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RESEARCH ARTICLE OPEN

Development of Tensor Product based Dynamic Phasor Estimation Algorithm Satisfying IEEE Standard of Synchrophasor Measurement

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ABSTRACT
In this paper a dynamic phasor estimation algorithm is introduced with the help of tensor product, the proposed algorithms has many qualities for dynamic conditions, it has great simplicity as well as great robustness for dynamic as well for pure sine waves, phasor estimation process follows some simple procedure based on tensor product and linear algebra. The phasor estimation algorithm is tested for different dynamic/noisy events as per IEEE C37.118.1-2011 standards, by observing the results it can be said that the algorithm performs well for dynamic/noisy conditions, the great advantages of algorithm is its simple procedure of implementation, it is also based on simple equations, which makes it easy to realize. For signal conditions like ramp event, modulation event, step event, as well noisy event the propose algorithm has tremendous robustness and performance. Its simplicity and robustness make the algorithm best suited for Wide Area Monitoring for measuring current as well voltage signal having various disturbances.

Index Terms**:** Tensor product, phasor measurement units, wide area monitoring, smart grids.

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I.INTRODUCTION

Now a days smart grid has become very important need for ensuring greater stability and reliability of power system, the performance of smart grid depends on WAM (Wide area measurement), as WAMS are important stages of smart grid, hence more focus is needed to get better efficiency and reliability of smart grids. Phasor measurement units are vital part of any WAM, PMU'S are used to get fundamental phasors from distorted as well pure sinusoidal waves, that means PMU'S are able to give fundamental magnitude, phase, frequency as well as rate of change of frequency from a input signal. The input signal may be distorted from modulation event, frequency ramp event, noise event, and step events also. These all disturbances have been taken into account, and algorithm is tested, as per IEEE **C37.118.1-2011**

standards.

There are numerous literatures [1], [2], [3], [4], [5], [6], [7], [8] present to estimate phasors for dynamic conditions, There are significant differences among them. The phasor estimation based on DFT and least error square algorithm are very old techniques and best suited for pure sinusoidal signal, but for dynamic events, the algorithm fails to get fundamental phasor, for dynamic events DFT and least square algorithms can be used with filters, then it will lead to huge cost requirement, all these demerits made above algorithms unsuitable for estimation of dynamic phasors.

In [1], algorithm based on taylor series expansion is discussed, the dynamic phasor within an observation data window is approximated by 2nd order taylor expansion.

In [2], a phasor estimation algorithm based on Hilbert transform and convolution is discussed, the algorithm is suitable for P-class PMU in protection application. In [3], dynamic phasor estimator based on subspace technique is proposed and high sampling rate and few modifications in the subspace-based techniques are suggested to estimate the voltage phasor with a fundamental frequency component without using antialiasing filter to the input signal. In [4], two fast and precise dynamic phasor estimation algorithms under oscillations and off nominal conditions are discussed, The methods use the signal model under these dynamic conditions, linearize them by using Taylor's series expansion, and estimate the phasor using least squares technique. Frequency and its rate of change are also calculated using adjacent phasors with minimum complexity.

Hence keeping all these requirements in consideration a novel phasor estimation technique is introduced in this paper, which does not require any filter for dynamic phasor estimation, also it gives very good performance and based on simple procedures.

II.DYNAMIC PHASOR ESTIMATOR

Tensors were introduced in 1940s and 1950s by G.kron for use in circuit theory only, tensors can also be used in areas of signal processing, image processing[1].

Here one of the properties of tensors is used for phasor estimation in PMU, normally tensors are multidimensional matrix having various informations. That property of tensor is used here to get fundamental informations from a signal, Here tensor product or kroncker product is discussed.

Assume a sinusoidal wave

$$X(t) = X_m \sin(2\pi f t + \theta) \quad (1)$$

Where $X(t)$=input signal, X_m=peak magnitude of the signal, θ=phase angle, f=fundamental frequency, t=time,

To prove the algorithm Taking N=4 samples

Assume $2\pi f = w$

In matrix form of above sine wave after sampling can be written as

$$x(n) = \begin{bmatrix} \sin(\theta) \\ \sin(wn_1 + \theta) \\ \sin(wn_2 + \theta) \\ \sin(wn_3 + \theta) \\ \sin(wn_4 + \theta) \end{bmatrix} \quad (2)$$

finding out Tensor product of equation (2) with itself

$$x(n) \otimes x(n) = \begin{bmatrix} \sin(\theta) \\ \sin(wn_1 + \theta) \\ \sin(wn_2 + \theta) \\ \sin(wn_3 + \theta) \\ \sin(wn_4 + \theta) \end{bmatrix} \otimes \begin{bmatrix} \sin(\theta) \\ \sin(wn_1 + \theta) \\ \sin(wn_2 + \theta) \\ \sin(wn_3 + \theta) \\ \sin(wn_4 + \theta) \end{bmatrix} \quad (3)$$

