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The historical aspects of origin and escalation of law of gravitation.

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ABSTRACT

In the elementary and higher textbooks, the law of gravitation is associated with Newton. To remove any doubt, it is believed that it is given in the Principia by Newton. However, it is not true. Newton has neither given definition nor equation of the law of gravitation. In Book III of the Principia Newton has given various individual propositions or theorems. Actually, these propositions do not lead to the law of gravitation as we understand now. Thus, this aspect can be seriously discussed how the law of gravitation is associated with Newton inconsistently? In the critical analysis the various interesting facts are revealed. The various historical aspects are revealed. B E Clotfelter stated Newton never gave law of gravitation (definition and equation, $F = GmM/r^2$) as we memorize it now and according to Rothleintner Newton never mentioned G in his works. The validity of the law is not questioned at all, but its historical origin is discussed. In the Principia Newton number of times wrote $F \alpha 1/r^2$ (or even $F \alpha 1/r$) and $F \alpha m$ but did not write $F \alpha Mm$. Primarily, the law of gravitation is discussed in Book III of the Principia in Propositions I-IX. Newton acknowledged in the Principia that Copernicus, Kepler, Hooke, Wren, Halley, Bullialdus, Borelli, Tycho Brahe etc have contributed to various aspects of the motion of celestial bodies at earlier different times individually. The various aspects of origin of law of gravitation lie in Newton's Principia in form of propositions, theorems etc. S D Poisson, Cornu, Konig and C V Boys (quoted prevalent definition and equation) are other scientists who have contributed to law after Newton's death. Poisson has stated material particles of bodies attract each other as direct 'ratio of masses' and inverse of square of the distances; but this method is not used now. If ratio of m and M is considered then it may be m/M or M/m. In past law of gravitation is physically described by different scientists in different ways, such issues are discussed.

Key words Principia, Gravitation, Newton, Boys, Falling bodies

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I. THE MATHEMATICAL AND GEOMETRICAL ASPECTS OF THE PRINCIPIA.

Sir Isaac Newton's masterpiece **The Mathematical Principles of Natural Philosophy** is the most wonderful scientific endeavor ever written **[1,2],** initiating physics as a subject separating it from natural philosophy. Newton's description has been different from current methodology (stating law, writing mathematical equations and confirming the predictions with experiments). If experimental deviations are found then correction in theory is sought. The theoretical origin of the law of gravitation and its equation is investigated here, in view of the existing contents in literature.

Structure of Principia: Newton has expressed scientific and mathematical contents in three books i.e. Book I, Book II and Book III of the Principia (1686) in form of propositions (a logic or a statement that affirms or denies something and is either true or false), theorems (a proposition deducible from basic postulates), problems, corollaries, lemmas, scholiums etc. In Book I Newton in 14 sections has given 98 propositions, 50 theorems, 48 problems, 27 lemmas; in Book II in 9 sections; 53 propositions, 41 theorems, 9 problems, 7 lemmas and in Book III in 5 units, 42 propositions, 20 theorems, 22 problems, 9 lemmas etc. In addition, in Book I Newton has given 8 definitions and 3 laws (a generalization that describes recurring facts or events in nature) or axioms (logic or a proposition that is not susceptible of proof or disproof; its truth is assumed to be self-evident) of motion. The various propositions, theorems etc. which are supposed to be related with law of gravitation are shown in Section (4.0).

First publication in the Latin: Newton originally wrote of the *Philosophiæ Naturalis Principia Mathematica* in Latin [3] in 1686 and first English translation of the book as *Mathematical*

Principles of Natural Philosophy was done by Andrew Mott in 1729 without any Newtonian hype at that time, as exists now. Andrew Mott has not given his own comments (so reader is free to understand) while translating the manuscript to English from the Latin, so the same may be regarded as original text. If required then original Latin version may be consulted. The various other subsequent translations (some of them may be traditionally commercial, and not fully research oriented) are also useful. The scientific terminology and vocabulary have changed in past over three centuries (practically Newton initiated physics) but essence of the text must remain same and should not be misinterpreted. Newton's text is clear to understand and interpret.

In *the Principia* Newton expressed the phenomena qualitatively, geometrically without mathematical equations, thus qualitatively in many cases. The various laws or axioms, propositions, theorems, perceptions or extrapolations are part and parcel of the science picked up from the Principia but the exact and original text of the Principia is not included in the curriculum or textbooks.

