RESEARCH ARTICLE OPEN ACCESS

Variable Structure Control for Stabilizing a Fourth order System.

Vibha Bhatnagar*, Gauri Gupta*
**(Department of Biomedical Engineering, Shri Govindram Institute of Technology & Science, Indore-01*

ABSTRACT
The Aim of feedback control is to make the outputs of a system to follow a certain desired function. There exist nonlinear control laws that ensure higher degree of stabilization along with better transients. A variable structure control algorithm is proposed in this paper, which stabilizes the system by switching the system control input among a set of feedback control laws. Each control law uses only one state variable with a fixed plus or minus gain at a time. Hence to realize the proposed Variable Structure Controller for n order system 2n feedback control laws are needed , simulation of fourth order system is done using MATLAB SIMULINK
***Keywords* –** Variable Structure Control, Sliding Mode, Sliding Sector, Extremum seeking, Lyapunov Function

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Date of Submission: 06-07-2020 Date of Acceptance: 21-07-2020

I. INTRODUCTION

The aim of feedback control is to make the outputs of a system to follow a certain desired function. To achieve such an aim it is necessary to derive error variables by instantaneously comparing outputs with the reference time functions. Thereafter the error variables are processed by so called a controller to generate actuating signals that shall drive the system (plant) so that the outputs tend towards the references. The applications of feedback control therefore change entirely the natural behavior so much so that there arises the question of closed loop stability. In this context of stability, the design of a controller poses two kinds of situations: (a) the nature of closed loop transient, and (b) degree of stability. There are several ways of formulating control laws such as proportional, derivative, integral and some linear combinations of these for linear dynamic systems. Theses control laws are linear in nature. Yet there exist non-linear control laws that ensure higher degree of stabilizability along with better transients. One such control as found in literature and has become very much popular, is variable structure control.

Literature survey revealed the work of researchers in this area.Vadim. I. Utkin [1] Elaborated on the design and analysis of a variable structure system having a suitable switching logic in his survey paper. Raymond A.DeCarlo. et.al [2] developed and surveyed the basic essential concepts of VSC, In Particular analysed the concept with regards to multi input case, and defined its scope in various fields of applications such as PWM Control Strategies, Robotics and Power Systems. Yaodong Pan , Katsuhisa Furuta [3] presented a robust and optimal VSS control law for a discrete time system, which enabled the system state to move into a sliding sector where the closed loop system is stable. Finally, to show system robustness bounded parameter uncertainty is considered. They employed pendulum control system for Simulation of the concept. K *S* Low.et.al [4] Proposed a chattering free control approach for position control of brushless dc drive using discrete time sliding mode with no high frequency switching term. Implementation was done with a DSP to control an experimental drive in real time. Results of which showed the response to be insensitive to the load disturbance and the inertia variation Andrzej Bartoszewicz [5]. Claim to have achieved better performance than previous algorithms. Which they verified theoretically and by simulation on an multi input multi output system. they proposed new discrete-time quasi-sliding mode control strategies based on the so-called reaching law approach. Which bounded the system state to stay in a band around the sliding hyperplane, but not to cross the hyperplane in each successive control step. Zoran M. Bučevac [6] Developed stabilizing state feedback control algorithm by Lyapunov's second method leading to the variable structure system with sliding modes. Essentially the system is driven from some arbitrary initial state to a predefined so-called sliding subspace S by the control algorithm in finite time. Inside the sliding subspace S the system is switched to the sliding mode regime and stay in it forever. Yaodong Pan.et .al [7]. Result of their VS controller with a generalized PR-sliding sector when applied on rotational inverted pendulum was chattering-free and with good control performance. The designed VS control system is quadratically stable even in presence of parameter uncertainties and external disturbances. Yaodong

Pan.et .al [8] proposed a VS controller for a SISO continuous time system, but could be extended to MIMO continuous time system and also can be implemented on Discrete time VS control & servo control. Their approach of VS controller with the sliding mode ensures that the state converges in a finite duration to the sliding mode and stays in it for ever. This produced stable, chattering free results. Yaodong Pan, Katsuhisa Furuta [9]. proposed a Quadratically stable controller for hybrid systems where subsystems may be unknown and need not be stable. Based on assumption that a stable convex combination of the subsystems exists a Lyapunov function is designed. The extremum seeking control algorithm is used to determine the switching rule such that the Lyapunov function tracks a pre-determined decreasing signal.[13][14][15] proposed different sliding mode approaches for nonlinear systems with uncertainties.