Size of tensor spectrum depends on number of samples taken like

Signal has $1 \times (N+1)$ size

Size of Tensor spectrum = 1×25

$$T(n) = \begin{bmatrix} \sin(\theta)\sin(\theta) \\ \sin(\theta)\sin(wn_1+\theta) \\ \sin(\theta)\sin(wn_2+\theta) \\ \sin(\theta)\sin(wn_3+\theta) \\ \sin(\theta)\sin(wn_4+\theta) \\ \sin(wn_1+\theta)\sin(\theta) \\ \sin(wn_1+\theta)\sin(wn_1+\theta) \\ \sin(wn_1+\theta)\sin(wn_2+\theta) \\ \sin(wn_1+\theta)\sin(wn_3+\theta) \\ \sin(wn_1+\theta)\sin(wn_4+\theta) \\ \sin(wn_2+\theta)\sin(\theta) \\ \sin(wn_2+\theta)\sin(wn_1+\theta) \\ \sin(wn_2+\theta)\sin(wn_2+\theta) \\ \sin(wn_2+\theta)\sin(wn_3+\theta) \\ \sin(wn_2+\theta)\sin(wn_4+\theta) \\ \sin(wn_3+\theta)\sin(\theta) \\ \sin(wn_3+\theta)\sin(wn_1+\theta) \\ \sin(wn_3+\theta)\sin(wn_2+\theta) \\ \sin(wn_3+\theta)\sin(wn_3+\theta) \\ \sin(wn_3+\theta)\sin(wn_4+\theta) \\ \sin(wn_4+\theta)\sin(\theta) \\ \sin(wn_4+\theta)\sin(wn_1+\theta) \\ \sin(wn_4+\theta)\sin(wn_2+\theta) \\ \sin(wn_4+\theta)\sin(wn_3+\theta) \\ \sin(wn_4+\theta)\sin(wn_4+\theta) \end{bmatrix} \quad (4)$$

And we are finding Tensor product of signal with itself, hence Tensor spectrum will have the size of $1 \times N^2$

Generalized equation to find out samples containing fundamental phase in Tensor spectrum of sine wave with itself -Assume N samples are taken hence

starting sample number = $\left((N+1) \times \frac{N}{2}\right) + 1$

Ending sample number $\left((N+1) \times \frac{N}{2}\right) + 1$+N,

Hence samples between starting sample and ending sample will contain fundamental signal

Starting sample number = $\left((4+1) \times \frac{4}{2}\right) + 1$ = 11

Ending sample number = $\left((4+1) \times \frac{4}{2}\right) + 1$+4 = 15

Samples containing fundamental = 11^{th} ,12^{th} ,13^{th} ,14^{th} ,and 15^{th} of T(n), Now adding the samples containing fundamental

$$F(n) = \sin(wn_2+\theta)\sin(\theta) + \sin(wn_2+\theta)\sin(wn_1+\theta) + \sin(wn_2+\theta)\sin(wn_2+\theta) + \sin(wn_2+\theta)\sin(wn_3+\theta) + \sin(wn_2+\theta)\sin(wn_4+\theta) \quad (5)$$

Taking out $\sin(wn_2 + \theta)$ common from equation (5) we can re-write the equation (5)

$F(n) = \sin(wn_2 + \theta)\,[\sin(\theta) + \sin(wn_1 + \theta) + \sin(wn_2 + \theta) + \sin(wn_3 + \theta) + \sin(wn_4 + \theta)]$ (6)

From equation (2) we can write equation (6) as

$F(n) = K\,x(n)$ (7)

Where $x(n)$ is fundamental sine wave, and $K = \sin(wn_2 + \theta)$ will have some constant value

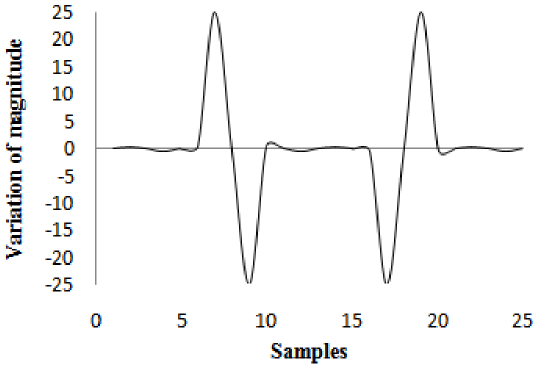


Fig.1: Tensor spectrum with 4 samples

From figure (1), it can be seen that the fundamental wave is contained within 11 to 15 samples, the minimum value of that small spectrum is to be found out and location of that minimum value is to be tracked which gives relationship between phase angle and location of small spectrum.

