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Analytical and Measuring Method for Calculating the Heat Flux through Inner Walls in a Multi-Apartment Building

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SUMMARY
The article sums up current methods for calculating energy consumption for the purpose of heating residential units in multi-apartment buildings. The article identifies the errors caused by overlooking the phenomenon of heat gains from in-house heat distribution systems and the transfer of energy between adjacent apartments. Nodal analysis, a method used in electrical engineering, is used to determine the consumption of energy for the purpose of heating the apartments. The heat flux exchanged between adjacent apartments with different indoor temperatures was calculated for the studied building. For this purpose, a classic method was used, based on correlations including the design coefficients for heat loss due to heat transfer through inner walls, temperature differences and the duration of the heating season. The results obtained were compared with the results of calculations conducted using an original method based on the actual consumption of energy in the building.

Key words: allocation of heating energy, heat cost allocator, heat transfer through walls, heat gains from pipes

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I. INTRODUCTION

The allocation of heating energy in multi-apartment buildings has been widely discussed in recent years. The debate was sparked by Directive 2012/27/EU of the European Parliament. Allocation of energy is necessary in order to calculate the cost of heating individual apartments. The above-mentioned Directive requires EU Member States to allocate the heating costs in multi-apartment buildings individually, on the basis of registered consumption [1], [2]. The Directive specifies heat meters or heat cost allocators as the devices fit for the purpose of measuring individual consumption of heat. However, a number of publications report significant errors in the allocation of heating costs, especially where heat cost allocators installed on radiators are used [3], [4], [5], [6], [7], [8], [9], [10]. These publications show that the errors are often connected with the quality of inner wall isolation in multi-apartment buildings and the layout of the heating system in the apartments. Both factors depend on the country in which heating energy costs are allocated. Taking into account the diversity in residential developments and the quality of heating systems in different EU Member States, the European Parliament included the following provision in Article 9 of the Directive [1]:"*Where multi-apartment buildings are supplied from district heating or cooling, or where own common heating or cooling systems for such buildings are prevalent, Member States may introduce transparent rules on the allocation of the cost of thermal or hot water consumption in such buildings to ensure transparency and accuracy of accounting for individual consumption. Where appropriate, such rules shall include guidelines on the way to allocate costs for heat and/or hot water that is used as follows:*

a) *hot water for domestic needs,*
b) *heat radiated from the building installation and for the purpose of heating the common areas (where staircases and corridors are equipped with radiators),*
c) *for the purpose of heating apartments."*

The source text of the proposed Directive, prepared by specialists at an initial stage, contained a requirement to take into account heat transfer through inner walls in the calculation and allocation of heating costs. That provision was excluded from the final version of the Directive adopted by the European Parliament because the reliable calculation of such heat transfers to adjacent apartments was problematic.

The above-quoted provision of the Directive gives the EU Member States a mandate to elaborate the applicable local provisions during the implementation of the Directive, especially with regard to the inclusion of heat gains from the building's in-house pipes system in the allocation of heating costs.

II. CURRENT STATE OF THE ART IN HEAT-TRANSFER THROUGH INNER WALLS

Andersson [3] has described the problems connected with the transfer of heat between adjacent apartments and their impact on the allocation of heating costs in multi-apartment buildings. As a solution, Andersson has proposed a method where the heating costs would depend on the indoor temperature set and maintained in the apartment. Linked to this, Siggelsten [4] has proposed a rational method for adjusting heat cost allocation in apartments for the purpose of correcting the errors caused by the transfer of energy through walls. The amount of energy was analytically calculated with the following equation:

$$E_{tot} = Q_{tot}\,\tau(T_{in} - T_e) \qquad (1)$$

where:

E_{tot}	-	total amount of consumed energy [kWh],
Q_{tot}	-	total heat flux compensating the heat losses due to heat transfer and ventilation [W/^{o}C],
τ	-	measurement duration [h],
T_{in}	-	average outdoor temperature [^{o}C],
T_e	-	weighted average temperature outside of the analysed apartment [^{o}C],

and was then compared with the energy consumption measured with heat cost allocators R_u and expressed in kWh. This enabled the calculation of heat transfer through the walls, which in turn allowed for an adjustment of the values read from the heat cost allocators.

Pakanen and Karjalainen [5] used ARMAX modelling for the dynamic calculation of heat transfers between adjacent rooms in a hotel with a variable residential profile. In their review of heat cost allocation methods, Yao, Liu and Lian [6] have pointed to the inability to describe heat transfers through walls to adjacent apartments in a quantitative manner. Likewise, Gafsi and Lefebvre [7] have shown that in specific cases, it is possible that 90% of the heating energy passes through inner walls and is not registered. Ling, Li and Xing [8] have described the impact of the location of an apartment in a building on the costs of heating. In case of heat cost allocators, heat gains from pipes supplying heat to radiators act as an additional source of error. Zoellner and Geisenheimer [9] have quantitatively described the phenomenon of recovering heat from pipes. They proposed that heat cost allocators should be installed on pipes and that heat gains should be subsequently estimated on the basis of their readings. Finally, Michnikowski [10] has described a method based on calculating the average indoor temperature in a multi-apartment building using specially programmed and certified heat cost allocators. By ascertaining the average indoor temperatures during the heating season, it was possible for Michnikowski to calculate the amount of energy necessary for heating the apartments to those temperatures or to adjust the readings of heat cost allocators installed on radiators.

