Dr Dev Raj Mishra. International Journal of Engineering Research and Applications
www.ijera.com
ISSN: 2248-9622, Vol. 10, Issue 7, (Series-IV) July 2020, pp. 11-15

RESEARCH ARTICLE OPEN ACCESS

A Brief Study on Covariance of Newtonian Mechanics

Dr Dev Raj Mishra
Department of Physics, R.H.Government Post Graduate College,
Kashipur, U.S.Nagar, Uttarakhand -244713 INDIA.

ABSTRACT
The possibility of a covariant formulation of Newton's second law of motion using Energy-Momentum four-vector was explored. This covariant formulation explains the relationships between the electromagnetic potentials and the electromagnetic fields. It successfully explains the observation of pseudo-forces in nature for example the observed up-thrust in a falling frame of reference or a falling lift and the observed centrifugal force in circular motion.
Keywords– Covariance,Electromagnetic Field Tensor,Four- vectors, Minkowski Space time, Relativity

---

Date of Submission: 06-07-2020 Date of Acceptance: 21-07-2020

I. INTRODUCTION

The invariance under Lorentz transformation expresses the proposition that the laws of physics are same for the observers in different frame of reference in relative motion to each other [1]. The laws of Physics should be expressible in what is known as a covariant form if it has to be independent of the choice of the frame of reference of the observer.Principle of Covariance requires the formulation of physical laws using only those variables the measurement of which the observer in different frames of reference can unambiguously correlate [2]. These are the quantities which have spatial as well as temporal components such as four-co-ordinates,four-vectorsand tensors [3].

The non-covariant form of Newton's law of Gravitation led to the formulation of Einstein's Field Equations in General theory of Relativity. These Field Equations use Einstein tensor, a derived form of Ricci's Tensor,is a tensor and therefore confirms to the laws of covariance. The laws of Electromagnetism were also brought to the format prescribed by the requirement of Covariance.Maxwell Equations are expressed in compact covariant forms using electromagnetic field tensor $F^{\mu\nu}$ and current density four-vector j^{μ}.

The other laws of physics of pre-relativistic eraneed to be modified so as to confirm to the principle of covariance. Newton's second law of Motion is one of the suitable candidates for this modification. As we know Newton's Second law of motion can be stated as "The rate of change of momentum is proportional to the applied force and takes place in its direction". It has two quantities – the applied force and the momentum. The applied force is also the gradient of potential energy. Therefore we have two quantities – the momentum and the energy, which are also part of a four-vector called four-momentum or Energy-Momentum four-vector. Therefore the covariant formulation of the second law, as we shall discuss below, can be accomplished using Energy-Momentum four-vector

II. RESULTS AND DISCUSSIONS

Let an event $\boldsymbol{E}$ of force $\mathbf{F} = F_x\hat{\mathbf{i}} + F_y\hat{\mathbf{j}} + F_z\hat{\mathbf{k}}$where F_x, F_y, F_z are components of the force, being applied to a particle P with spatial co-ordinates (x, y, z) occursat time, say t. Therefore the four co-ordinates of event $\boldsymbol{E}$ aret, x, y and z, denoted in terms of four-coordinate as $x^{\mu} = (x^0, x^1, x^2, x^3)$choosing system of units such that the velocity of light, c=1.

The separation of two events in Minkowski space – time using Einstein summation convention, is given by

$$ds = g_{\mu\nu}\,dx^{\mu}dx^{\nu} \qquad (1)$$

The tensor$g_{\mu\nu} = g^{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$ is called metric of the space-time frame. Alsodx^{μ}denote the separation of two events along four-coordinate x^{μ}.

Newton's Second law of motion, as stated above, tells us that rate of change of momentum$\mathbf{p}$is proportional to the applied force $\mathbf{F}$and takes place in its direction. Mathematically we can write it as

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} \qquad (2)$$

Here$\mathbf{p} = p_x\hat{\mathbf{i}} + p_y\hat{\mathbf{j}} + p_z\hat{\mathbf{k}}$is the momentum and p_x, p_y and $p_z$are its spatial components.As we know, the units of force are so defined to make the constant of proportionality in above equationas unity. Then x-component of the force is given by

$$F_x = \frac{\partial p_x}{\partial t} \quad (3)$$

Of course this force can be a fieldand therefore possessing a unique value at each point.