Law of Gravitation: The validity of Newton's law of gravitation is not questioned at all but the developments regarding its historical origin are critically, scientifically and impartially analyzed on the basis of existing facts in the literature. The law of gravitation is not defined in the Principia and no equation is given.

It is agreed by expert scientists on the basis available scientific facts in literature Newton neither stated the law of gravitation nor perceived constant of gravitation [4-8]. This issue is being discussed here impartially and logically.

1.1 The method of ratios used by Newton.

Newton neither defined law of Gravitation as we understand now nor gave any equation for the law involving universal gravitational constant, G. The gravitational constant was not even mentioned by Newton in his works [4-8].

Newton never stated the universal gravitation in the Principia as we memorize it; also, there is no suggestion in the Principia that Newton ever wrote the statement of the law of gravitation in form of an equation, B E Clotfelter [4]. It was stated by Rothleitner [5] that Newton gave law by 'methods of ratios' as laws were not formulated in terms of equations and proportionalities at legend's time:

"In fact, when Newton wrote the law of gravitation, he did not introduce this proportionality factor because at that time, laws were formulated as ratios rather than as equations. Hence, G was of no significance to Newton."

But this statement is not justified on the basis of existing doctrines even quoted in the Principia. If physical quantities are considered in form of ratio, then it means comparison of magnitude of quantities. Basically, in geometry and trigonometry the theorems and propositions are expressed in form of ratios. In the interpretation of diagrams there are projections and extrapolations. In this method all important universal gravitational constant G does not come in picture.

All the phenomena need to be expressed quantitatively, even then 'perception of description of phenomena as ratios' need to be expressed in form of mathematical equations. Otherwise, explanation is qualitative, in Newton's time interpretation was not done in form of equations as those were initial days of physics.

(a) Further ratios may be in terms of forces, masses, radii, quantities of motion etc (it must be clearly specified). If the ratios are taken in terms of masses then it may be m/M or M/m, both have entirely different magnitudes (M is mass of the earth, m is mass of stone). Poisson has used ratio of masses. However, product is commutative i.e. mM=Mm. It is not clear whether Newton used ratio as m/M or M/m, in determination of force of attraction. If force is regarded as proportional to product of masses of bodies (mM) then issue of m/M or M/m does not arise.

(b) Now-a-days the law of gravitation is expressed in proportionality form, and universal constant G is its inseparable part. Practically the definitions of physical laws are precise, exact and realistic; their predictions from mathematical equations are scientifically confirmed experimentally. The value of G which is all important in law of gravitation does not come in picture by this method. This 'method of ratio' based on geometrical interpretation is not used now, as proportionalities and equations are more significant. It means comparison of various physical quantities. When proportionality is removed then constant (in form of G here) comes in picture as in eq. (1).

1.2 The proportionalities existed in Newton's time, contrary to Rothleitner's quote.

Rothleitner [5] mentioned that Newton could not perceive all important factor G as proportionalities were non-existent at that time. However contrary to claim proportionalities existed at that time, it can be illustrated with examples.

In the existing literature there are numerous examples that laws were expressed in forms of proportionality and equality (rather than ratios) before or during Newton's time. Kepler's third law (T^2 α r^3) in 1619 i.e. 67 years before publication of *the Principia*, Boyle's law (P α 1/V) in 1662 were expressed in proportionality forms leading to equations. Newton even expressed Kepler's second law (radii draw areas proportional to times of description) of motion in proportionality form in Proposition XIII, p. 234 Book III of *the Principia.*

Similarly, Newton himself quoted various laws (second law of motion, Newton's law of cooling, speed of sound in media) in *the Principia* in proportionality form which can be expressed in terms of equations to draw quantitative conclusions. In addition, third law of motion is in direct equality form, Action=Reaction. The proportionality can be changed to equality.

Newton stated many laws in proportionality and equality form in *the Principia* itself. But Newton did not express the law of gravitation even in form of proportionality as we do now i.e. eq. (1).

II. INITIAL DEVELOPMENTS FOR GRAVITATION AND BOYS' PREVALENT DEFINITION (1894)

C V Boys[9], Sim´eon Denis Poisson [10] in 1809, Cornu [11] in 1873 and Konig [12] in 1885 has mentioned about the various aspects of law of gravitation in one way or other. K A Tomilin [13] has mentioned that G appeared for first time in publication of by K¨onig and Richarz [12]. Thus, the law of gravitation has been gradually developed and its origin and escalation are different from Newton's basic three laws or axioms of motion. Also, it is different from a postulate. The genuine origin and history of development of such significant law need to be logically studied.