Present work is devoted to the design of set of feedback control laws using VSS (Variable Structure system) such that within each feedback control law a Lyapunov function decreases. Linear time –invariant fourth order system is considered in which samples only one state variable in each time interval, The Lyapunov function based on convex combination of the feedback laws is used to define the sliding sector whereby a switching logic is developed using extremum seeking control where the common Lyapunov function tracks a predefined decreasing reference signal and the system asymptotically converge to zero Four examples are presented using MATLAB-SIMULINK software to illustrate the principle

II. VARIABLE STRUCTURE CONTROLLER

Variable structure control, or VSC, is a discontinuous nonlinear control. The method alters the dynamics of a nonlinear system by application of a high-frequency switching control. The state-feedback control law is not a continuous function of time; it transits from one smooth condition to another. Based on the position of the state trajectory; the control switches from one smooth control law to another and possibly at very fast speeds (e.g., for a countable infinite number of times in a finite time interval) and hence the structure of the control law varies.

Variable structure control (VSC) and associated sliding mode behavior was first investigated in early 1950s in the Soviet Union by Emelyanov and several co researchers. Variable structure systems consist of a set of continuous subsystems together with suitable switching logic. In variable structure systems the control changes its structure, according to one to another member of a set of possible continuous functions of the state. The next design step is to define the switching logic. for variable structure by selecting the parameters of the structure. A variable structure system has a characteristic of new properties not present in any of the structure used. For instance, two structures neither of which is asymptotically stable may form an asymptotically stable system.

Most important virtue of a VSC system is its robustness. Under certain conditions, the sliding mode of a VSC system more than just being robust is also invariant with respect to system perturbations and external disturbances. Proper design of switching function guarantees the sliding mode of a VSC to be asymptotically stable.

Elementary adaptive systems are those in which some characteristic of the system displays a unique maximum, which is directly measurable, and the object of the control process is to maintain the operating conditions of the system at, or in the vicinity of, the maximum. Such systems determine in some manner the slope of the characteristic and endeavor to operate substantially at the zero slope condition corresponding with maximum output, and are frequently termed “Extremum –Seeking Regulators” Extremum seeking regulators make use of techniques such as switching methods in which the system hunts continuously about the maximum. Various self-driving methods in which information about the slope of the characteristic is obtained directly and used to drive the system towards the maximum, and perturbation methods in which slope information is obtained from a continuous small perturbation signal.

III. PROBLEM DESCRIPTION

Yaodong Pan and Katsuhisa Furuta [13] work is taken as base for realization of a fourth order time invariant system. According to which the methodology is revisited in section 3.1,3.2,3.3. Consider an n^{th}-order system with 2n feedback control laws. Only one state variable with a fixed plus or minus feedback gain is used by each control law. A Variable Structure Feedback Control algorithm is used to stabilize the system by switching the system control input among the 2n feedback control laws.

The concept of Sliding Sector shows that there exists an area for each control law, inside which a Lyapunov function decreases with the control law, although the system cannot be stabilized by it alone. A common Lyapunov Function for all control laws exists, if the system is stable. Sliding mode is defined based on the Extremum seeking control algorithm such that the common Lyapunov function tracks a pre-determined decreasing function ..

Consider a single input linear time-invariant system as described by the state space equation:

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (1)$$

Where x(t) is the state vector, u(t) is the input variable, $A \in \Re^{n\times n}$ *and* $B = \Re^{n\times 1}$ are system matrices, and (A, B) is a controllable pair.