Further, we know that force is a gradient of the potential energy V of the particle. Mathematically

$$\mathbf{F} = -\nabla V \quad (4)$$

where$\nabla = \frac{\partial}{\partial x}\hat{\mathbf{i}} + \frac{\partial}{\partial y}\hat{\mathbf{j}} + \frac{\partial}{\partial z}\hat{\mathbf{k}}$ is the gradient operator.

The x-component of the force from Eq. (4)is given by

$$F_x = -\frac{\partial V}{\partial x} \quad (5)$$

Comparing the two equations, we get

$$-\frac{\partial V}{\partial x} = \frac{\partial p_x}{\partial t} \quad (6)$$

This result, however is not covariant in the sense $\mathbf{p}$ is a three vector. The corresponding four - vector is$\vec{p} = (E,\ p_x, p_y, p_z)$, E being the total energy of the particle. The total energy E is the sum of potential and kinetic energies and also kinetic energy is not a function of co-ordinates. Therefore we can safely replace V by Ein above equation

$$-\frac{\partial E}{\partial x} = \frac{\partial p_x}{\partial t} \quad (7)$$

Replacing coordinates $x = (t, x, y, z)$ by$x^\mu = (x^0, x^1, x^2, x^3)$ and $\vec{p} = (E,\ p_x, p_y, p_z)$by $p^\mu = (p^0, p^1, p^2, p^3)$, we can write above equation as

$$-\frac{\partial p^0}{\partial x^1} = \frac{\partial p^1}{\partial x^0} \quad (8)$$

Further using notation∂_μ for$\frac{\partial}{\partial x^\mu}$, Eq. (8) becomes

$$\partial_\mu p^\nu + \partial_\nu p^\mu = 0 \quad (9)$$

for$\mu = 0$ and $\nu = 1$.

Lowering indices of four-momentum p^μ using metric tensor$g^{\mu\nu}$, Eq. (9) can be written as

$$\partial_\mu (g^{\nu\alpha} p_\alpha) + \partial_\nu (g^{\mu\beta} p_\beta) = 0 \quad (10)$$

But all off-diagonal elements of the metric $g^{\mu\nu}$are zero; therefore$g^{\nu\alpha} p_\alpha = g^{\nu\nu} p_\nu$and $g^{\mu\beta} p_\beta = g^{\mu\mu} p_\mu$.we get fromEq. (10)

$$g^{\nu\nu} \partial_\mu p_\nu + g^{\mu\mu} \partial_\nu p_\mu = 0 \quad (11)$$

for$\mu = 0$ and $\nu = 1$.

But $g_{\mu\nu} = g^{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$, therefore $g^{\mu\mu} = 1$ *and* $g^{\nu\nu} = -1$for$\mu = 0$ and $\nu = 1$. We get from Eq. (11)

$$\partial_\mu p_\nu - \partial_\nu p_\mu = 0 \quad (12)$$

for$\mu = 0$ and $\nu = 1$.

We see that left hand side of above equationis $\mu\nu$ component of an anti-symmetric tensor $\partial \Lambda$ pfor$\mu = 0$ and $\nu = 1$. Let us generalize this result assuming it true for allμ and ν,we can write the covariant form substitute for Newton's Second Law of Motion as

$$\boldsymbol{\partial}\, \boldsymbol{\Lambda}\, \mathbf{p} = \mathbf{0} \quad (13)$$

The tensor $\boldsymbol{\partial}\,\boldsymbol{\Lambda}\,\mathbf{p}$ has six components given by$(\boldsymbol{\partial}\,\boldsymbol{\Lambda}\,\mathbf{p})_{\boldsymbol{\mu\nu}} = \partial_\mu p_\nu - \partial_\nu p_\mu$for $\mu,\ \nu =\ 0,1,2,3$.