This short review of the relevant available literature shows that there is an urgent need to describe, both qualitatively and quantitatively, the phenomenon of heat transfer through inner walls in order to determine the actual consumption of energy for the purpose of heating individual apartments. The proposed method, apart from precisely describing the phenomenon, should also meet the criterion of easy implementation to the heating cost allocation system in multi-apartment buildings.

This publication aims at describing the phenomenon in a classic manner and proposes a new approach towards important issues connected with the calculation of energy consumption for heating purposes in multi-apartment buildings.

III. OBJECT OF THE ANALYSIS

The object of the analysis was a 5-storey building presented on Fig. 1. The building is divided into two sections. One section features ten apartments, each with an area of 51.7 sq m; the other section consisted of a further 10 apartments, each with an area of 62.9 sq m. Table 1 shows the thermophysical parameters of the building.

Fig. 1 The analysed building

Table 1: Heat-transfer coefficients for walls in the analysed building

wall isolation	outer wall	U=0,270 W/m^{2}K
`	inner wall	U=1,400 W/m^{2}K
	intermediate floor	U=1,030 W/m^{2}K
	basement ceiling	U=0,886 W/m^{2}K
	flat roof	U=0,216 W/m^{2}K
	windows	U=2,100 W/m^{2}K

Fig. 2 shows a floor plan with visible vertical heating risers supplying individual radiators in the apartments with heat. All of the radiators in the apartments have heat cost allocators installed on them, with the exception of the hallways between the staircase entry doors and apartment doors. The heating substation in the basement contains a control and measurement system that enables the calculation of both the heat supplied to the building for central heating purposes and production of domestic hot water.

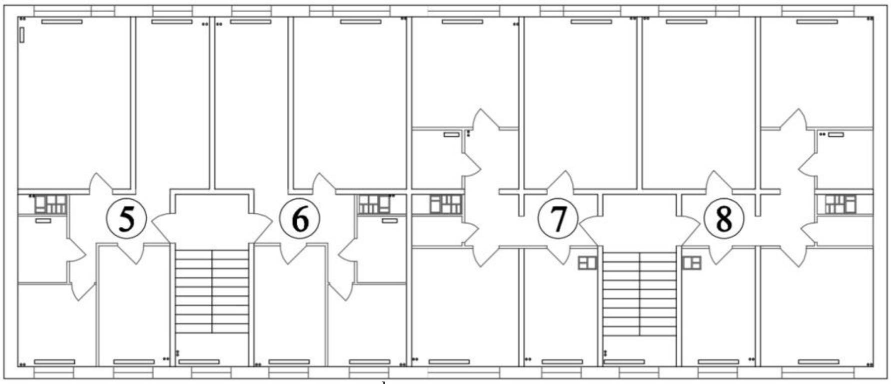

Fig. 2 Floor plan of 2nd storey with apartment numbering

The mathematical analogy between a heat-flow network and an electrical power grid enables using similar methods for solving such systems of equations [11], [12]. In this analysis, heat-flow equations are solved by way of nodal analysis. A heat-flow modelling network for the whole building includes branches connected into nodes. The flow of heat through the branches of the network is caused by differences in temperature which are attributed to various network nodes. The building's node model with heat conductivity symbols is presented in Fig. 3.

Nodes 1-20 represent the apartments in the building, nodes 21-25 represent the corridors on different floors, node 26 represents the basement and node 0 the outdoor air. The conductivity between node 21, representing the corridor, and node 26, the basement, was not included in the analysis, as the temperature difference between these areas was too small and the impact of heat losses on the global heat balance of the building too insignificant.

The analysis was conducted over a period of 12 months, from 1 October 2014 to 30 September 2015, in which the heating season lasted 223 days and the average outdoor temperature that season equalled 4,8^{o}C.

IV. IDENTIFICATION OF HEAT FLUX BETWEEN ADJACENT APARTMENTS

4.1. Nodal analysis

The building's node (circumferential) model presented in Fig. 3 is linear. The model was solved using nodal analysis known from electrical engineering [13]. The following was attributed to nodes *i*, *j* = 1,2,…,26:

- θ_i - temperature increases in comparison with a reference temperature, which can be either the design temperature for a given climate zone or the average outdoor temperature in the heating season [K],
- H_{ij} - own heat conductivities ($i = j$) and mutual conductivity between the nodes ($i \neq j$) [W/K].
- ϕ_i - source heat loads in nodes [W].

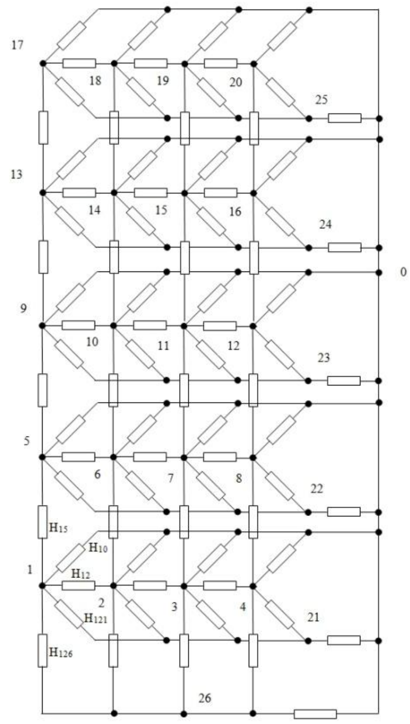

Fig. 3 Nodal model of the building

The following vectors were created for the purpose of calculating the heat flux between adjacent apartments:

$\boldsymbol{\theta} = [\theta_i]_{26x1}$ - temperature increases vector [K],
$\boldsymbol{H} = [H_{ij}]_{26x26}$ - heat conductivities matrix [W/K]
$\boldsymbol{\phi} = [\phi_i]_{26x1}$ - source heat loads in nodes vector [W].