Like this if N=8 samples fundamental sine wave can be found in tensor spectrum by adding samples from

Starting sample number = $\left((8+1) \times \frac{8}{2}\right) + 1 = 37$

Ending sample number = $\left((8+1) \times \frac{8}{2}\right) + 1 + 8 = 45$

Here tensor spectrum will have size of $1 \times (9 \times 9) = 1 \times 81$

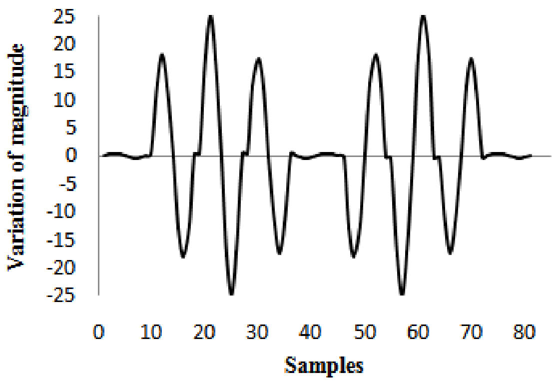


Fig.2: Tensor spectrum with 8 samples

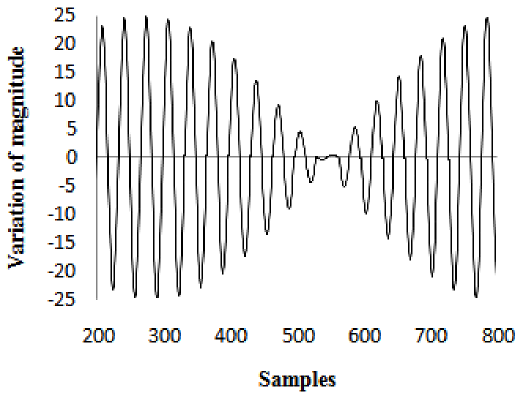


Fig.3: Tensor spectrum with 32 samples

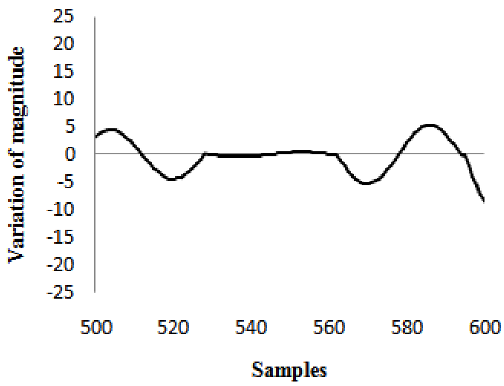


Fig.4: Fundamental view of tensor spectrum with 32 samples

In this paper N=256 is taken hence

Starting sample number = $\left((256+1) \times \frac{256}{2}\right) + 1 =$ 32897

Ending sample number = $\left((256+1) \times \frac{256}{2}\right) +$ 1+256= 33153

Here tensor spectrum will have size of

$1 \times (257 \times 257) = 1 \times 66049$

Here F(n) can be found by adding samples from sample number 32897 to sample number 33153

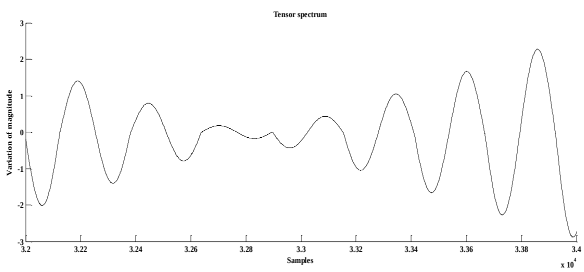


Fig.5: Fundamental view of tensor spectrum with 256 samples

So it can be witnessed from above tensor spectrum that peak value of the spectrum is constant which is square of fundamental magnitude taken, hence the magnitude can be found by taking square root of peak value of tensor spectrum and also by taking minimum value of F(n) and finding out its location in X-axis with respect to phase change, relation between phase change and location of small value of fundamental spectrum in X-axis can be obtained as it can be seen it gives linear relation.

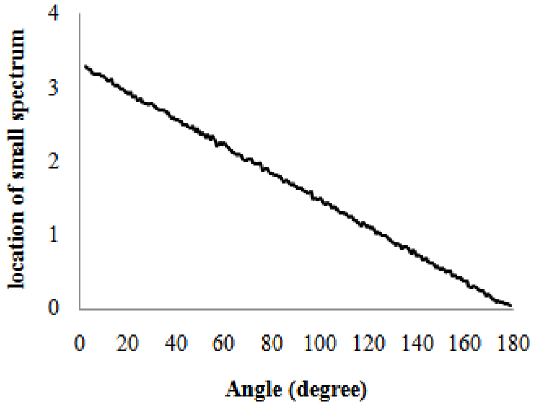


Fig.6: Location of small spectrum for pure sine wave

Fig (6) shows the relation between change of location of fundamental spectrum with respect to phase ange variation for pure sinusoidal wave. It can be observed from fig (7),(8),(9),(10) that for dynamic signal also the algorithm gives linear relation with phase change which is desirable.