The modern form of law of gravitation and its mathematical equation has been given afterwards by C V Boys [9] in 1894, in a lecture at Royal Society published in prestigious journal *Nature.* *"Every particle the of the universe attracts any other particle towards itself with force which is proportional to product of their masses divided by square of distance between their centers."* The bodies are of distinct masses m and M (may be smaller or heavenly bodies), separated by distance d (between their centers) then force is given by,

$$\text{Force} = G\frac{mass \times mass}{d^2}$$

$$= G\frac{m \times M}{d^2} \qquad (1)$$

C V Boys credited the law of gravitation to Newton but did not quote the source for above definition and equation.

B E Clotfelter [4] has clearly stated that Newton never wrote law of gravitation, and according to Rothleitner [3] and Lally [5] universal constant was insignificant to Newton as he never wrote equation involving G. These are two significant deductions regarding historical understanding of origin and escalation of the law of gravitation. Obviously more noble facts may surface on critical analysis of literature.

If method of ratios is successful as quoted by Rothleitner [3] then there should have been no necessity of eq. (1) or eq. (2). With help of eq. (1), equation for height of satellite above the surface of the earth as given in eq. (8) is determined. Also, the equation for escape velocity, orbital velocity, gravitational potential energy is written with help of eq. (1) and also involve G. These are not determined with help of method of ratios as in the existing literature, as not mentioned in the literature.

III. The previous existing related works and Newton's acknowledgements.

Prior to Newton (1642-1727), the heliocentric model of Nicolaus Copernicus (1473-1543) was supported by Galileo (1564-1642) with help of telescopic observations, thus centuries old geocentric model of Claudius Ptolemy (100-170AD) was practically abandoned. Copernicus [14] stated in 1543 in his book *On the Revolutions of the Heavenly Spheres* that planets revolve around the sun in spherical orbits, then Tycho Brahe and Kepler (1609) predicted the elliptical orbits of planets. Neither Ptolemy nor Copernicus gave any reason for movement of heavenly bodies around the central body. In *the Principia* Newton has also frequently re-quoted existing laws of Kepler (1571-1630), which are based on experimental observations of Tycho Brahe (1546-1610).

Newton's acknowledgements.

It is stated on the basis of scrutiny of literature [6] that "by the late 1670s" the assumption of an "inverse proportion between gravity and the square of distance was rather common and had been advanced by a number of different people for different reasons."

Newton acknowledged contributions of Wren, Hooke (who vigorously disputed priority of the law) and Halley in this connection in the Scholium to Proposition 4, p.66 in Book I of the Principia [1] regarding inverse square law.

Further Newton gave credit of law of gravitation in the Principia to two people [7]. Firstly to Bullialdus who has shown in 1645 but without demonstration, that there was an attractive force from the Sun in the inverse square proportion to the distance of planet [7]. The mean distances of the earth and planets from the sun are measured by Bullialdus have been re-quoted by Newton in Phenomenon I p. 210-11, Book III of the Principia [2]. These distances are comparable to those measured by Kepler and method of the periodic times. So, Newton was aware of work of Bullialdus and quoted the same in the Principia which was done about 41 years before publication of the Principia i.e. in 1645.

Secondly Borelli [6] who illustrated in 1666 that there was a centrifugal tendency in counterbalance with a gravitational attraction towards the Sun so as to make the planets move in orbits. The result of these forces is similar to a stone's orbit when tied on a string. Thus, Borelli also assumed attractive force between planet and sun, which is counter balanced by outward force. Thus, Borelli was aware of attractive gravitational force and centrifugal force and published in 1666 i.e. 20 years before Newton's Principia. D T Whiteside has described the contribution to Newton's thinking that came from Borelli's book, a copy of which was in Newton's library at his death [15]. Further Newton has quoted the data already given by Bullialdus, and acknowledged works of others.

So, there was already considerable scientific basis present at Newton's time, and he re-quoted or complied all while writing *the Principia*. Thus astronomical observations were initiated by Tycho Brahe and others about a century before Newton, and more than three centuries before the eq.(1). The previous works may not be regarded as research contributions of Newton. These are shown in Table I. The scientific developments took place gradually. Even then law of gravitation was not properly described by Newton.