The matrix equation of the circumference from Fig. 3 is as follows:

$$\boldsymbol{\phi} = \boldsymbol{\theta} \text{ x } \boldsymbol{H} \tag{2}$$

which can be written as:

$$\begin{bmatrix} \phi_1 \\ \vdots \\ \phi_{26} \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \vdots \\ \theta_{26} \end{bmatrix} \text{x} \begin{bmatrix} H_{11} & -H_{12} & \dots & -H_{126} \\ -H_{21} & H_{22} & \dots & -H_{226} \\ \vdots & \vdots & \ddots & \vdots \\ -H_{261} & -H_{262} & \dots & H_{2626} \end{bmatrix} \tag{3}$$

where:

- $H_{11} = H_{10} + H_{12} + H_{15} + H_{121} + H_{126}$ - node 1 own conductivity [W/K],
- $H_{22} = H_{20} + H_{21} + H_{23} + H_{26} + H_{221} + H_{226}$ - node 2 own conductivity [W/K],
- $H_{2626} = H_{260} + H_{261} + H_{262} + H_{263} + H_{264}$ - node 26 own conductivity [W/K],
- $H_{10} = H_{tr\,10} + H_{ve10}$ - total conductivity: transfer and ventilation between the node and outdoor space [W/K],
- H_{12} or H_{21} - mutual conductivity of nodes 1 and 2 [W/K].

4.2. Design heat loads

In the case of own conductivities of individual nodes to outdoor air, losses due to ventilation should be added to the losses due to heat transfer, which can be written down as: $H_{i0} = H_{tri\,0} + H_{vei\,0}$. The result of the equation (3) is a column matrix $\boldsymbol{\phi} = [\phi_i]_{26x1}$ of the heat loads resulting from a difference in temperatures in apartments from 1 to 20, corridors from 21 to 25 and basement 26, with the design temperature in node 0. For example, design temperature increases in individual nodes in comparison with the design temperature for a given climate zone in node 0, e.g. -18^{o}C, will be substituted in the column matrix $\boldsymbol{\theta} = [\theta_i]_{26x1}$ In nodes 1 to 20, representing respective apartments, the design temperature equalled 20^{o}C[14]. In nodes 21 to 25, representing the corridors, the design temperature was 8^{o}C and in node 26, i.e. the basement, also 8^{o}C. It resulted in the following increases: $\theta_1, \dots, \theta_{20}$= 38^{o}C,

$\theta_{21},\dots,\theta_{26}$= 26°C. The result obtained through multiplication (3) is a column matrix $\boldsymbol{\phi} = [\phi_i]_{26x1}$ of design heat loads, which are presented in Table 2.

In cases where the heat loads need to be calculated only for apartments 1 to 20, the matrix calculus can be simplified to a matrix with 21 rows and 21 columns. Nodes 21-25, representing the corridors, and node 26, representing the basement (as they have the same temperature of 8°C) were included together in node 21. The heat conductivity matrix takes the following form:

$$\begin{bmatrix} H_{11} & -H_{12} & \dots & -H_{121} \\ -H_{21} & H_{22} & \dots & -H_{221} \\ \vdots & \vdots & \ddots & \vdots \\ -H_{211} & -H_{212} & \dots & H_{2121} \end{bmatrix} \quad (4)$$

where column 21 of the matrix, i.e. $-H_{121},\dots,-H_{2021}$, represents the heat conductivities of apartments 1 to 20 in comparison with node 21. Row 21 of the matrix (4), i.e. $-H_{211},\dots,-H_{2120}$ represents the heat conductivity in node 21 in comparison with nodes 1 to 20. In the last row and column of the matrix, a symbol of H_{2121} marks the own heat conductivity of node 21, which is a sum of conductivity to node 0 (heat transfer and ventilation) and to particular nodes from 1 to 20.

Table 2: Analytical data: design heat loads, heating energy, exchange of energy between apartments

Apart ment no.	Area [sq m]	Design heat loads ϕ [W]	Average indoor temperatures t_i [°C],	Actual heat loads ϕ_r [W]	Actual heating energy E_r [kWh]	Energy (without transfers to adjacent units) [kWh]	Heat transfer between units [kWh]
1	2	3	4	5	6	7	8
1	51,7	3739	18,43	1709	9147	9233	-86
2	51,7	3598	18,72	1503	8042	9227	-1185
3	62,9	3801	20,74	2320	12419	11411	1008
4	62,9	4061	19,19	1823	9759	10748	-989
5	51,7	3382	18,60	1258	6733	7642	-908
6	51,7	3232	21,09	1985	10626	8801	1824
7	62,9	3353	20,07	1406	7525	8434	-909
8	62,9	3613	21,49	2169	11611	9903	1707
9	51,7	3382	20,80	2046	10951	8939	2012
10	51,7	3232	19,16	1182	6328	7700	-1372
11	62,9	3239	20,66	1741	9316	8524	791
12	62,9	3613	19,31	1326	7095	8543	-1448
13	51,7	3382	16,55	822	4398	6433	-2035
14	51,7	3232	19,85	1685	9017	8094	922
15	62,9	3353	20,70	1663	8899	8804	96
16	62,9	3613	20,87	1854	9920	9516	404
17	51,7	3984	18,00	1645	8802	8405	398
18	51,7	3760	18,01	1243	6652	8039	-1387
19	62,9	3987	21,09	2141	11457	10523	934
20	62,9	4363	21,30	2186	11702	11527	175