TABLE-1 signals used

Signals	Equations
Sine wave	$x(t) = X_m \sin(2\pi ft + \theta)$
Step change event	$x(t) = X_m(1 + K_{xs}U_1(t))\sin(2\pi ft + K_{as}U_1(t) + \theta)$
Frequency ramp event	$x(t) = X_m \sin(2\pi ft + \pi R_f t^2 + \theta)$
Modulation event	$x(t) = X_m(1 + K_{xm}\sin(2\pi f_m + \theta))\sin(2\pi ft + K_{am}\sin(2\pi f_m t) + \theta)$
Noise event	$x(t) = X\sin(2\pi ft + \theta) + \varepsilon$

In table (1) x(t)=input signal, X_m=peak magnitude of the signal, θ=phase angle, f=fundamental frequency, t=time, K_{xm}=modulation index, K_{am}=phase sensitivity, f_m=modulation frequency, R_f=frequency ramp rate, K_{xs}=magnitude step size, K_{as}= phase step size, $U_1(t)$=unit step signal, ε=Gaussian noise present in the signal

TABLE-2 specifications used

Parameter	Notation	Specifications
Nominal magnitude	X_m	5 volts
Nominal frequency	f	50Hz
Phase angle	θ	30 Degree
Phase angle sensitivity	K_{am}	0.1
Modulation frequency	f_m	0.2 to 2 Hz
Step change size	K_{xs}	0.1
Phase step size	K_{as}	0.1
Noise	ε	15 db to 50 db SNR

In this work, the following specifications as shown in Table 2 are taken to test proposed phasor estimation algorithms. The proposed algorithm is able to estimate one phasor per cycle at a sampling rate of 256 samples per cycle.