Therefore assigning different values to$\boldsymbol{\mu}$ and $\boldsymbol{\nu}$and substituting $p_0 = g_{00} p^0 = g_{00} E = E$ and$p_1 = g_{11} p^1 = g_{11} p_x = -p_x$ and so on for other four-momentum components, we obtain the following six relations between four components of Energy-Momentum four-vector.

$$-\frac{\partial E}{\partial x} = \frac{\partial p_x}{\partial t} \quad \text{(i)}$$

$$-\frac{\partial E}{\partial y} = \frac{\partial p_y}{\partial t} \quad \text{(ii)}$$

$$-\frac{\partial E}{\partial z} = \frac{\partial p_z}{\partial t} \quad \text{(iii)}$$

$$\frac{\partial p_y}{\partial x} = \frac{\partial p_x}{\partial y} \quad \text{(iv)}$$

$$\frac{\partial p_z}{\partial x} = \frac{\partial p_x}{\partial z} \quad \text{(v)}$$

$$\frac{\partial p_z}{\partial y} = \frac{\partial p_y}{\partial z} \quad \text{(vi)}$$

(14)

The first three relations are between spatial variation of energy and the temporal rate of change of three-momentum. These relations prove the fact that whenever there is a spatial variation in Energy; a force appears in the direction of the decrease in energy. This force we usually call as pseudo- force. The examples of these forces are upward force in a falling lift or frame of reference, centrifugal force on a particle doing circular motion etc. The last three relations just correlate the spatial variations of the components of the three- momentum.

Following are the illustrative examples of these results.

III. ILLUSTRATIVE EXAMPLES

The illustrative examples chosen here have kinetic energy variations which are the prominent energy variations in these cases but tensor Eq. (13) is equally and actually valid for the total energy variations only.

3.1. Relation between Electromagnetic potentials

Let an electron having charge –e is placed in an electromagnetic field. The electric field $\mathbf{E} = E_x\hat{\mathbf{i}} + E_y\hat{\mathbf{j}} + E_z\hat{\mathbf{k}}$ and magnetic field $\mathbf{B} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}$are expressed in terms of the electric potential φ and the magnetic potential $\mathbf{A}$by the following equation

$$\mathbf{E} = -\nabla\varphi - \frac{\partial \mathbf{A}}{\partial t} \text{ and } \mathbf{B} = \nabla \times \mathbf{A} \quad (15)$$

Further Energy E and Momentum $\mathbf{p}$ of the electron in presence of electromagnetic fields modifies to

$$E \to E - e\varphi, \qquad \mathbf{p} \to \mathbf{p} - e\mathbf{A} \tag{16}$$

We have from Eq. (14(i))

$$-\frac{\partial E}{\partial x} = \frac{\partial p_x}{\partial t}$$

Let us assume the variation in Energy is only due to the electromagnetic fields, then, from Eq. (16) in x - direction

$$-\frac{\partial(-e\varphi)}{\partial x} = \frac{\partial(p_x - eA_x)}{\partial t}$$

Rearranging terms, we get

$$\frac{1}{e}\frac{\partial p_x}{\partial t} = \frac{\partial \varphi}{\partial x} + \frac{\partial A_x}{\partial t} \tag{17}$$

But from Eq.(15), x-component of electric field E_x is given by

$$E_x = -\frac{\partial \varphi}{\partial x} - \frac{\partial A_x}{\partial t} \tag{18}$$

Combining Eqs.(17) and (18), we can write

$$\frac{1}{e}\frac{\partial p_x}{\partial t} = \frac{\partial \varphi}{\partial x} + \frac{\partial A_x}{\partial t} = -E_x$$

Therefore we conclude that the Electric field in x-direction is given by

$$E_x = -\frac{1}{e}\frac{\partial p_x}{\partial t}$$

It is therefore, the rate of change of momentum classically speaking the force experienced by a unit positive charge. Note here the charge is –e. This is also the known definition of E_x.

Reordering above equation

$$-E_x = \frac{1}{e}\frac{\partial p_x}{\partial t} = \frac{\partial \varphi}{\partial x} + \frac{\partial A_x}{\partial t} \tag{19}$$

Writing in terms of the four-potential $A^\mu = (A^0, A^1, A^2, A^3) = (\varphi, A_x, A_y, A_z)$ and the four co-ordinates, we get

$$-E_x = \frac{i}{e}\frac{\partial p_x}{\partial t} = \frac{\partial A^0}{\partial x^1} + \frac{\partial A^1}{\partial x^0}$$

Further using notation ∂_μ for $\frac{\partial}{\partial x^\mu}$ on right hand side, we get

$$-E_x = \frac{i}{e}\frac{\partial p_x}{\partial t} = \partial_1 A^0 + \partial_0 A^1 \tag{20}$$

But $\quad \partial_1 A^0 + \partial_0 A^1 = \partial_1(g^{0\alpha}A_\alpha) + \partial_0(g^{1\alpha}A_\alpha)$