IV. INTRODUCTION OF G IN MATHEMATICAL EQUATIONS.

Universal constant G is inseparably associated with modern form law of gravitation. But neither equation for law of gravitation nor G is mentioned in *the Principia.* Thus, G has empirical nature [8], for first time it was introduced by Boys [9] in 1894 and distinguished from g.

(i) Newton: There are no evidences that ever Newton gave prevalent definition of law of gravitation and its mathematical equation [4]. G was of no significance to Newton [5], which is soul of law of gravitation. Thus, the above critical study hints that Newton has not made significant contribution in law of gravitation, which we read and teach now.

In general Newton did not give any mathematical equation in *the Principia* [4] and explained the phenomena geometrically with propositions, theorems, mathematical projections or extrapolations etc. The origin of eq. (1) is independent of Newton's geometrical methods of ratios of masses. The definition and application of modern form of law of gravitation cannot be perceived without gravitational constant (G). Thus, Newton was ignorant of all important universal constant G, however now all the celestial data is based on G, as evident from eqs. (6-8).

Thus, origin and development of law of gravitation is different from Newton's first, second and third laws; these laws are given once for all times in the Book I, p.19-20 of the Principia [1]. Newton initially stated that speed of sound in media is isothermal in nature but later on it was observed that it is adiabatic in nature. But status of origin and development of the law of gravitation is entirely different.

The scientific breakthroughs were made before Newton, he did so and continued after him. The credit for scientific works available before *the Principia* must be given to preceding scientists and that of Newton's discoveries to Newton. The various other scientists also contributed towards law of gravitation.

Newton's contribution: In the Principia Newton number of times wrote $F \alpha 1/r^2$ (or even $F \alpha 1/r$, Proposition, XI p. 229, Book III of the Principia); in both cases force decreases with distance but magnitudes are different.

B E Clotfelter [4] has stated that in Proposition VII, p. 225-26 of Book III of the Principia Newton has mentioned about the law ($F \alpha m$, at various places $F \alpha 1/r^2$ or $F \alpha m/r^2$). This proportionality does not involve the product of masses of heavenly body and mass of body ($F \alpha Mm$) which is basic tenet in law of gravitation. G was not of any significance to Newton [5]. Thus, Newton's contribution in this regard may be regarded as limited. Had Newton stated eq. (1), and its corresponding definition like that of Boys [9]; then there would have no need for this discussion.

(ii) Sim´eon Denis Poisson:

Definition : Possibly the text of law of gravitation was first stated by the French physicist and mathematician Sim´eon Denis Poisson [10] in "A Treatise on Mechanics" (1809) — at least historians have not found any earlier works. Poisson has stated that

"The material points of all bodies attract each other mutually, in the direct ***ratio of the masses,*** *and in the inverse of the square of the distances."*

This great law of nature, which was discovered by Newton, is a necessary consequence of observation and calculation; according to Poisson [10]. He quoted the statement on behalf of Newton but did not quote the source of first part of statement for clarification. Further ratio of masses m and M can be m/M or M/m, which are entirely different (M is mass of earth, m is mass of stone, say) however product Mm or mM is the same. In those days books were published un-reviewed as manuscript was given by author to publisher. Newton did not give any such definition and equation in *the Principia*.

Equation: If above statement may be considered in equality form (as no direct or inverse 'proportionality' is mentioned in the definition), then f does not come in picture. This deduction implies that value of f must be unity. Poisson in 1809 has given following equation in his book *A Treatise of Gravitation* (at p. 376) as

$$F = f\, mM/r^2 \qquad (2)$$

Poisson has justified that if masses of bodies (m and M) are unity and placed at unit distance apart, then f is equal to attractive force (F =f). In 1809, the unit of force was not identified, as the dyne (in CGS system) was defined in 1873 [4]. Thus, definition is different than well-known and customary definition given by Boys [9] afterwards, the equation precisely follows from Boys definition.