It is assumed that only one state variable is available for the feedback. Thus, a sampled data control may be realized with a high sampling frequency because there is no time delay as sampling of all state variables is avoided and only one state variable is active at a time. It is also assumed that the feedback control laws corresponding to every state variable are designed as $\pm kx_i(t), (i = 1,2, \ldots n)$ where k is a positive constant. The 2n control laws are rewritten as

$$u_i(t) = F_i x(t),\ (i = 1,2,\ldots\ldots, 2n) \quad (2)$$

where the feedback gain $F_i\ (i = 1,2,\ldots, 2n)$ are defined as :

$$F_{2i-1} = kI_{ni}$$
$$F_{2i} = -kI_{ni} \quad (i = 1,2,\ldots,n)$$

I_n (i=1, 2,….,n) is the i-th row of an n x n identity matrix

The aim of the control system is to asymptotically stabilize the system (1) by switching the control input u (t) among the feedback control laws $F_i x(t),\ (i = 1,2,\ldots\ldots, 2n)$ in the sequence such that a measurable Lyapunov Function:

$$L(T) = x^T(t)Px(t) \quad (3)$$

For some positive definite matrix $P \in \Re^{n\times n}$ decreases with a VS control input:

$$u(t) = F_{\sigma(t)}x(t) \quad (4)$$

In each period when the control input is switched once through all the feedback control laws, determined by a VS control rule taking values from a finite set $\sum := \{\sigma : \sigma = 1,2,3,\ldots.2n\}$ in the sequence of $\{1,2,\ldots 2n,1,2\ldots 2n,1,\ldots.\}$

3.1. Lyapunov Function

The (A, B) pair are considered to be controllable, hence there exists a feedback control law:

$$u(t) = Fx(t), \qquad F = [f_1\ f_2 \ldots f_n] \in \Re^{1\times n} \quad (5)$$

Such that the closed loop system

$$\dot{x}(t) = (A + BF)x(t) \quad (6)$$

Is stable Where F is denoted as

$$F = \begin{bmatrix} F_1 \\ F_2 \\ . \\ . \\ . \\ F_{2n-1} \\ F_{2n} \end{bmatrix} = \begin{bmatrix} k\ 0\ldots 0\ 0 \\ -k\ 0\ldots 0\ 0 \\ . \\ . \\ . \\ 0\ 0\ldots 0\ k \\ 0\ 0\ldots 0 -k \end{bmatrix} \in \Re^{2n\times n} \quad (7)$$

One such solution to the following linear algebraic equations

$$\beta F = \mathbf{F} \quad (8)$$

can be obtained as

$$\beta = \mathbf{F}(F^T F)^{-1} F^T \quad (9)$$

where $\beta \in \Re^{1\times 2n}$ is a parameter vector defined as

$$\beta = [\beta_1\ \beta_2 \ldots \beta_{2n}] \quad (10)$$

By calculating, elements of β are determined by

$$\beta_{2i-1} = -\beta_{2i} = \frac{f_i}{2k}, (i = 1,2,\ldots, n) \quad (11)$$

If k is chosen to be large enough, it is possible to find a positive constant γ such that

$$\gamma = \frac{1}{2n}(1 - \sum_{i=1}^{2n}|\beta_i|) = \frac{1}{2n}(1 - \sum_{i=1}^{n}\frac{|f_i|}{2k}) \succ 0$$

Define

$$\alpha_i = \beta_i + |\beta_i| + \gamma \succ 0,\ (i = 1,2,\ldots.,2n)$$

A convex combination of the feedback control laws, $F_i x$ (t) (i = 1, 2, …,2n), which stabilizes the system (1) is obtained as

$$u(t) = \sum_{i=1}^{2n} \alpha_i F_i x(t) = Fx(t) \quad (12)$$

Because positive constant $\alpha i\ (i = 1,2,\ldots.,2n)$ satisfy the following coefficient conditions of a convex combination

$$0 \prec \alpha_i \prec 1, i = 1,2,\ldots,2n$$
$$\sum_{i=1}^{2n} \alpha_i = \sum_{i=1}^{2n} |\beta_i| + 2n\gamma = 1$$

And equation 12 holds with

$$\sum_{i=1}^{2n} \alpha_i F_i = \sum_{i=1}^{2n} (\beta_i + |\beta_i| + \gamma)F_i$$
$$= \sum_{i=1}^{2n} \beta_i F_i = \beta F = F$$