All off-diagonal elements of the metric $g^{\mu\nu}$ are zero, therefore $g^{0\alpha}A_\alpha = g^{00}A_0$ and $g^{1\alpha}A_\alpha = g^{11}A_1$. We get from the above equation

$$\partial_1 A^0 + \partial_0 A^1 = \partial_1(g^{00}A_0) + \partial_0(g^{11}A_1)$$

But as we know $g^{00} = 1$ and $g^{11} = -1$, we get

$$\partial_1 A^0 + \partial_0 A^1 = \partial_1 A_0 - \partial_0 A_1 = F_{10}$$

and also $\qquad F_{10} = g_{11}g_{00}F^{10} = -F^{10} \qquad (21)$

Here F^{10} is the x-t component of the Electromagnetic Field Tensor $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$

Therefore from Eq.(20), we get

$$-E_x = \frac{i}{e}\frac{\partial p_x}{\partial t} = -F^{10} \tag{22}$$

Similarly it can be shown that $F^{20} = E_y$ and $F^{30} = E_z$ using equations (14)(ii) and (14)(iii) respectively.

Therefore the first three equationsof the formulationexpressed by Eq.(14) successfully correlate the force experience by a charged particle with the corresponding electric field components of the electromagnetic field tensor $F^{\mu\nu}$. Therefore if the formulation is complete, the last three equations should correlate the motion of the particle with the magnetic field components of the tensor.Let us consider Eq. (14)(iv)

$$\frac{\partial p_y}{\partial x} = \frac{\partial p_x}{\partial y}$$

Howeverthe components of the momentum modifyin presence of electromagnetic potentials in accordance with theEq. (16). Therefore we get

$$\frac{\partial(p_y - eA_y)}{\partial x} = \frac{\partial(p_x - eA_x)}{\partial y}$$

Rearranging terms, we get

$$\frac{\partial p_y}{\partial x} - \frac{\partial p_x}{\partial y} = e\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) \tag{23}$$

According to Eq. (15), magnetic field $\mathbf{B}$ is related to the vector potential $\mathbf{A}$by the equation$\mathbf{B} = \nabla \times \mathbf{A}$, therefore

$$\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} = B_z \text{ i.e. z component of } \mathbf{B} \tag{24}$$

Also the left hand side of Eq.(23)can be written as

$$\frac{\partial p_y}{\partial x} - \frac{\partial p_x}{\partial y} = m\left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right) \tag{25}$$

Here m is the mass of the electron and $\mathbf{v} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} + v_z\hat{\mathbf{k}}$is its velocity.But we know from calculus

$$\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} = (\nabla \times v)_z = 2\omega_z$$

Here ω_z is the z-component of the angular velocity of the particle. Therefore from Eq. (25)

$$\frac{\partial p_y}{\partial x} - \frac{\partial p_x}{\partial y} = 2m\omega_z \tag{26}$$

Substituting results from Eqs.(24)and (26), in Eq. (23), we get

$$2m\omega_z = e\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) = eB_z$$

Dividing throughout by charge e and rearranging, we get

$$B_z = \frac{2m}{e}\,\omega_z = \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \tag{27}$$

As $A^\mu = (A^0, A^1, A^2, A^3) = (\varphi, A_x, A_y, A_z)$ we can write

$$\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} = \frac{\partial A^2}{\partial x^1} - \frac{\partial A^1}{\partial x^2}$$

Further using notation ∂_μ for $\frac{\partial}{\partial x^\mu}$ on right hand side, we get

$$\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} = \partial_1 A^2 - \partial_2 A^1$$

Therefore from Eq. (27), we get

$$B_z = \frac{2m}{e}\,\omega_z = \partial_1 A^2 - \partial_2 A^1 \quad (28)$$

But $\partial_1 A^2 - \partial_2 A^1 = \partial_1(g^{2\alpha}A_\alpha) - \partial_2(g^{1\alpha}A_\alpha)$

But all off-diagonal elements of the metric tensor$g^{\mu\nu}$ are zero, therefore $g^{2\alpha}A_\alpha = g^{22}A_2$and $g^{1\alpha}A_\alpha = g^{11}A_1$. Therefore we have

$$\partial_1 A^2 - \partial_2 A^1 = \partial_1(g^{22}A_2) - \partial_2(g^{11}A_1)$$