4.3. Energy needed to heat the apartments to actual temperatures E_r

If the temperature increases between the actual temperatures registered during the heating season in individual apartments t_i and the average outdoor temperature during that season, e.g. 4,8°C, are substituted in the vector equation $\boldsymbol{\theta} = [\theta_i]_{21x1}$, the result of the multiplication will be a column matrix of actual heat loads in nodes $\boldsymbol{\phi_r} = \left[\phi_{r,i}\right]_{21x1}$ (Table 2). Heat loads will include the heat losses in nodes resulting from the transfer of heat outdoors and between the nodes, as well as ventilation losses.

The data necessary to create a vector of temperature increases can be obtained from a special heat cost allocator, which has already been described in this journal [10]. As we can see, the maximum relative error in the temperature readings in a room is lower than $\pm$5%. When the room temperature equals 20°C, the indoor temperature fluctuations are lower than $\pm$1°C.

By multiplying the column matrix of average heat loads and the duration of the heating season τ, it is possible to calculate the energy $\boldsymbol{E_r}$necessary to heat individual units to a given indoor temperature according to the following equation:

$$\boldsymbol{E_r} = \boldsymbol{\phi_r} \,\mathrm{x}\, \tau \quad (5)$$

where $\boldsymbol{E_r}$ is a column vector $[E_{r,i}]_{21x1}$. The above values $\boldsymbol{E_r}$ include the amount of heat necessary to compensate heat losses to outdoor air and adjacent apartments with different temperatures, as well as ventilation losses (Table 2).

4.4. Adjustment of energy caused by heat transfers between adjacent apartments

In compliance with the currently applicable Polish law[15], the heating costs for individual apartments should be calculated on the basis of the consumption of heat for heating purposes. Such consumption includes the heat necessary to compensate for the ventilation and heat-transfer losses to outdoor air, as well as heat losses through inner walls into the staircase (corridors) and, depending on the location of the apartment, to the basement. The issues then arises as to how the additional heat transfers between apartments with different indoor temperatures can be eliminated from the procedure of calculating the energy consumption for heating.

A classic solution to this problem is a matrix calculus (3) with a heat conductivity matrix that would comprise only losses to outdoor air (due to heat transfer and ventilation) and to indoor common areas, i.e. the corridors and the basement. Such a matrix takes the following form:

$$\begin{bmatrix} H_{11} & 0 & \cdots & -H_{121} \\ 0 & H_{22} & \cdots & -H_{221} \\ \vdots & \vdots & \ddots & \vdots \\ -H_{211} & -H_{212} & \cdots & H_{2121} \end{bmatrix} \quad (6)$$

In the matrix (6), the mutual conductivities of apartments 1 to 20 equal zero. The result obtained through multiplication (3) is a column matrix $\boldsymbol{\phi_e} = [\phi_{e,i}]_{21x1}$ in which the heat loads in individual apartments, necessary to compensate the ventilation and heat-transfer losses outdoors and into corridors and basements, were calculated. The differences between the registered average indoor temperatures during the heating season t_i and the average outdoor temperature in that period was substituted in the temperature increases matrix $\boldsymbol{\theta} = [\theta_i]_{21x1}$. Having taken into account the duration of the heating season τ and substituted it to the equation (5) we can obtain a column vector of energy. The difference between column matrices (5), that were determined taking into account:

- all of the heat losses in the apartments (also to adjacent apartments),
- lack of losses due to heat transfer to adjacent apartments,

is a column matrix of energy exchanged between the apartments. The results with regard to energy exchanged between the apartments with a specific indoor temperature are presented in Table 2.

The same results as the ones presented above can be obtained directly from a matrix quotient (3), by way of substituting, in the conductivity matrix (4), own conductivities $H_{11}, H_{22}, \ldots, H_{2020}$ with conductivities between nodes. Conductivities to outdoor space should be omitted. The conductivity matrix could also be limited to 20 rows and 20 columns, ignoring node 21, i.e. the corridor and the basement. In the column matrix $\boldsymbol{\theta} = [\theta_i]_{20x1}$, individual rows are filled with increases between the actual average indoor temperatures of the apartments and the average outdoor temperature during the heating season.

Table 2, column 3 shows the design heat loads ϕ for indoor temperatures in the apartments of 20^{o}C, 8^{o}C at the corridors and basement, and an outdoor temperature of -18^{o}C. Column 4 shows the average indoor temperatures registered in the apartments t_i in the season 2014/15. Column 5 shows the actual heat loads ϕ_r for the average indoor temperature in the heating season of 4,8^{o}C and the average indoor temperatures registered in the apartmentst_i. The next column 6 shows the actual energy E_r used for heating the apartments, taking into account that there were 223 heating days in the 2014/15 season. In that column, the energy is the sum of heat used to compensate the heat losses (due to ventilation and heat transfer) to outdoor air as well as losses due to heat transfer to adjacent apartments. Column 7 shows the energy consumption in the case when no transfer to adjacent apartments was observed. The last column shows the energy resulting only from the transfer of heat through the walls to adjacent apartments with different indoor temperatures during 223 heating days. A minus sign (-) denotes gains from the transfer of energy through walls, whereas a plus sign (+) denotes energy losses in the apartment.