For the stable closed loop system (6), there exists a Lyapunov function defined as

$$L(t) = \|x\|_P^2 = x^T Px \succ 0, \forall x \in \Re^n, x \neq 0 \quad (13)$$

Whose derivative satisfies

$$\dot{L}(t) = x^T(A *^T P + PA *)x = x^T Rx \leq 0, \forall x \in \Re^n, \quad (14)$$

Where $A *= A + BF, P \in R^{n\times n}$, is a positive definite symmetric matrix P,$R = C^T C \in R^{nxn}$ a positive semi definite symmetric matrix, $C \in R^{l \times n}$, $l \geq 1$ *and* (C, A^*),are observable pair. Sliding Sectors will be defined by the Lyapunov Function defined in equation (13) based on the convex combination in (12)

3.2. SLIDING SECTOR

The system (1) with one of the feedback control laws $u(t) = F_i x(t), (i = 1,2,\ldots,2n)$ is determined by

$$\dot{x}(t) = A_i x(t) A_i = A + BF_i (i = 1,2,\ldots,2n) \quad (15)$$

Consider the Lyapunov Function as in Eq (13) the inequality:

$$\dot{L}(t) = x^T (A_i{}^T P + PA_i) x \leq -x^T R x \forall x \in \mathfrak{R}^n$$

The mentioned inequality does not hold true in general because the stability of the closed loop system in (15) is not guaranteed. The state space for each control law is decomposed into parts such that one part satisfies the condition

$$\dot{L}(t) = x^T (A_i{}^T P + PA_i) x \succ -x^T R x \text{ for some}$$

elements $x \in \mathfrak{R}^n$,and another part satisfies the condition

$$\dot{L}(t) = x^T (A_i{}^T P + PA_i) x \leq -x^T R x$$

for some other elements $x \in \mathfrak{R}^n$,elements in this group form a special subset in which the Lyapunov function L (t) decreases.

A P-norm denoted by $\|x\|_P$ is defined as the square root of the Lyapunov function L (t) in (13)

$$\|x\|_P = \sqrt{L(t)} = \sqrt{(x^T P x)}, x \in \mathfrak{R}^n \quad (16)$$

With the i th control law the P norm $\|x\|_P$ decreases inside this special subset.

$u(t) = F_i x(t) \quad as$

$$\frac{d}{dt}\|x\|_P{}^2 = x^T (\mathrm{A}_i{}^T \mathrm{P} + \mathrm{PA}_i) x \leq -x^{\mathrm{T}} R x \qquad (i-1,2,\ldots 2n)$$

This special subset is called as PR_i sliding sector because the matrices P and R together with the system parameter $A_i = A + BF_i$ (i=1, 2,…, 2n) define the property of this subset.

For the system in equation (15) the PR_i sliding sectors defined on the sate space $\mathfrak{R}^n$ is

$$Si = \{x | x^T (A_i{}^T P + PA_i) x \leq -x^T R x, x \in \mathfrak{R}^n\} \quad (17)$$

inside which the P norm of the system decreases and satisfies

$$\frac{d}{dt}\|x\|_P{}^2 = \frac{d}{dt}(x^T(t) P x(t)) = x^T (A_i T P + PA_i) x + PA_i) x$$
$$\leq -x^T(t) R x(t) \forall x(t) \in S_i$$

Where P and R are the matrices used in the definition of the Lyapunov function (13)

3.3. VARIABLE STRUCTURE CONTROLLER

Concept of extremum seeking control is used to define a VS control rule to switch the control input among all feedback control laws is which is given by:

$$\sigma(t) = 1 + [\sigma(t-) + \bar{\sigma}(t)] \bmod 2n \qquad (18)$$

Where

$$\bar{\sigma}(t) = \frac{|sgn \sin(h\Pi s(t)/\alpha_{\sigma(t-)} - sgn \sin(h\Pi s(t-)/\alpha_{\sigma(t-)}|}{2}$$

$\sigma(t-)$ is the previous discrete time signal to any time instant t , h is a positive constant, sgn () is the signum function and s(t) defined as

$$s(t) = L(t) - g(t) \qquad (19)$$

Equation (15) defines the Lyapunov function $L(t)$ and decreasing signal $g(t)$ is determined by

$$g(t) = -\varepsilon^{\cdot}(g(0) = 0) \qquad (20)$$

ε is a large enough positive constant.