(iii) Cornu and Baille : Cornu [11] in 1875, has given first apparent reference of gravitational constant and given direct value of f in terms of average density of the Earth (D_e) as

$$f = 3g^2/ 4\pi RD_e \qquad (3)$$

Now f has dimensions $M^{-1}L^4T^{-4}$, it is inconsistent. Thus eq.(3) is dimensionally inconsistent, if it is G (G has dimensions $M^{-1} L^3T^{-2}$). The process of units and dimensions was initiated by Fourier [15] in 1822 and Martin [16] and existed in Cornu's time. Thus, direct equation of f in terms of density of Earth as given by eq. (3) is dimensionally inconsistent.

(vi) A. Konig : K A Tomilin [13] has mentioned that Poisson has introduced constant of proportionality in law of gravitation initially; but gravitational constant appeared for first time in publication of by K¨onig [12] in the year 1885 i.e. 200 years after publication of the Principia.

(v) C V Boys: The definition given by Boys [9] and eq.(1) has been quoted in the section(2.0). The universal gravitational constant G for first time was introduced by C V Boys in 1894 i.e. 209 years after publication of *the Principia,* he distinguished between G and g. Further Boys had not given any previous reference or origin of the statement of the law and eq. (1) that why credit has been given to Newton? The eq. (1) and eq. (2) are identical, they differ by symbols f and G only.

From works of predecessors of Newton, Poisson, Cornu, Konig, Boys etc. it follows that law has been refined and developed gradually. It is different from Newton's three laws of motion given in Book I p.19-20 of the Principia [1], which are perfectly given in single shot. It is like lighting one lamp from the other.

4.1 Application of universal gravitational constant, G.

Rothleitner [5] has stated that G was insignificant for Newton as he never knew anything about it. But now all the celestial data is based on G, so it is exceptionally significant constant.

$$W = mg \qquad (4)$$

or $$F = GmM /R^2 = mg \qquad (5)$$

or $$G = gR^2 / M = 3g/4\pi RD_e. \qquad (6)$$

The standard value of gravity was measured $9.8806 m/s^2$ in France in 1888. These equations are equally applicable for all heavenly bodies.

Newton has speculated in *the Principia* that average density of earth is 5-6 times that of water, Proposition p.230-231, Book III of the Principia. Many scientists made attempts to measure density of earth. Cavendish [18], improving the experimental set up devised John Michell (geologist) and measured density of earth very accurately equal to $5.448 gm/cm^3$. Thus, it became possible to measure the value of G, hence mass of the earth.

$$M = gR^2/G \qquad (7)$$

Even altitude of geosynchronous satellite is calculated with help of G.

$$h = (T^2R^2g/4\pi^2)^{1/3} - R \qquad (8)$$

THE FEASIBILITY OF LAW OF GRAVITATION IN BOOK III (THE SYSTEM OF THE WORLD) OF THE PRINCIPIA.

It is believed that Newton had given law of gravitation in the Propositions 1-IX, p.223-230 Book III of the Principia [2]. Also, in Proposition IX, p.229 of Book III of the Principia Newton had written that force decreases with distance (r) which implies $F \alpha 1/r$. The various propositions of the Principia which appear to describe inverse square law and fact that force is proportional to mass of body. But the real issue is that force is proportional to product of various masses i.e. $F \alpha Mm$ is not implied by any of propositions. These issues need to be carefully addressed.

Proposition I Theorem I

That the forces by which the circumjovial planets are continually drawn off from rectilinear motions, and retained in their proper orbits, tend to Jupiter's centre; and are reciprocally as the squares of the distances of the places of those planets from that centre.

(Force α reciprocally as square of distance between planet and Jupiter)

or $F \alpha\ r^{-2}$)

Force decreases with distance.

Proposition II Theorem II

That the forces by which the primary planets are continually drawn off from rectilinear motions, and retained in their proper orbits, tend to the sun; and are reciprocally as the squares of the distances of the places of those planets from the sun's centre.

($F \alpha\ r^{-2}$)

r : distance between centers of planet and sun.

Proposition III Theorem III

That the force by which the moon is retained in its orbit tends to the earth; and is reciprocally as the square of the distance of its place from the earth's centre.

($F\ \alpha\ r^{-2}$)

Proposition IV Theorem IV

That the moon gravitates towards the earth, and by the force of gravity is continually drawn off from a rectilinear motion, and retained in its orbit.

Moon retains orbit by force of gravity.

Proposition V Theorem V

That the circumjovial planets gravitate towards Jupiter; the circumsaturnal towards Saturn; the circumsolar towards the sun; and by the forces of their gravity are drawn off from rectilinear motions, and retained in curvilinear orbits.