V. IMPLEMENTATION OF MEASUREMENT DATA TO ANALYTICAL DATA

The length and diameter of the pipes in the apartments, determined by way of a survey conducted during the installation of heat cost allocators, were used for the calculations below. The following linear conductivities $\dot{\lambda}_p$ were used for pipes of the diameter of ϕ10, ϕ15 and ϕ20, respectively: 0,64, 0,72 and 0,85 W/(mK). The average water temperature in the in-house piping during the heating season was calculated on the basis of an hourly schedule of outdoor temperature in the season 2014/15 and weather regulation characteristics of the heating substation in the building, and equalled t_p = 48,8^{o}C. The heating season lasted 223 days, i.e. 5352 hours. The energy from pipes in individual apartments is specified in Table 3.

Table 3, column 2 shows, again, the average indoor temperatures registered in the 2014/2015 heating season; column 3 shows the standardised readings from heat cost allocators in the apartments; column 4 shows the same readings in corresponding units of consumption; column 5 shows the energy from pipes estimated on the basis of equations (7); column 6 shows the sum of columns 4 and 5; column 7 shows the heat transfers between apartments. A minus sign (-) denotes gains from the transfer of energy through walls, whereas a plus sign (+) denotes energy losses in the apartment. Column 8 shows the energy in an apartment after adjustment. The adjustment consisted of adding together columns 6 and 7, if the latter contained a minus sign (-). If column 7 contains a plus sign (+), the result in column 8 is a difference between the values specified in column 6 and 7. Column 6 was additionally summed up and as a result, a sum of usable energy delivered to the apartments was calculated that equalled 74819 kWh. The total final energy supplied to the building, registered by the heat meter at the entrance to the building, equalled 78310 kWh. This means that the energy supplied to the apartments constituted 95,5% of the energy supplied to the building.

Table 3: Compilation of analytical data and measurement data on the energy delivered for heating purposes to individual apartments.

Apartment no.	Average indoor temperatures t_i [°C],	Heat cost allocator readings in apartments [-]	Readings in energy units [kWh]	Energy from in-house piping system E_p[kWh]	Sum of energy from radiators and pipes E_s [kWh]	Heat transfer between adjacent apartments E_t[kWh]	Energy delivered to an apartment after adjustment E_k [kWh]
1	2	3	4	5	6	7	8
1	18,43	4530	1756	2318	4074	-86	4160
2	18,72	49	19	2289	2308	-1185	3493
3	20,74	8085	3134	2087	5221	1008	4213
4	19,19	5514	2137	2241	4378	-989	5367
5	18,60	1513	586	2071	2657	-908	3565
6	21,09	4944	1916	1847	3763	1824	1939
7	20,07	538	209	1938	2147	-909	3056
8	21,49	13493	5230	1811	7041	1707	5334
9	20,80	4123	1598	1873	3471	2012	1459
10	19,16	692	268	2020	2288	-1372	3660
11	20,66	4310	1671	1885	3556	791	2765
12	19,31	1732	671	2006	2678	-1448	4126
13	16,55	121	47	2260	2307	-2035	4342
14	19,85	4399	1705	1957	3662	922	2740
15	20,70	4250	1647	1882	3529	96	3433
16	20,87	3287	1274	1866	3140	404	2736
17	18,00	3419	1325	850	2176	398	1778
18	18,01	5325	2064	850	2914	-1387	4301
19	21,09	8467	3282	739	4021	934	3087
20	21,30	22593	8757	731	9488	175	9313
Σ		101385	39296	35523	74819	0	74819

One thing in Table 3 requires greater explanation. The indoor temperature registered in apartment no. 13 equalled 16,55°C, while the consumption of heat registered by the heat-cost allocators was low and, when converted to heat units, equalled 47 kWh. The total energy after adjustment equalled 4342 kWh, with 2260 kWh from pipes and 2035 kWh from adjacent units. Such an anomaly, i.e. low indoor temperature and high usable energy collected for heating purposes, could result from the fact that the temperature on thermostatic radiator valves was set to 15-17°C, as well as increased ventilation (gravitational). The amount of energy transferred between the apartments is determined on the basis of theoretical assumptions: the difference in temperatures, insulating power of the walls, and the duration of the heating season, without taking into account the total amount of energy supplied to the apartments. Taking into account the energy gains from internal and external sources, it is hard to establish whether apartment no. 13 could have absorbed that much heat from adjacent units.

VI. CALCULATION OF ENERGY FLUX BETWEEN ADJACENT APARTMENTS TAKING INTO ACCOUNT THE ACTUAL DATA CONSUMPTION OF HEAT

In the previous section, it has been pointed out that the results, obtained using the analytical method, on the exchange of energy between adjacent apartments, are debatable, as they were obtained on the basis of theoretical considerations, completely separate from the actual results of the measurement of heat consumption in the building. Below, an analytical and measuring method is proposed, which enables more realistic results to be obtained. This is closely connected with the actual consumption of heat for the whole building and the manner of using the apartments in terms of ventilation and heat gains.

