first eigenvalue λ_1, the standard procedure is used. First the sums of the rows in the table 2 are determined, and the largest value is taken as the first approximation for λ_1. Then all sums are divided by the largest value and the obtained normalized values are taken in the first approximation as components of the first eigenvector. The process is repeated until the value of the vector becomes constant. It should be noted that a total of 4 to 5 iterations are required to determine λ_1, i.e. less than with numerical calculations using traditional methods. This is understandable, since smoothed data is used. In addition, random errors within the gradation have practically no effect on the result. The calculations give $\lambda_1 = H/M^2$, which corresponds to a numerical value $\lambda_1 = 7VL/25(VL)^2 = 2.8$. Hereinafter, numerical estimates are given for convenience of comparison with the traditional method. The eigenvalue λ_1 corresponds to the eigenvector having seven components X_1 = ((M-H), M, H, H, VL, (H-VH), (H-VH)). The matrix of correlation coefficients of the first parameter (factor) is given in table 3.

Table 3
The matrix of the correlation coefficients of the first parameter

X_1	M-H	M	H	H	VL	H-VH	H-VH
M-H	L-M	L	L-M	L-M	VL	M	M
M	L	VL-L	L-M	L-M	VL	L-M	L-M
H	L-M	L-M	M	M	VL	M-H	M-H
H	L-M	L-M	M	M	VL	M-H	M-H
VL	VL	VL	VL	VL	VVL	VL	VL
H-VH	M	L-M	M-H	M-H	VL	M-H	M-H
H-VH	M	L-M	M-H	M-H	VL	M-H	M-H

To exclude the influence of the first parameter, we subtract from the matrix in table 2 matrix represented in table 3, which gives the first residual matrix after the exclusion of the first parameter. The resulting matrix is represented in table 4. According to the data of this matrix, the second eigenvalue λ_2 and the corresponding vector X_2 are calculated. Calculations are performed similarly to calculations for λ_1 and X_1. We have λ_2 = H/M, which corresponds to a numerical value $\lambda_2 = 7VL/5VL = 1.4$. The eigenvector X_2, corresponding to λ_2, has the form X_2 = ((M-H), (L-M), L^-, VL^-, (H-VH), $(VL\text{-}L)^-$, L^-); 5 or 6 iterations are enough to calculate it. Further calculations are performed similarly to the previous one. The correlation matrix of the coefficients of the second parameter (factor) is given in table 5. To exclude the second parameter, subtract from the matrix in table 4 matrix represented in table 5, which gives the residual matrix after eliminating the second parameter. The resulting matrix is given in table 6. Table 6 shows that the sum of the elements in the rows of this matrix is one or two gradations; then it is within the error. Moreover, none of the matrix elements exceeds the gradation H (high value), i.e. reliability condition (2) is not satisfied (see above). Therefore, further calculations are unreliable, and it is sufficient to restrict by two eigenvalues, the contribution of which is $(H/M^2 + H/M)/(H/VL) = 0,6$. The error of estimates in a matrix increases with increasing parameter number. It has a particularly strong effect on the accuracy of determining eigenvectors, starting with X_3; the error affects the accuracy of the calculation of eigenvalues less, since they are determined by the maximum value of the sum in the rows of the corresponding matrix. Therefore, to estimate the order of values, we considered it sufficient to calculate only the eigenvalues. Calculations give λ_3 = M/M, λ_4 = H, λ_5 = L-M. The last value is below the confidence threshold, so the calculations of λ_6 and λ_7 do not make sense.

Table 4
Residual matrix after exclusion of the first parameter

	K^1	K^2	K^3	K^4	K^5	K^6	K^7
K^1	M-H	VL	$(VL\text{-}L)^-$	$(VL\text{-}L)^-$	L	VL^-	$(VL\text{-}L)^-$
K^2	VL	H-VH	VVL	$(VL\text{-}L)^-$	VVL	L^-	$(VL\text{-}L)^-$
K^3	$(VL\text{-}L)^-$	VVL	M	VL^-	$(VL\text{-}L)^-$	VL^-	VL^-
K^4	$(VL\text{-}L)^-$	$(VL\text{-}L)^-$	VL^-	M	VVL	$(VL\text{-}L)^-$	VVL
K^5	L	VVL	$(VL\text{-}L)^-$	VVL	VVH	VL^-	$(VL\text{-}L)^-$
K^6	VL^-	L^-	VL^-	$(VL\text{-}L)^-$	VL^-	L-M	VL
K^7	$(VL\text{-}L)^-$	$(VL\text{-}L)^-$	VL^-	VVL	$(VL\text{-}L)^-$	VL	L-M