Planets retain orbits due to force of gravity.

Proposition VI Theorem VI

That all bodies gravitate towards every planet; and that the weights of bodies towards any the same planet, at equal distances from the centre of the planet, are proportional to the quantities of matter which they severally contain.

(Weights of bodies α quantity of matter which they severally contain

or $F \alpha\ m$)

Proposition VII Theorem VII

The current form of law of gravitation is proportional to product of masses of attracting bodies (mM). B E Clotfelter [4] has quoted that law of gravitation only appears in this proposition. But directly this proportionality (force is proportional to product of masses of bodies) does not follow from any of propositions given by Newton. However, it is inconsistently believed that it (proportionality i.e. $F \alpha$ Mm in definition and mathematical form) follows from Proposition VII Theorem VII in Book III of the Principia. This proposition is simultaneously studied in original Latin as given by Newton and subsequent translations by authors.

Proposition VII Theorem VII in Latin in the Principia (1686)

Gravitatem in corpora universa fieri, eamque proportionalem esse quantitati materiæ in singulis.

Translation of Proposition VII Theorem VII by Andrew Motte in 1729

"That there is a power of gravity tending to all bodies, proportional to the several quantities of matter which they contain."

(Power of gravity or force of attraction α quantities of matter of various bodies

or $F \alpha\ m$)

It implies that force of attraction is proportional to mass of each body; it does not mean force is product of masses. But it does not imply $F \alpha$ mM, as there is conceptual and mathematical difference between $F \alpha\ m$ and $F \alpha$ mM

Translation of Proposition VII Theorem VII by I Bernard Cohen [19] in 1999

"Gravity exists in all bodies universally and is proportional to the quantity of matter in each.

The force of gravity (power of gravity) is proportional to product of masses of attracting and attracted body (mM) i.e.

$$F\ \alpha\ Mm$$

(M, m are masses of bodies).

does not follow from this said proposition. For critical analysis the proposition(s) may be critically translated purposely.

Proposition VIII Theorem VIII

In two spheres mutually gravitating each towards the other, if the matter in places on all sides round about and equi-distant from the centers is similar, the weight of either sphere towards the other will be reciprocally as the square of the distance between their centers.

(Weight of sphere α reciprocally as the square of the distance between their centers

or W α r^{-2})

Proposition IX Theorem IX

This proposition also implies that force of attraction is proportional to r^{-1}, normally force of attraction is regarded as proportional to r^{-2}. Thus, initially proposition IX is stated in Latin as quoted by Newton and its English translations are discussed.

Prop. IX Theor. IX in Latin

"Gravitatem pergendo `a superficiebus Planetarum deorsum decrescere in ratione distantiarum `a centro quam proxim`e."

Translation of Prop. IX Theor. IX by Andrew Motte in 1729

"That the force of gravity, considered downward from the surface of the planets, decreases nearly in the proportion of the distances from their centres."

Force decreases in proportion to distance.

(Force of gravity α 1/ distance of the centers of bodies or F α 1/r)

Translation of Prop. IX Theor. IX by Bernard Cohen in 1999

Prop. IX: *"In going inward from the surfaces of the planets, gravity decreases very nearly in the ratio of the distances from the center."*

Force decreases in ratio of distance.

(Force of gravity α 1/ distance of the centers of bodies or F α 1/r)

Thus mathematical expression for both translation is the same. None of the translation implies inverse square law.

(Force of gravity α 1/ square of distance of the centers of bodies or F α $1/r^2$)

The force decreases nearly inversely proportional to distance between centers of bodies. In other cases, Newton has specifically mentioned that force varies reciprocally as square of distance between bodies. Apparently in this case there is deviation from inverse square law of gravity, in Newton's own propositions. In Newton's Principia there is variation about dependence of force of gravity on distance. Currently ratio is understood between two quantities as A/B for quantities A and B; if Newton's perception is expressed in terms of ratio then then two quantities must be clearly defined.

Comparison of translations of Andrew Motte and I Bernard Cohen.

Andrew Motte was the first scientist who translated the Principia from the Latin and he was well versed with Latin language. Whereas Bernard Cohen's translation may be quotation from other's work, as from the record available in public domain Cohen was not well versed in Latin. So, Cohen has used the previous translations to compile his masterpiece. The translations used by I Bernard Cohen are based on Andrew Motte's translation. Or purposely the propositions can be translated from Latin for understanding.