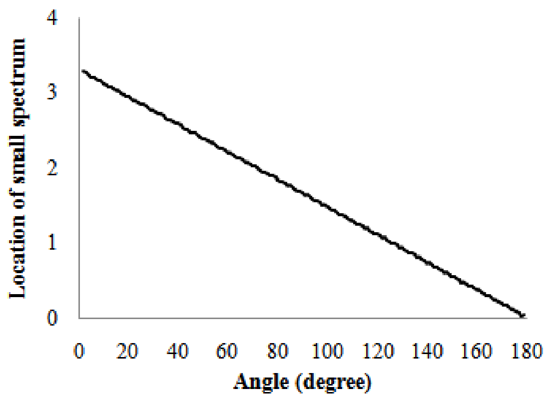


Fig.7: Change of location of small spectrum for frequency ramp event

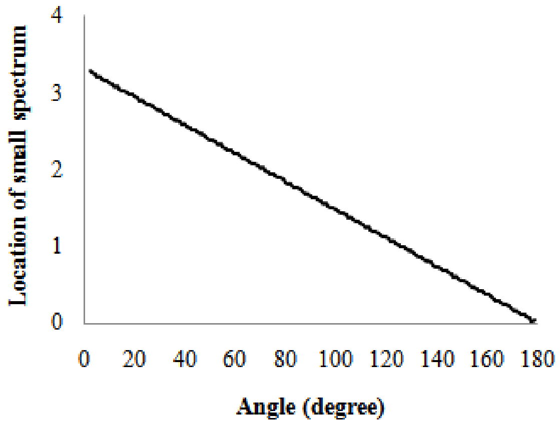


Fig.8: Change of location of small spectrum for noise event

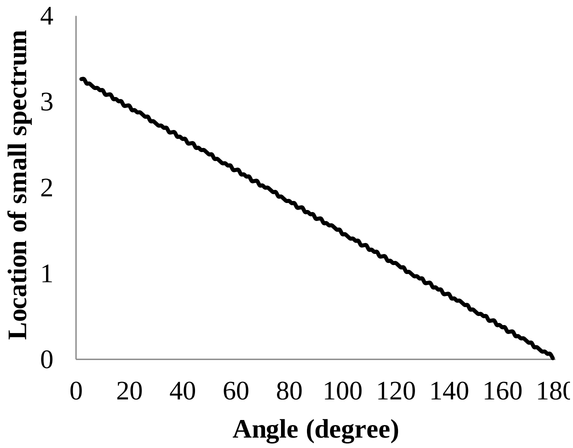


Fig.9: Change of location of small spectrum for modualtion event

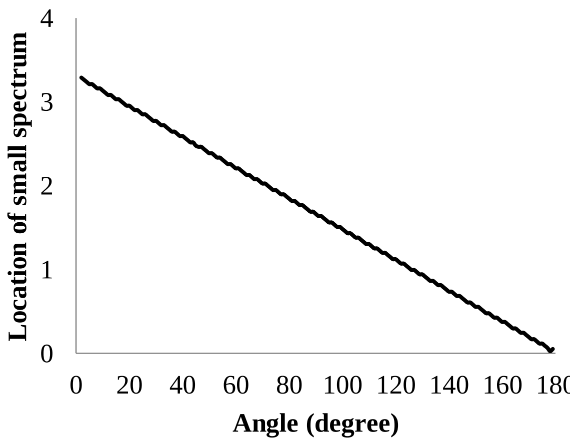


Fig.10: Change of location of small spectrum for step event

III.SIMULATION RESULTS

A.For pure sinusoidal wave

In power system voltage and current can be represented as sinusoidal waveform shown in equation (1) for normal operating condition.

$$x(t) = X_m \sin(2\pi f t + \theta) \quad (8)$$

Where X_m is peak magnitude of sine wave , f is nominal frequency and θ is the phase angle. PMU'S must be able to detect magnitude and phase angle of pure sine wave efficiently for different phase angles and for different cycles.The performance of any PMU can be estimated by calculating TVE with respect to phase angle change as well for different phasors. The figures shown below can be seen to find TVE at different conditions.

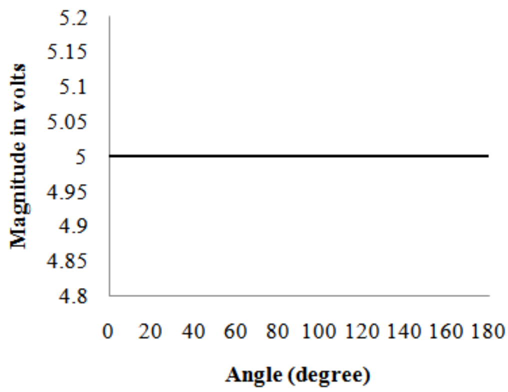


Fig.11: Magntitude vs phase angle for pure sinusoidal signal

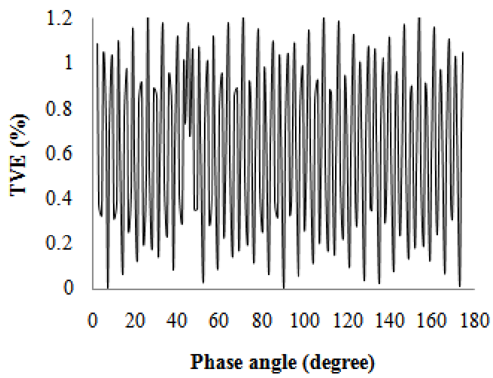


Fig. 16: TVE vs phase angle for pure sine wave.

Figure (16) shows the TVE with respect to change in phase angle, the phase angle is varied here from 0 to 180 in degree, For a phasor estimation technique there must not be much change in magnitude and TVE with respect to phase angle change, hence the pure sine wave is tested for the same. As we can see the tensor algorithm is performing within IEEE standards as TVE is around 1% only.

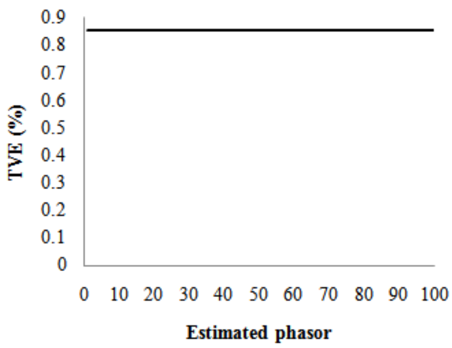


Fig. 17: TVE vs estimated phasor for pure sine wave.

Figure (17) shows the TVE for different estimated phasors, each cycles of time contains 1 phasor hence for 50 phasors separate TVE is there, as it can be seen TVE is restricted upto 0.9 for pure sine wave and satisfying IEEE standards for pure sine wave.

B. For sinusoidal wave with modulation event

There are various abnormal conditions in power system because of that there is some undesirable change in fundamental magnitude and phase of sinusoidal wave. These changes must not affect the phasor estimation by PMU.

PMU'S must be always able to find fundamental magnitude and phase from various abnormalities. Power swings in power systems is one the major abnormality. These power swings are caused due to generator outages, switching of lines, use of lumped load and also overloaded tie lines, these all changes in power system causes oscillations in machine rotor angles in power swing. These power swings can be modeled as modulated sine wave and it causes abnormal change in magnitude and phase of the pure sine wave.