Table 5
The matrix of the correlation coefficients of the second parameter

X_2	M-H	L-M	L⁻	VL⁻	H-VH	(VL-L)⁻	L⁻
M-H	L-M	VL-L	(VL-L)⁻	VL⁻	M	VL⁻	(VL-L)⁻
L-M	VL-L	VL-L	VL⁻	VVL	L	VL⁻	VL⁻
L⁻	(VL-L)⁻	VL⁻	VL	VVL	(VL-L)⁻	VL	VL
VL⁻	VL⁻	VVL	VVL	VVL	VL⁻	VVL	VVL
H-VH	M	L	(VL-L)⁻	VL⁻	M-H	(VL-L)⁻	(VL-L)⁻
(VL-L)⁻	VL⁻	VL⁻	VL	VVL	(VL-L)⁻	VVL	VL
L⁻	(VL-L)⁻	VL⁻	VL	VVL	(VL-L)⁻	VL	VL

Table 6
Residual matrix after elimination of the second parameter

	K^1	K^2	K^3	K^4	K^5	K^6	K^7
K^1	VL-L	VL⁻	VVL	VL⁻	(VL-L)⁻	VVL	VVL
K^2	VL⁻	M-H	VL	(VL-L)⁻	L⁻	(VL-L)⁻	VL⁻
K^3	VVL	VL	L-M	VL⁻	VVL	(VL-L)⁻	(VL-L)⁻
K^4	VL⁻	(VL-L)⁻	VL⁻	M	VL	(VL-L)⁻	VVL
K^5	(VL-L)⁻	L⁻	VVL	VL	L-M	VL	VVL
K^6	VVL	(VL-L)⁻	(VL-L)⁻	(VL-L)⁻	VL	L-M	VVL
K^7	VVL	VL⁻	(VL-L)⁻	VVL	VVL	VVL	L

V. DISCUSSION OF THE RESULTS

Thus, from the above consideration it follows that the first two eigenvalues and the corresponding eigenvectors are determined reliably. This establishes an objective criterion for completing the calculation procedure. When using fuzzy gradations, all results are obtained with fewer iterations than when using numerical data or data using a membership function, much less laborious, which is important when processing large data arrays. As for the interpretation of the causes of relationships, it is determined by the subject area. For example, when analyzing the quality of learning, the hidden parameters can be abilities and interest in learning. When analyzing product quality, hidden parameters can be the level of technology and the quality of raw materials. In diagnosing diseases – heredity and systemic changes in the body. In technical diagnostics – wear of materials and defects in subsystems. In the analysis of the composition of mineral samples – the rate of crystallization of ores and the type of rocks being replaced. When analyzing economic problems (market), the hidden parameters can be solvency and the level of price regulation (pricing).

Similar calculations were performed using the traditional numerical method and show good agreement with the results of this article (see [13]). The disadvantages of the traditional method are the laboriousness of the calculations, the considerable time costs, the strong dependence of the result on the errors of the initial data and the absence of an objective criterion for the end of the calculation procedure.

VI. CONCLUSION

The proposed approach makes it possible to use all advantages of fuzzy data representation and, at the same time, preserve clarity and certainty in the interpretation of the results obtained. The approach based on Zadeh generalization principle is practically inapplicable, or at least very difficult in correlation analysis. The main time consumption in our approach is the representation of the initial data in the form of fuzzy gradations. The results of calculations and conclusions obtained in the article depend mainly on the structure of the initial data. As directions for further research, it should be noted: study of the influence of the degree of data inconsistency on the results of the analysis; determination of the limiting capabilities of the proposed method; comparison of the calculation results and conclusions for the initial matrices, represented by gradations of different levels, namely, a low level (VL or L), an average level (L-M, M or M-H) and a high level (H or VH). The proposed approach based on fuzzy gradations can be applied in the methods of factorial and cluster analysis. This approach can also be used for time series analysis; in this case, relation (5) with corresponding changes is written for autocorrelation.

Conflict of interest

The author declares that there is no conflict of interest.

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