But as we know$g^{11} = g^{22} = -1$, we get

$$\partial_1 A^2 - \partial_2 A^1 = \partial_2 A_1 - \partial_1 A_2 = F_{21}$$

Also $F_{21} = g_{22}g_{11}F^{21} = F^{21} = -F^{12} \quad (29)$

Finally,from Eqs.(28) and (29), we get

$$B_z = \frac{2m}{e}\,\omega_z = F^{21} = -F^{12} \quad (30)$$

Therefore B_z is associated with the angular motion of the particle and is (2, 1) component of the Electro-magnetic Field Tensor$F^{\mu\nu}$. Similarly it can be shown that $F^{13} = -F^{31} = B_y = \frac{2m}{e}\,\omega_y$ and $F^{32} = -F^{23} = B_x = \frac{2m}{e}\,\omega_x$ using Eqs.(14(v)) and(14(vi)) respectively.

Let us consider an electron moving with velocity $\boldsymbol{v}$enters a perpendicular magnetic field **B**. It experiences a Lorentz force which provides for the necessary centripetal force. The electron therefore, undergoes a circular motion with an angular velocity ω, given as

$$Bev = \frac{mv^2}{r} \quad \text{or} \quad B = \frac{m}{e}\,\omega \quad (31)$$

Therefore we have arrived at the angular frequency (see Eq. (30)which is half of the expected value in Eq. (31). This requires an appropriate explanation which we shall reserve for our future communication. Our present discussion, however, successfully illustrates the fact that the magnetic field causes rotation and the angular velocity is proportional to the applied magnetic field.

Thus various components forElectromagnetic Field Tensor using Minkowskimetric $g_{\mu\nu} = g^{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$can be arranged in the matrix form as below

$$F^{\mu\nu} = \begin{bmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{bmatrix} \quad (32)$$

This is exactly the matrix predicted from other methods.Therefore we have obtained all the components of Electromagnetic tensor starting from ourbasic covariant Eq. (14).

3.2. Pseudo force in a freely falling frame

Consider an object of mass m in a frame of reference falling vertically downward along z-axis with acceleration $-\mathbf{a_z}$ (negative sign to show that the acceleration is along negative z axis) as shown in Fig.1.

Let the frame falls from an initial height h. The change in energy at a height z is equal to the gain in the kinetic energy$\frac{1}{2}m\dot{z}^2$ of the object. As we know from elementary kinematics

$$0 - \dot{z}^2 = -2a_z(h - z). \quad (33)$$

Therefore theincrease in kinetic energy of the objectis given by $E = \frac{1}{2}m\dot{z}^2 = ma_z(h - z)$.

Therefore

$$\frac{\partial E}{\partial z} = \frac{\partial(ma_z(h - z))}{\partial z} = -ma_z \quad (34)$$

We get from Eq.(14(iii))

$$\frac{\partial E}{\partial z} + \frac{\partial p_z}{\partial t} = 0 \quad (35)$$

Therefore

$$\frac{\partial p_z}{\partial t} = -\frac{\partial E}{\partial z} = ma_z \quad (36)$$

This shows that the object experiences a force ma_z along positive z direction which is the actually observed pseudo force in a frame moving downward

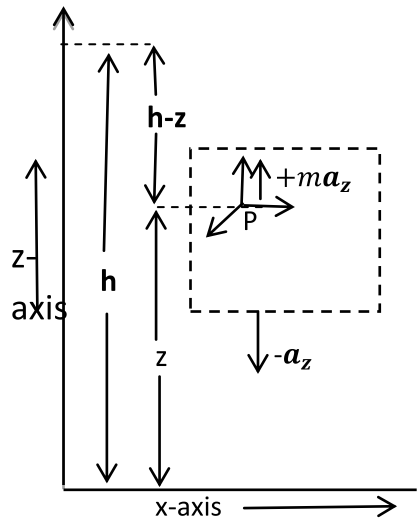

Fig.1. Frame at particle P in a container moving downwards with acceleration $\boldsymbol{a_z}$. The particle experiences a force m$\boldsymbol{a_z}$ upwards.

with accelerationa_z.