Proposition XIII Theorem XIII

The planets move in ellipses which have their common focus in the centre of the sun; and by radii drawn to that centre, they describe areas proportional to the times of description.

Thus, Proposition XIII is nothing but other form or duplication of Kepler's first and second laws. Thus, Newton had also complied the main doctrines of existing literature in the Principia. These propositions are simply statements (discrete) expressing motion of bodies.

Thus, Newton neither gave prevalent form of definition of law of gravitation nor equation.

5.1 **Mathematical Equations**

Newton did not give any equation in the Principia, so attempt can be made to formulate equations for above Propositions.

(i) Proposition VII , p. 225 , Book III of the Principia

F α mass of body being attracted or F α m

or $F = k_1 m$ (9)

The various propositions imply

F α $1/r^2$

In case both proportionalities are simultaneously considered

F α m/r^2 or $F = k\, m/r^2$ (10)

where k_1 and k are coefficient or constant of proportionality.

(ii) In Proposition IX . p 229 Book III of the Principia

Newton wrote, Force α 1/r

F α 1/r

Thus combining above proportionalities we get

F = Km/r (11)

Thus various propositions in Book III of the Principia imply different forms of law of gravitation if critically analyzed . None of propositions given by Newton implies that force is proportional to product of masses

F α Mm

But above proportionality is clearly indicated by Boys [9] in definition and equation. Earlier Poisson [10] has implied similar proportionality in eq. (2). Newton's propositions lead to eq. (10) or eq. (11). Newton never gave any equation for law of gravitation. The universal gravitational constant G which is the most significant term in law of gravitation does not appear here. Thus eq. (1) i.e. $F = G\ mM/r^2$ does not follow from Newton's propositions; however, it follows definition given by Boys [9]. Whereas Poisson has written similar equation i.e. eq. (2) but has given different definition (method of ratios).

V. APPARENT INCONSISTENCY OF DEFINITIONS WITH EQUATIONS AT DIFFERENT TIMES.

The initial definition given by Poisson [10] does not seem to be consistent with mathematical eq. (2) quoted by him. The force of attraction is in direct ratio of masses (m/M or M/m). But the eq. (2) does not involve ratio (m/M or M/m); as it involves product of masses mM or Mm only.

In view of eq. (2), the statement should have been that bodies should attract each other directly as product of masses (Mm) of bodies, and inversely proportional to square of distance; as given by Boys [9]. There is consistency between definition and equation given by Boys. It is strange that Boys [9] and poisson [10] have given identical equations ($f \equiv G$) but definitions are different. Newton has mentioned inverse square law number of times but never mentioned $F \alpha Mm$ in *the Principia*.

B E Clotfelter [4] has written that Newton did not write any equation for law of gravitation; Rothleitner [5] has expressed the law by 'method of ratios' (prevalent in geometry and trigonometry) and further stated that Newton has no significance for G. Thus, they restricted credit to Newton.

Prior to Newton many scientists have laid down foundations of motion of heavenly bodies. In 17th century Newton has expressed the law in form of propositions without mathematical equation. In 19th (Poisson), late 19th (Boys), late 20th (Clotfelter and Lally) and 21st (Rothleitner) centuries have enriched law by their contributions. Further in Proposition IX, p.229 Book III of the Principia Newton implied $F \alpha 1/r$, which is different from inverse square law.

Further Newton's propositions do not lead to eq.(1). However, in the Principia Book I, p. 19-20 has given laws of motion precisely for all times. So, these issues can be logically discussed as noble results are expected for understanding of universal law of gravitation. Now it is open question among the scientists whether or not Newton be given complete credit for law of gravitation. The Frenchman S D Poisson in 1809, has given first equation for law of gravitation which is close to modern form of law of gravitation (only difference is of f and G) but it is not consistent with definition. Thus, Newton had privilege to compile contribution of various scientists. Apparently, it is not prudent to give credit to single scientist for law of discovery gravitation which is result of contributions of various scientists.

VI. SCHOLIUM OF COROLLARY VI AT PAGE 31 OF THE PRINCIPIA

"When a body is falling, the uniform force of its gravity acting equally, impresses, in equal particles of time, equal forces upon that body, and therefore generates equal velocities."