The VS control rule given in equation (18) is used by the control input to switch to the next control law at any time instant, when the trigonometric function sin (hπs (t)/ά$_i$) changes its sign. In general the switching function s(t) is an increasing function with a large enough positive constant ε, this enables the control input switching among all the feedback control laws in the sequence (1,2,….2n,1,2…)

The above VS control rule ensures that the Lyapunov function $L(t)$ decreases and asymptotically converge to zero by tracking the decreasing signal $g(t)$ in every switching period for

$$\sigma(t) = \{1,2,\ldots.2n\}$$

The interval for the control input to be switched to the next control law is very short and can be approximately given by

$$\Delta t_i = \frac{\alpha_i}{2\varepsilon h}, \ (i = 1,2,\ldots,2n)$$

And the total time interval for the control input to be switched once among all the control laws is

$$\Delta t = \sum_{i=1}^{2n} \frac{\alpha_i}{2\varepsilon h}$$

Which is also a very short interval.

As the change rate is different, the exact time interval for each control law is given by

$$\overline{\Delta t_i} = \frac{\alpha_i}{2(\dot{L_i} + \varepsilon)h}, (i = 1,2,\ldots 2n)$$

Where it is assumed that the parameter ε is chosen to be large enough to satisfy

$$|\dot{L}_i| < \varepsilon, (i = 1,2,\ldots 2n)$$

This means that the control law will be used for a longer time interval, if the Lyapunov function for this control law decreases otherwise use it for a shorter time interval. In this way, in every switching period from the control law $F_1 x$ to the control law $F_{2n} x$,the Lyapunov function will decrease. No matter whether $\dot{L}_i(t)$, $(i = 1,2 \ldots 2n)$ is positive or negative. In other words, with the proposed VS control rule even though the Lyapunov function may increase in some time intervals it will decreas in every switching period .

IV. SIMULATION

Simulation of fourth order system is done using MATLAB- SIMULINK. Consider Fourth order system as

$$x(t) = Ax(t) + Bu(t) \tag{21}$$

Where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 20.601 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -0.4905 & 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0.5 \end{bmatrix}$$

Eight feedback control laws are

$$F1 = \begin{bmatrix} k & 0 & 0 & 0 \end{bmatrix}, \ F1 = \begin{bmatrix} -k & 0 & 0 & 0 \end{bmatrix}$$
$$F2 = \begin{bmatrix} 0 & k & 0 & 0 \end{bmatrix}, \ F4 = \begin{bmatrix} 0 & -k & 0 & 0 \end{bmatrix}$$
$$F5 = \begin{bmatrix} 0 & 0 & k & 0 \end{bmatrix}, \ F6 = \begin{bmatrix} 0 & 0 & -k & 0 \end{bmatrix}$$
$$F7 = \begin{bmatrix} 0 & 0 & 0 & k \end{bmatrix}, \ F8 = \begin{bmatrix} 0 & 0 & 0 & -k \end{bmatrix}$$

Where the positive constant k is chosen as k=200.

Eigen values of the system (21) with each control law

eig(A+B*F1)= {0 , 0 , 99.1996 , 0.8004}
eig(A+B*F2)={0 , 0 , -101.1918 , 1.1918}
eig(A+B*F3) = {0 , 0, 17.6809 , -117.6809}
eig(A+B*F4) = {0 , 0 , 28.5407 , 71.4593}
eig(A+B*F5)={100.4927, 4.5784, -4.6084, -0.4627}
eig(A+B*F6)= {-99.4924, 4.5752, -4.6159, -0.4669}
eig(A+B*F6) = {0 , -4.0067 , 4.9956 , 49.0111 }
eig(A+B*F6) = {0 , 4.9084 , -3.9197 , -50.9887}

Hence the system cannot be stabilized by any of the control laws alone.