The proposed method is divided into two stages[19]. **In the first stage**, the energy E_a necessaryto heat apartment a to a given indoor temperature $t_{i,a}$ is calculated where there are heat losses to be compensated that result from ventilation and transfer of heat to outdoor air of the average temperature of t_e during the heating season, as well as heat-transfer losses to common areas, i.e. the corridors and basements of indoor temperature of $t_{e,i}$ and adjacent apartments which have a temperature of t_i. **In the second stage**, the energy $E_a^{'}$ necessary to heat apartment a to a given indoor temperature $t_{i,a}$ is calculated where there are heat losses to be compensated that result from ventilation and transfer of heat to outdoor air, as well as heat-transfer losses to common areas, i.e. the corridors and basements, yet without adjacent apartments. The difference between the two energy values, calculated from the following equation:

$$E_{t,a} = E_a - E_a^{'} \quad (7)$$

enables the calculation the transfer of energy between apartment a and adjacent apartments.

The energy E_a necessary to heat apartment a is a product of multiplying the average heat load ϕ_a of apartment a and the quotient of the sum of the usable energy E of all of the apartments in the building and the sum of average heat loads $\phi = \sum_{a=1}^{l} \phi_a$($l$ – total number of apartments in a building)of all of the apartments in the building calculated from the following equation:

$$E_a = \phi_a \frac{E}{\phi} \quad (8)$$

The average heat load ϕ_a of apartment a is a product of multiplying the conductivity H_a and temperature increase θ_a in apartment a and is calculated as follows:

$$\phi_a = H_a \theta_a \quad (9)$$

The conductivity H_a of apartment a is the sum of the following conductivities: total (ventilation, heat transfer) conductivity$H_{e,a}$ of apartment a to outdoor air, conductivity through heat transfer $H_{i,a}$ to indoor areas (corridors, basements) and the total conductivity through the transfer of heat $H_{i,i,a,s}$ to adjacent apartments, and is calculated as follows:

$$H_a = H_{e,a} + H_{i,a} + \sum_{s=1}^{g} H_{i,i,a,s} \quad (10)$$

where:

- s - index denoting the adjacent apartment,
- g - index denoting the maximum number of adjacent apartments.

The increase in temperatures θ_a is a difference between the average indoor temperature $t_{i,a}$ of apartment a and a weighted average temperature outside of the apartment $t_{e,av,a}$ and is calculated as follows:

$$\theta_a = t_{i,a} - t_{e,av,a} \quad (11)$$

The weighted average temperature outside of the apartment $t_{e,av,a}$ is a quotient of: a sum of the products of multiplying conductivity and the average temperatures outside of apartment a in the numerator, and the total conductivity of the apartment to indoor and outdoor areas in the denominator, calculated as follows:

$$t_{e,av,a} = \frac{t_e H_{e,a} + t_{e,i} H_{i,a} + \sum_{s=1}^{g} t_{i,a,s} H_{i,i,a,s}}{H_a} \quad (12)$$

where:

- $t_{e,i}$ - average temperature of indoor areas (corridors, basement),
- $t_{i,a,s}$ - average temperature of adjacent apartments.

The second stage follows the same model, except for (10) and (12), which take the following form:

$$H_a^{'} = H_{e,a} + H_{i,a} \quad (10a)$$

$$t_{e,av,a}^{'} = \frac{t_e H_{e,a} + t_{e,i} H_{i,a}}{H_a} \quad (12a)$$

Table 4 shows the method of calculating the energy E_a in the case of heat losses to adjacent apartments, and Table 5 shows the method of calculating the energy $E_a^{'}$ in case of heat losses to outdoor and indoor (corridors and basement) areas, without taking into account the adjacent apartments. The data pertaining to the building and the heating season are the same as in section 3 (Table 1). The vertical cross-section of the building with apartment numbers, average indoor temperature for the heating season and the floor plan of adjacent

Table 4: Example of calculating the energy E_a in the case of heat loss to adjacent apartments

No.	$H_{e,a}$ [W/K]	$H_{i,a}$ [W/K]	H_1 [W/K]	H_2 [W/K]	H_3 [W/K]	H_4 [W/K]	H_a [W/K]	$t_{i,a}$ [K]	$t_{e,av,a}$ [K]	θ_a [K]	ϕ_a [W]	E_a [kWh]
1	2	3	4	5	6	7	8	9	10	11	12	13
1	79	61			26	52	218	18,43	10,64	7,79	1697	3777
2	75	62		26	53	52	268	18,72	13,17	5,55	1489	3313
3	79	68		53	26	63	289	20,74	12,74	8,00	2308	5137
4	85	69		26		63	243	19,19	11,74	7,45	1809	4028
5	79	31	52		26	52	240	18,60	13,39	5,21	1251	2785
6	75	32	52	26	53	52	290	21,09	14,26	6,84	1981	4409
7	79	31	63	53	26	63	314	20,07	15,62	4,45	1399	3115
8	85	31	63	26		63	268	21,49	13,43	8,06	2164	4816
9	79	31	52		26	52	240	20,80	12,30	8,50	2042	4545
10	75	32	52	26	53	52	290	19,16	15,11	4,05	1174	2614
11	79	31	63	53	26	63	314	20,66	14,98	5,67	1783	3969
12	85	31	63	26		63	268	19,31	14,40	4,91	1319	2936
13	79	31	52		26	52	240	16,55	13,16	3,39	814	1811
14	75	32	52	26	53	52	290	19,85	14,06	5,79	1679	3737
15	79	31	63	53	26	63	314	20,70	15,43	5,27	1657	3689
16	85	31	63	26		63	268	20,87	13,99	6,88	1847	4112
17	95	31	52		26		204	18,00	9,96	8,03	1639	3648
18	89	32	52	26	53		252	18,01	13,10	4,91	1237	2754
19	96	31	63	53	26		268	21,09	13,11	7,98	2143	4769
20	105	31	63	26			225	21,30	11,62	9,68	2180	4853
Σ											33612	74819