The modulated sine wave can be represented as equation

$$x(t) = X_m\big(1 + K_{xm}\sin(2\pi f_m t + \theta)\big)\sin(2\pi f t + K_{am}\sin(2\pi f_m t) + \theta) \quad (9)$$

Where X_m is peak magnitude of sine wave , f is nominal frequency and θ is the phase angle, K_{xm} modulated amplitude, f_m is modulation frequency, K_{am} phase sensitivity.

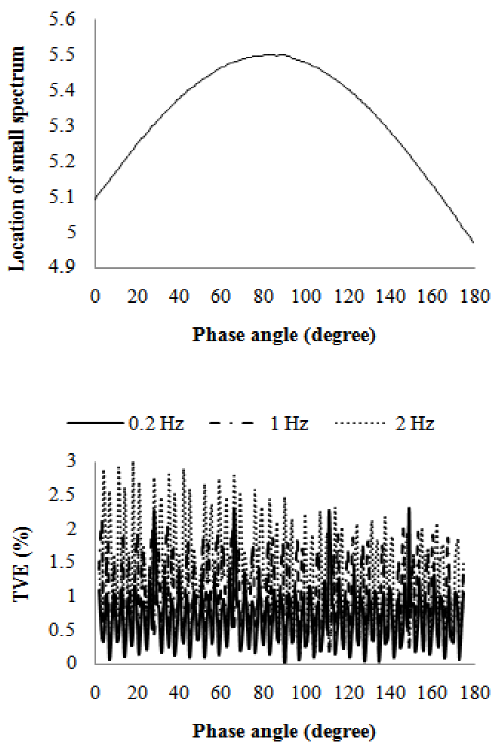


Fig.18: TVE vs phase angle with modulation event.

Figure(18) shows TVE with respect to phase angle variation, here phase angle is varied from 0 degree to 180 degree and TVE is estimated, the modulation frequency taken here are 0.2 Hz, 1Hz and 2 Hz, as it can be seen that there is more TVE for 2 Hz modulation frequency as compared to 0.2 Hz and 1Hz, and all the conditions are meeting IEEE standards.

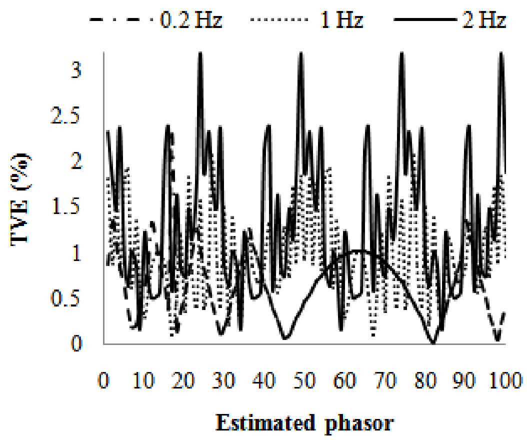


Fig.19: TVE with modulation event.

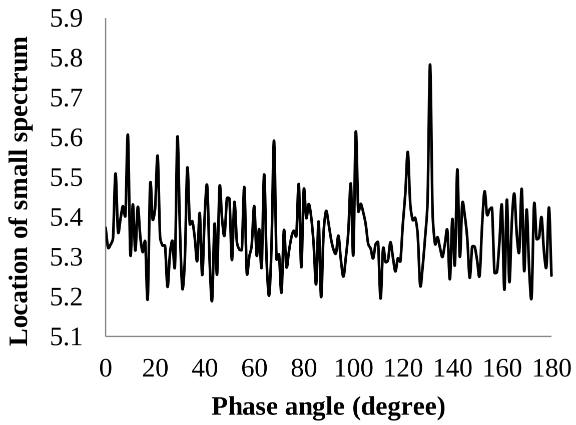
Figure (19) shows TVE with respect to estimated phasor for different modulation frequencies, in power system modulation frequency varies from 0.2 Hz to 2 Hz hence the signal is tested for 3 modulation frequencies, 0.2 Hz, 1Hz and 2 Hz. As we can see there is maximum TVE for 2 Hz modulation frequency and less TVE for 0.2 Hz, TVE for all the cases within 3 % as per requirements of IEEE standards.

C. For sinusoidal wave with noise event

The use of capacitor banks as well as some capacitive loads causes noise in power system, also while receiving signals in PMU may cause pure sine wave to get distorted hence the algorithm is also tested for noise event for different Signal to Noise ratios e.g. 20 db, 15 db and 50 db.

$$x(t) = X_m\sin(2\pi f t + \theta) + \varepsilon \quad (10)$$

Equation (10) represents sine wave distorted with white Gaussian noise, where X_m is peak magnitude of sine wave , f is nominal frequency and θ is the phase angle, and ε is white Gaussian noise.