The velocity of moving body is considered with respect to reference point A or frame of reference. **Newton implies that in** case of a falling body (w.r.t reference frame) force of gravity generates equal velocity or due to gravity the bodies fall with equal velocity in the interval (equal or uniform or constant). At every point body has velocities but of equal magnitude. Thus, Newton implies that body falls with equal velocities in the interval (with uniform velocity or zero acceleration). It is not consistent as bodies are confirmed to fall with constant acceleration.

7.1 Proposition XLI, General Scholium of the Book III of the Principia.

"*Bodies projected in our air suffer no resistance but from air. Withdraw the air, as done in Mr. Boyle's vacuum, and the resistance ceases; for in this void a bit of fine down and piece of solid gold descend with equal* ***velocity.****"*

Let the body falls from reference point A with respect to which distance and time to be measured as required in equation $S = Vt$ as body descend with equal velocity. But the velocities are always in classical region not relativistic region when measured with respect to reference point A. It is equally true for bodies moving upward and downward.

The velocity of an object is the rate of change of its position with respect to a frame of reference, and is a function of time. The distances of same bodies are different from different points, so reference point is needed for quantitative measurements.

Here Newton discussed both types of bodies projected upwards and descending downwards. In case of falling bodies a light feather of length 1mm or less and 20 kg (say) or more may be dropped in vacuum in chamber. Then results can be noticed. Similarly, the upward motion of bodies can be studied in evacuated chambers. Theoretically distances travelled by bodies under different on various planets can be studied and predictions are shown in Table II and Table III.

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Table I: Developments of Law of Gravitation (definition and equation)

Sr. No	Scientist/Astronomer	Scientific perception	Reason for motion
1	Claudius Ptolemy (100-170 AD)	Geocentric Theory (Earth centered model)	No reason for movement of heavenly bodies
2	Nicolaus Copernicus (1473-1543)	Heliocentric Theory Sun centered model	No reason
3	Galileo Galilee (1564-1642)	Telescopic support to heliocentrism	-----
4	Kepler (1609)	Heliocentric Theory	No reason
5	Bullialdus (1645)	Heliocentric Theory	Force of attraction exists
6	Borelli (1666)	Heliocentric Theory	Three forces act on the planets
7	Various scientists* Till 1670s	Heliocentric Theory	Force α $1/r^2$
8	**Isaac Newton (1686)**	**Heliocentric Theory**	**F α $1/r^2$, F α 1/r, F α m F α Mm (not given) No prevalent definition, No equation.**
9	Poisson (1809)	Heliocentric Theory	$F = f\, mM/r^2$ Definition in form of ratios
10	C V Boys (1894)	Heliocentric Theory	Definition of prevalent law of gravitation and equation ($F= GmM/r^2$)

* Newton himself acknowledged contributions of Wren, Christopher and Hooke regarding inverse square law (Force α $1/r^2$), and other scientists in *the Principia.*

Table II Comparison of distances travelled (S) in time (t) in the vacuum chamber, surface of Earth, Moon and Mars.

Sr No	Time	$S_{earth} = 1/2\, g_{earth}\, t^2$ $=1/2\, (9.8)\, t^2$ (m)	$S_{moon} = 1/2\, g_{moon}\, t^2 =$ ½ $(1.62)\, t^2$ (m)	$S_{mars} = 1/2\, g_{mars}\, t^2 =$ ½ $(3.7)\, t^2$ (m)
1	0	0	0	0
2	1	4.9	0.81	1.85
3	1.5	11.03	1.82	4.16
4	2	19.6	3.24	7.4
5	2.5	30.63	5.06	11.56
6	3	44.1	7.29	16.65
7	3.5	60.03	9.92	22.68
8	10	490	81	185

Table III The comparison of various velocities of bodies on the Earth, Moon and Mars.

Sr. No	Time (s)	$v_{earth} = g_{earth}\, t$ = 9.8t (m/s)	$v_{moon} = g_{moon}\, t$ = 1.62t (m/s)	$v_{mars} = g_{mars}\, t$ = 3.7t (m/s)
1	0	0	0	0
2	1	9.8	1.62	3.7
3	1.5	14.7	2.43	5.55
4	2	19.6	3.24	7.4
5	2.5	24.50	4.05	9.25
6	3	29.4	4.86	11.1
7	3.5	34.3	5.67	12.95
8	4	39.2	6.48	14.8
9	4.5	44.1	7.29	16.65
10	10	98	16.2	37