Table 5: An example of calculating the energy $E_a^{'}$ in case of a lack of heat loss to adjacent apartments

No.	$H_{e,a}$ [W/K]	$H_{i,a}$ [W/K]	$H_a^{'}$ [W/K]	$t_{i,a}$ [K]	$t_{e,av,a}^{'}$ [K]	θ_a [K]	ϕ_a [W]	$E_a^{'}$ [kWh]	E_a [kWh]	$E_{t,a}$ [kWh]	E_s [kWh]	E_k [kWh]
1	2	3	4	5	6	7	8	9	10	11	12	13
1	79	61	140	18,43	6,19	12,24	1713	3813	3777	-36	4074	4110
2	75	62	137	18,72	6,25	12,46	1712	3810	3313	-497	2308	2805
3	79	68	147	20,74	6,29	14,45	2119	4716	5137	421	5221	4800
4	85	69	154	19,19	6,23	12,97	1994	4440	4028	-412	4378	4790
5	79	31	110	18,60	5,70	12,90	1422	3165	2785	-379	2657	3036
6	75	32	107	21,09	5,75	15,34	1638	3647	4409	763	3763	3000
7	79	31	109	20,07	5,70	14,37	1570	3494	3115	-380	2147	2527
8	85	31	116	21,49	5,66	15,83	1844	4105	4816	711	7041	6330
9	79	31	110	20,80	5,70	15,10	1664	3704	4545	841	3471	2630
10	75	32	107	19,16	5,75	13,41	1432	3188	2614	-574	2288	2862
11	79	31	109	20,66	5,70	14,96	1634	3638	3969	331	3556	3225
12	85	31	116	19,31	5,66	13,65	1590	3539	2936	-603	2678	3281
13	79	31	110	16,55	5,70	10,85	1196	2662	1811	-850	2307	3157
14	75	32	107	19,85	5,75	14,10	1506	3352	3737	384	3662	3278
15	79	31	109	20,70	5,70	15,00	1639	3648	3689	41	3529	3488
16	85	31	116	20,87	5,66	15,21	1772	3944	4112	168	3140	2972
17	95	31	126	18,00	5,59	12,41	1564	3482	3648	166	2176	2010
18	89	32	121	18,01	5,64	12,37	1496	3329	2754	-576	2914	3490
19	96	31	126	21,09	5,58	15,51	1960	4363	4769	406	4021	3615
20	105	31	136	21,30	5,53	15,77	2148	4781	4853	73	9488	9415
Σ							33612	74819	74819	0	74819	74819

⑰ 18,00°C	⑱ 18,01°C	⑲ 21,09°C	⑳ 21,30°C
⑬ 16,55°C	⑭ 19,85°C	⑮ 20,70°C	⑯ 20,87°C
⑨ 20,80°C	⑩ 19,16°C	⑪ 20,66°C	⑫ 19,31°C
⑤ 18,60°C	⑥ 21,09°C	⑦ 20,07°C	⑧ 20,49°C
① 18,43°C	② 18,72°C	③ 20,74°C	④ 19,19°C

Fig. 4 Floor plan on vertical cross-section of the building

apartments was presented in Fig. 4. Apartments 6, 9, 11, and 14 were marked as adjacent to apartment 10. Table 4, column 2 shows the sum of conductivities through heat transfer and ventilation outside, whereas column 3 shows the conductivities through heat transfer to indoor areas
(corridors, basement); columns from 4 to 7 show conductivities through heat transfer to adjacent apartments; column 8 shows the total conductivity; column 9 the registered indoor temperature of an apartment; 9 weighted average temperature outside of the apartment; column 10 shows the temperature increase between the apartment's indoor temperature and the average temperature outside; column 12 shows the heat load, and column 13 the final amount of energy necessary to compensate all of the heat losses in an apartment, outdoors and indoors, including adjacent apartments.

Table 5, column 4 shows the summary conductivity through heat transfer without taking into account the adjacent units; column 9 shows the amount of energy necessary to compensate all of the losses without heat transfer to adjacent apartments; column 10 shows the amount of energy E_a transcribed from table 4; and column 11 shows the transfer of energy between apartments as calculated from the equation (8). Additionally, column 12 shows the amount of energy read from heat cost allocators and the estimated energy from pipes (E_s from Table 3), whereas column 13 shows the adjusted amounts of energy for each apartment that can be converted to adjusted readings of heat cost allocators by multiplying them by 2,58.
The procedure for calculating the heat flux through inners walls to adjacent apartments for the first time utilized the actual duration of such heat transfer; previously the duration of the heating season was used. This enables the inclusion of factors which were not taken into account in similar analyses:

- thermal inertia of walls,
- amount of internal and external heat gains,
- user behavior.