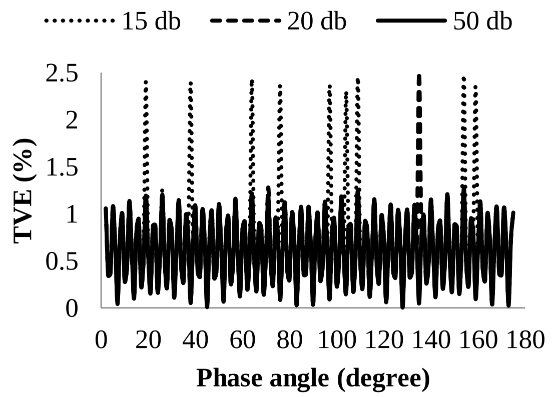


Fig. 20: TVE vs phase angle for noise event.

Figure (20) shows TVE variation with respect to phase angle in degree. As we can see there is more TVE for noise15 db SNR and least TVE for 50 db SNR, here also all the cases meeting IEEE standards as TVE is within 3 % only.

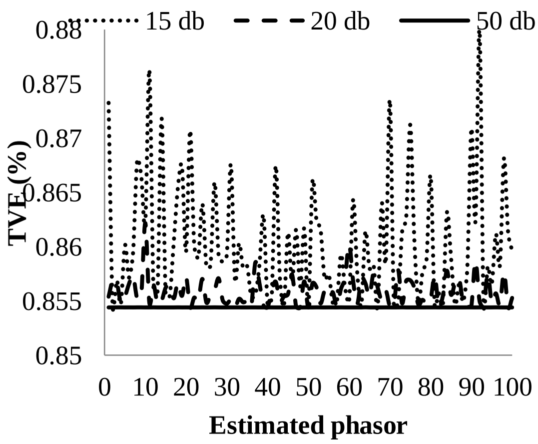


Fig.21: TVE with noise event.

Figure(21) shows variation of TVE with respect to estimated phasors for various cycles, here algorithm is tested for noise with15 db, 20 db and 50 db. the algorithm proposed here performs very well for noise event and TVE is restricted less than 1% which is one of the major advantage of proposed algorithm.

D.For sinusoidal wave with frequency ramp rate event

In power system to meet the load demands the generating power has be adjusted, load demand is not constant in power system it changes time to time, hence to meet all these automatic generation control adjusts the speed of generators, due to that there is sudden increase and decrease in frequency of power system. This phenomenon can be represented as sinusoidal wave with ramp event and it can be mathematically shown as

$$x(t) = X_m \sin(2\pi ft + \pi R_f t^2 + \theta)$$
(11)

where X_m is peak magnitude of sine wave , f is nominal frequency and θ is the phase angle, and R_f is ramp constant.

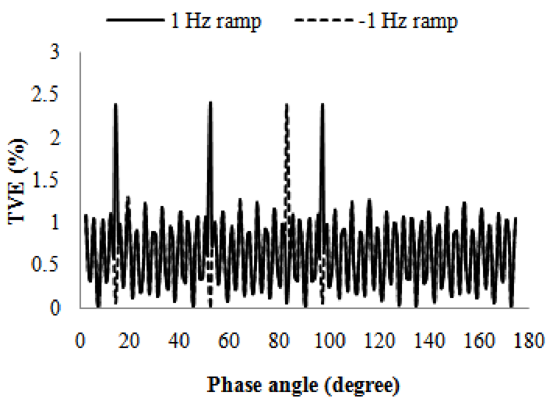
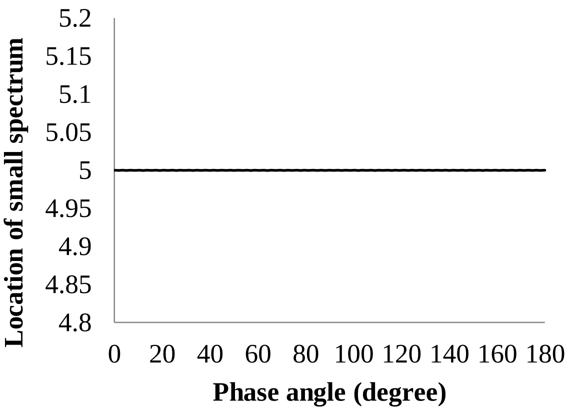


Fig.22: TVE vs phase change with ramp event.

Figure (22) shows the variation of TVE with respect to change in phase angle, it can be seen that for various phases the TVE is less than 3%. The signal is tested for positive ramp as well as negative ramp of 1 ramp constant. In both the cases the algorithm is meeting IEEE standards.