The feedback law employing pole assignment method to stabilize the system (21) with the eigenvalues of {-2 , -4 , -5 , -8} is given by

F= F= [-45.0145, -10.9279, 32.6198, -35.0663];

Choosing R as

R=[1 0 0 0;0 1 0 0; 0 0 1 0;0 0 0 1]

The positive definite solution P of the Lyapunov Equation

$$(A + BF)^T P + P(A + BF) = -R$$

Is given by

$$P = \begin{bmatrix} 6.110 & 1.4128 & -5.8772 & 3.5970 \\ 1.4128 & 0.4388 & -1.3197 & 1.0413 \\ -5.8772 & -1.3197 & 10.9862 & -12.880 \\ 3.5970 & 1.0413 & -12.8880 & 19.9580 \end{bmatrix}$$

By the calculation

$$\alpha_i = \beta_i + |\beta_i| + \gamma \succ 0, \ (i = 1,2,\ldots.2n),$$

Where, elements of β are determined by

$$\beta_{2i-1} = -\beta_{2i} = \frac{f_i}{2k}, (i = 1,2,\ldots, n)$$

It is possible to find a positive constant γ such that it satisfies below mentioned condition, if k is chosen to be large enough,

$$\gamma = \frac{1}{2n}(1 - \sum_{i=1}^{2n} |\beta_i|) = \frac{1}{2n}(1 - \sum_{i=1}^{n} \frac{|f_i|}{2k}) \succ 0$$

$\alpha_1 = 0.2728$, $\alpha_2 = 0.0477$, $\alpha_3 = 0.1023$, $\alpha_4 = 0.0477$,
$\alpha_5 = 0.0477$, $\alpha_6 = 0.2108$, $\alpha_7 = 0.22306$, $\alpha_8 = 0.0477$

Set the sampling Interval as 0.0001 second and the initial condition as

$$x(0) = \begin{bmatrix} 0.1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Other parameters of the controller are chosen as
$\varepsilon = -20.0, \quad h = 5.0$

The simulation results with the proposed VS controller are as presented in Fig 1to 7

It is observed that the time response where the continuous state $x(t)$ converges to the origin as indicated in Fig.1. The focused result on [19.5 20] shown in fig.2. indicates that the state can reach the origin with high accuracy although there exists vibration around the origin.

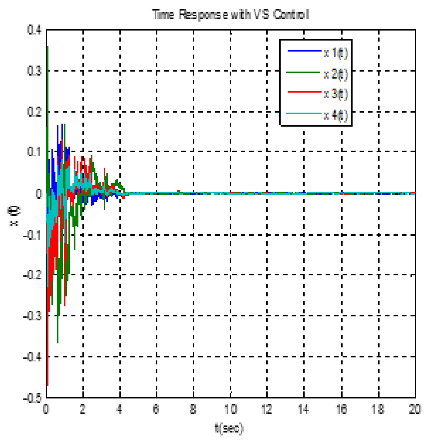

Figure 1- Evolution of continuous state variables $x_1(t), x_2(t), x_3(t) \& x_4(t)$

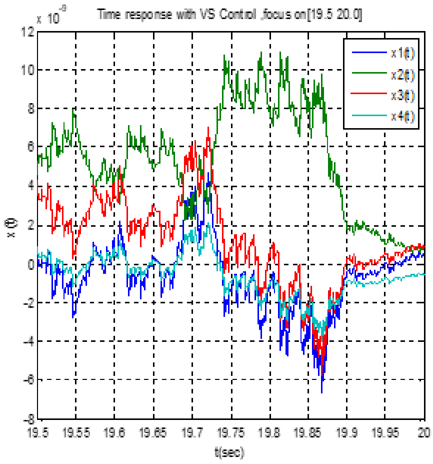

Figure 2- Evolution of continuous state variables $x_1(t), x_2\,(t), x_3\,(t) \& x_{4,}(t)$ focus on [19.5,20]

The change of the Lyapunov function, which keeps decreasing to zero is indicated in the simulation results in Fig 3