VII. ANALYSIS OF THE RESULTS

In the research, the amount of energy exchanged between adjacent apartments was calculated using two methods. The first method was based on equations that take into account the coefficients for losses due to heat-transfer through walls separating the apartments, differences in temperatures and the duration of the heating season. The method did not take into account the actual conditions of using the object, weather conditions, as well as indoor and outdoor heat gains. The second method took into account both current operating conditions of the object, as well as the operating conditions characteristic for a given period of time. The data obtained from both methods differ significantly. The amount of energy lost or gained from heat transfers between apartments calculated using the first method were presented in column 8 of Table 2 and column 7 of Table 3. The amount of energy from heat transfers between apartments calculated using the second method were presented in column 11 of Table 5.The amount of energy exchanged between adjacent apartments calculated using the second method, as described in section 6, is almost 2,5 times smaller. Moreover, the data pertaining to individual apartments, especially the relative error caused by the transfer of energy to adjacent apartments, show significant differences.

In order to assess the relative error in the calculation of energy needed to heat an apartment, caused by not taking into account the transfer of heat between apartments, the correct value of such energy has to be known. In the analysis of the error, the sum of readings from heat cost allocators, converted to units of heat, and the estimated amount of energy from pipes after heat-transfer adjustment, was adopted as the correct value.
The error in the amounts calculated on the basis of the first method, caused by not taking into account the heat transfer, equalled +94,07% for apartment 6 and +137,9% for apartment 9, and so it is by this percentage that the energy values obtained by the first method exceed the correct value.In the case of apartment 13, the amount of energy was lower than the correct amount by -46,87%.
In the case of the second method, the correct error values were as follows:

- for apartment 6 +25,43%,
- for apartment 9 +31,98%,
- for apartment 13 -26,92%,

and so, were thus significantly lower than in the case of the first method.

VIII. CONCLUSION

The above article has employed the analogy between the transfer of heat in a network and the flow of electrical power in a linear system.In order to describe the transfer of heat in multi-apartment buildings, a nodal analysis was applied. The apartments were replaced with network nodes connected via branches. The flow of heat through the branches of the network is caused by differences in temperature which are attributed to network nodes.

Design heat loads, which occur in the case of a difference in temperatures between the design indoor temperature of the apartments and the design (lowest) temperature outside, were calculated using matrix equations from the nodal analysis. Similar

calculations were performed for the actual temperatures of the apartments on the basis of appropriate measurements.
The energy for heating the apartments during the heating season was calculated
for two cases:

- with transfer of energy between adjacent units,
- without transfer of energy, taking into account only the losses to outdoor and indoor areas (corridors, basements).

This enabled a quantitative determination of the amount of energy exchanged between adjacent apartments. Relatively high values of such transfers provoked doubts as to the reliability of the results.

A new method was thus proposed for calculating the value of heat transfers between adjacent apartments, correlated with the amount of energy used for heating the whole building. The calculations performed using the new method gave, on average, 2,5 times lower results than when the classic method, based on theoretical description of the phenomenon of an exchange of energy between adjacent apartments, was applied.

[1] European Parliament, Directive 2012/27/EU, 2012.
[2] S. Siggelsten, S. Olander, Individual metering and charging of heat and hot water in Swedish housing cooperatives, Energy Policy 61 (2013) 874-860.
[3] M. A. Anderson, Energy cost allocation in multifamily buildings using comfort devices, ASHRAE Transactions 99(1) (1993) 899–906.
[4] S. Siggelsten, Reallocation of heating costs due to heat transfer between adjacent apartments, Energy and Buildings 75 (2014) 256-263.
[5] J. Pakanen, S. Karjalainen, Estimating static heat flow in buildings for energy allocation systems, Energy and Buildings 38 (2006) 1044–1052.
[6] Y. Yao, S. Liu, Z. Lian, Key technologies on heating/cooling cost allocation in multifamily housing, Energy and Buildings 40 (2008) 689–696.
[7] A. Gafsi, G. Lefebvre, Stolen heating or cooling energy evaluation in collective buildings using model inversion techniques, Energy and Buildings 35 (2003), 293-303.
[8] J. Ling, Q. Li, J. Xing, The influence of apartment location on household space heating consumption in multi-apartment buildings, Energy and Buildings 103 (2015) 185-197.
[9] G. Zoellner, W. D. Geisenheimer, Bedeutung der Rohrwaermeabgabefuer die verbrauchsabhaengigeAbrechung, HLH 6 (2002) (In German).
[10] P. Michnikowski, Allocation of heating costs with consideration to energy transfer from adjacent apartments, Energy and Buildings 139 (2017) 224-231.
[11] P. Narowski, Modelowanie cieplne budynków uogólnioną metodą potencjałów węzłowych, CIEPŁOWNICTWO OGRZEWNICTWO WENTYLACJA, 9(2015), 345-354 (In Polish).
[12] EN ISO 13790:2008, Energy performance of buildings - Calculation of energy use for space heating and cooling.
[13] C. Łucyk, Elektrotechnika podstawowa, Warszawa 2006, str.78-80.
[14] Ordinance of the Minister of Infrastructure of 12 April 2002 on technical conditions for buildings and their locations, Polish Journal of Laws Dz.U. No. 75, item 690, 2002.
[15] Act of 10 April 1997 – Energy Law.
[16] EN834:2012, Heat cost allocators for the determination of the consumption of room heating radiators – Appliances with electrical energy supply.
[17] VDI 2077, Energy consumption accounting for the building services – Method for the consideration of heat loss through Pipes, March 2009, Beuth Verlag Duesseldorf.
[18] HandbuchzurverbrauchsabhaengiegenHeiz- and Warmwasserkostenabrechnung VHKA, BundesamtfuerEnergie Bern 2004.
[19] P. Michnikowski, Calculation of heat transfers through inner walls, patent application no. P.423290.