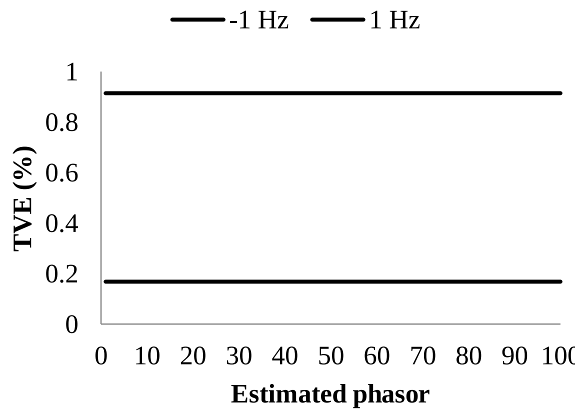


Fig.23: TVE vs estimated phasor with ramp event.

Figure (23) show TVE with respect to estimated phasor for positive as well as negative ramp constants as we can see TVE is constantly maintained less than 1%, which is very much lesser than desirable TVE for IEEE standards. for dynamic events 3% TVE is acceptable but proposed algorithm has very less TVE which is one of the biggest advantage of proposed algorithm.

E. For sinusoidal wave with step change event:

In power system due to lightening surges and also switching phenomenon the transients occurs, these transients occurs for some cycles and after that it can be cleared also, step wave in power system is the best way to represents these surges. PMU's must be able to identify such events and to give satisfactory TVE under these conditions.

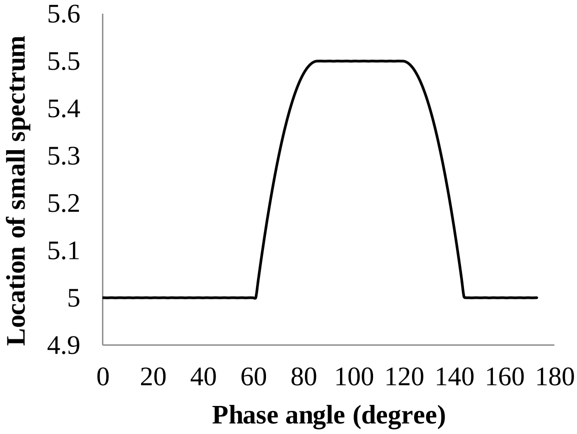
Mathematically sine wave with step event can be represented as:

$$x(t) = X_m\big(1 + K_{xs}U_1(t)\big)\sin(2\pi ft + K_{as}U_1(t) + \theta) \quad (12)$$

where X_m is peak magnitude of sine wave , f is nominal frequency and θ is the phase angle, K_{xs}=amplitude step size, K_{as}=phase step size

$$U_1(t) = \begin{cases} 0 \text{ if } t < t_1 \\ 1 \text{ if } t > t_2 \end{cases} \quad (13)$$

t_1 = step change occurance time,



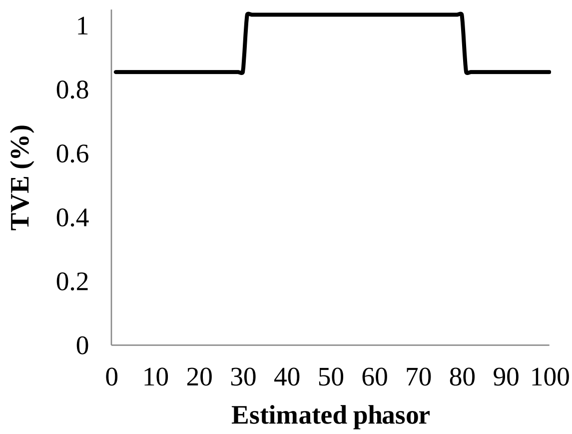


Fig. 24: TVE with step event.

Figure(24) shows variation of TVE with respect to estimated phasor , step change of 10% in magnitude and phase is created from 30th to 80th estimated phasor, as it can be seen from figure (24) that proposed algorithm has robust performance for step event, it is maintaining less than 1% TVE which is desirable for IEEE standards.

IV.CONCLUSION

This paper presents simplest and robust dynamic phasor estimation algorithm based on tensor product of signal with itself, the algorithm has been successfully tested under compliance test recommended by IEEE **C37.118.1-2011** standards. The simulation results shows that the algorithm is suitable for dynamic phasor estimation and also suitable for pure sinusoidal wave as it gives TVE less than 3% for dynamic events and TVE less than 1% for pure sinusoidal waves. The proposed algorithm is simplest fast and robust for various dynamic conditions, surely the implementation of algorithm in PMU,S will increase the efficiency reliability of smart girds.

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