3.3. Centrifugal Force

Let a particle of mass m is moving in a circle of radius r with an angular velocityω, as shown in Fig.2. The magnitude of the velocity of the particle at any instant is$v = \omega r$. The kinetic energy, which is the only variable part of the total energy of the particle under non-relativistic conditions, is given by

$$E = \frac{1}{2}mv^2$$

Therefore $$\frac{\partial \mathrm{E}}{\partial \mathrm{r}} = \mathrm{mv}\frac{\partial \mathrm{v}}{\partial \mathrm{r}} \qquad (37)$$

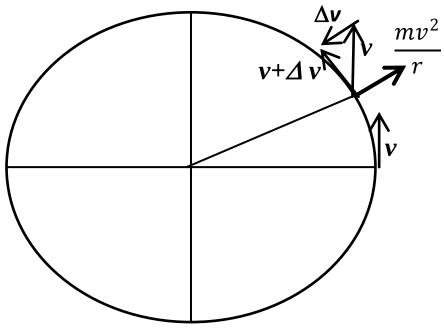

Fig. 2: Particle rotating in circular path with velocity v experiences an outward force of$\frac{mv^2}{r}$.

The change in velocity is $\Delta \mathrm{v} = \omega \Delta \mathrm{r}$ and is along $-\Delta \mathrm{r}$ i.e. points towards the center of the circle. Therefore $\Delta \mathrm{v} = -\omega \Delta \mathrm{r}$. For small variations, this becomes

$$\frac{\partial \mathrm{v}}{\partial \mathrm{r}} = \lim_{\Delta \mathrm{r} \to 0} \frac{\Delta \mathrm{v}}{\Delta \mathrm{r}} = -\omega$$

Therefore from Eq.(37), we have

$$\frac{\partial \mathrm{E}}{\partial \mathrm{r}} = -\mathrm{mv}\omega \qquad (38)$$

Writing again Eq. (14(i))

$$\frac{\partial \mathrm{E}}{\partial \mathrm{x}} + \frac{\partial \mathrm{p_x}}{\partial \mathrm{t}} = 0 \qquad (39)$$

Let the particle is at an infinitely small displacement from the x-axis,we can safely replace x by r, to get

$$\frac{\partial \mathrm{E}}{\partial \mathrm{r}} + \frac{\partial \mathrm{p_r}}{\partial \mathrm{t}} = 0$$

Then from Eq. (38), we get

$$\frac{\partial \mathrm{p_r}}{\partial \mathrm{t}} = -\frac{\partial \mathrm{E}}{\partial \mathrm{r}} = \mathrm{mv}\omega \qquad (40)$$

Therefore

$$\frac{\partial \mathrm{p_r}}{\partial \mathrm{t}} = \frac{\mathrm{mv}^2}{\mathrm{r}} \text{ as } \omega = \frac{\mathrm{v}}{\mathrm{r}} \qquad (41)$$

This is obviously outward force acting along positive r direction that is along$p_r$and is known as centrifugal force.

These were few examples to illustrate the validity of $\boldsymbol{\partial \Lambda p = 0}$ as covariant form substitute for Newton's Second law.

IV. CONCLUSION

Thecovariant form of Newton's Second Law of motionsuccessfully explain the various pseudo-forces observed in nature and also confirmthe known relations between the electromagnetic potentials,fields and the field tensor components.The relationship between spatial temporal variations of Energy-Momentum four-vector components indicate that the variation in one component is related to that in the other component. This relationship can lead to new results in theoretical physics.

ACKNOWLEDGEMENT

Author will like to thank Head, Department of Physics, Kumaun University, Nainital, Uttarakhand, India and Principal, R.H. Govt PG College, Kashipur (U.S. Nagar), India for their generous support in pursuing these studies.

REFERENCES

[1]. A. Einstein A Brief Outline of the Development of the Theory of Relativity *Nature 106, 1921*, 782-784.
[2]. R.Adler,M.Bazin, , &M. Schiffer, *Introduction to General Relativity(Second Edition)* (McGraw-Hill, New York, 1975).
[3]. Bernard F. Schutz*A First Course in General Relativity (Second Edition)* (Cambridge University Press, New York, 2009).
[4]. Weblink: www.cambridge.org/9780521887052

DrDev Raj Mishra Gold Medallist at Graduation, did M.Sc. from M.D. University, Rohtak (Haryana) India. Did his Ph.D. in High Temperature Superconductivity from Cryogenics Group in National Physical Laboratory and University of Delhi, Delhi, India. Joined as Lecturer in the Department of Higher Education, Government of Uttarakhand in Jan. 2000. Presently working as Professor at Department of Physics, R.H. P.G. College, Kashipur, U.S.Nagar, India.