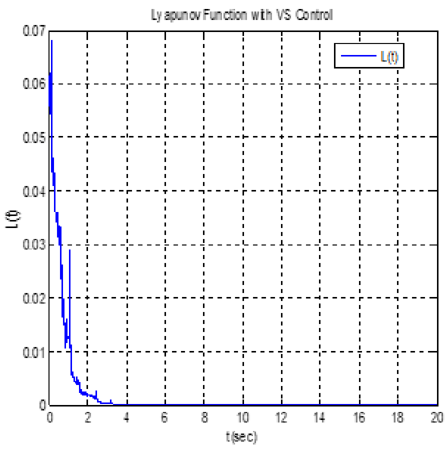

Figure 3: Evolution of Lyapunov Function $L(t)$

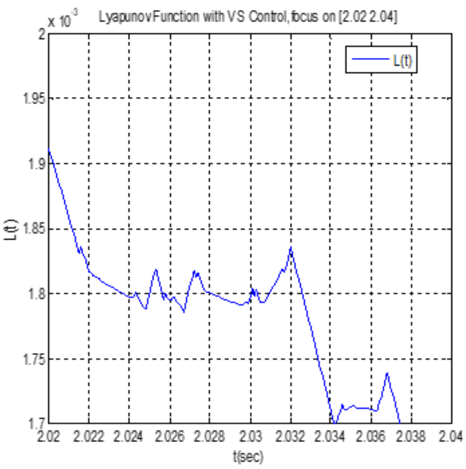

Figure 4 -Evolution of Lyapunov Function $L(t)$

Fig 5, is u(t) the control input, The control input is switched among all the control laws is shown in Fig 6 -u(t) with focus at [0.04,0.06]. Fig 7 shows the discrete sigma function focus on [4, 4.02].

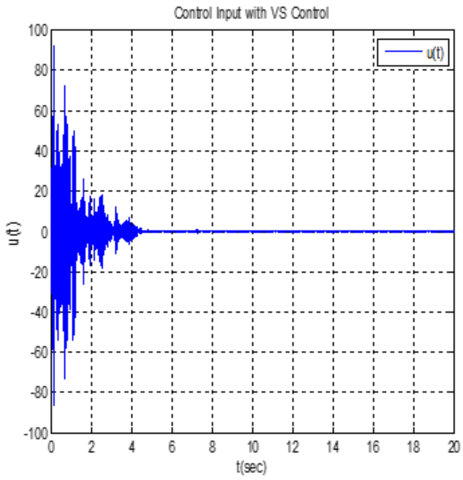

Figure 5: - Evolution of control Input $u(t)$

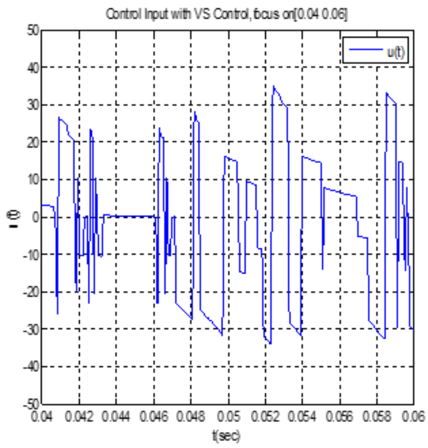

Figure 6: - Evolution of control Input u(t)focus [0.04,0.06]

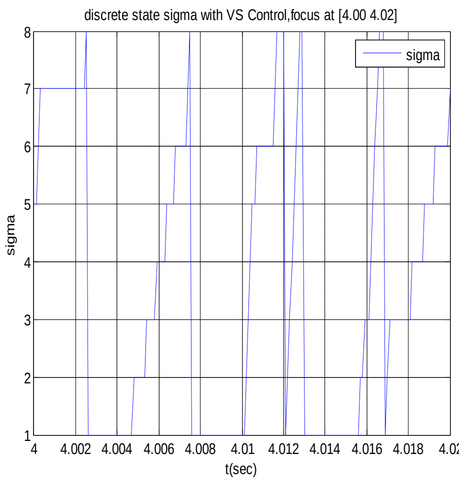

Figure 7: - Evolution of Discrete state variable $\sigma(t)$ focus on [4.0 4.02]

V. CONCLUSION

In this paper, simulation of fourth- order system, variable structure controller using sliding sector and extremum seeking algorithm was designed with some simple control laws. Only one state variable is used at a time by one control law. The simulation results show the efficiency of the Variable Structure Controller and justify that those control laws which ensure the decrease of the Lyapunov function are used for a longer time interval and others are used for a shorter time interval. The resultant VS system is quadratically stable.

Future scope of the method is to check VSC performance on higher order nonlinear and stochastic systems to achieve stable system performance. .

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