Analytical Study of Solute Transport in Porous Media with a Periodic Flow

R. R. Yadav, Dilip Kumar Jaiswal and Gulrana
Department of Mathematics and Astronomy, Lucknow University
Lucknow-226007, India

Abstract

Analytical models require significant simplification of the real-world system, especially in the case of solute transport in subsurface groundwater. This article presents an analytical study of one-dimensional non-reactive solute transport in a homogeneous finite porous medium. The governing advection-dispersion equation, which includes retardation factor, is included for solute transport. The solute is initially introduced from periodic point source from right end of the domain i.e., $x = L$. It is assumed that the flow is one-dimensional with periodic velocity nature and pulse type periodic source pollutants are entering in the domain from right end of the boundary. The second boundary condition is of flux type at sub domain $x = L_1$. Transport equation is solved analytically by using Laplace transformation technique. The developed solution should be applicable to a broad variety of solute transport problems, especially those in homogeneous porous media. Alternate as an illustration; solutions for the present problem are illustrated by numerical examples and graphs.

Key words- Advection, Dispersion, Periodic flow, Porous medium, Retardation.

I. INTRODUCTION

Subsurface solute transport is generally described and predicted using solute transport models. Solute transport by groundwater results from complex interactions between physical, chemical, and biological processes occurring in natural aquifers. The movement of water and solute through the unsaturated zone has been of importance in traditional applications of groundwater hydrology, soil physics and engineering. Actually, the groundwater flow in aquifers can be classified into three flow states: steady-state flow, unsteady state flow, and periodic flow, which are induced by steady-state, non-periodic, and periodic forcing, respectively. The contaminants tend to enter the groundwater system and travel seaward in the ambient groundwater flow. They not only pollute the groundwater, but also endanger the environment of coastal beaches. The understanding and modeling of contaminant transport in coastal aquifers are vital to good management of the coastal environment.

A large number of physical and mathematical models have been developed and deployed to study the hydrodynamic processes involved in groundwater and surface water. Advective transport due to a falling or fluctuating water table has received little attention in the literature. When the water table falls, the originally saturated soil becomes unsaturated allowing contaminated soil gas to enter the pores and contact the residual draining water. Contaminants from surface sources enter the groundwater system with rainfall recharge and travel towards water table. The instabilities may generated at the fresh water interface due to the high concentration difference and to the water fluctuation of the contaminants carried out by the fresh water and may lead to the attenuation or enhancement the contaminant spread, depending on physical properties of porous medium. Velocity fluctuations at the scale of pores cause a swarm of solute particles to spread about their mean position. This spreading is described at the Darcy scale through dispersion coefficients. The transport of migrants in the subsurface water proceeds by molecular diffusion, advection and hydrodynamic dispersion. The naturally occurring mixing effect due to river fluctuations is an important process to consider in the assessment of contaminant transport from the confined disposal facility. Jacob [1] was the first to extend periodic solutions from heat flow to groundwater flow for use in quantifying aquifer response to tidal fluctuations. Ferris [2] derived an equation to describe the change of groundwater head in a confined aquifer in response to sinusoidal oscillations in sea level. Carr and Van Der Kamp [3] developed a method to estimate hydraulic conductivity and the storage coefficient separately in aquifers with tidal boundary fluctuations on the basis of amplitude and phase lag. Fang et al. [4] simulate the tidal fluctuation of the groundwater table, numerically, by using a two-dimensional finite element model. Flow was considered in a simplified domain with a vertical beach face. The cause of the spatial variations in flow velocity is normally attributed to spatial variations in hydraulic conductivity. The study of flow against dispersion in non-adsorbing porous media was presented by Marino [5] in which the flows were opposite to dispersion. The flows were assumed to be one-dimensional in a horizontal direction and the average velocities were taken to be constant throughout the flow field. Al-Niami and Rushton [6] studied dispersion in the direction opposite to flow in one-dimensional flow. Latter on Kumar [7] studied similar problem but with an exponential change in concentration at the source of the dispersion and an unsteady one-dimensional flow. Gelhar et al. [8] and Matheron and De'Marsily [9] studied solute transport in stratified aquifer of infinite thickness. They calculated dispersion under the assumption that the permeability of each layer is random and the flow is in a direction parallel to the layers. Molz et al. [10] have suggested that it may be better to try and incorporate the spatial variations in hydraulic conductivity rather than try to represent the mixing with mechanical dispersion. Goode and Konikow [11] demonstrated, however, that spatial variations in hydraulic conductivity are not the only cause of spatial variations in groundwater flow velocity. They concluded that dispersion also could be caused by temporal variations in flow velocity. Koch and Zhang [12] investigated the effects of contaminant density on its movement also in a steady horizontal flow field by performing a series of numerical exercises. Zhang and Neuman [13] derived first-order expressions of macro-dispersion accounting for temporal variability in the velocity fields.

Attinger et al., [14] and Dentz et al., [15-16] studied the temporal behavior of dispersion coefficients in a stochastic modeling framework developed up to the second order in the fluctuations of the random fields, for a chemically and physically heterogeneous medium respectively under steady state flow conditions. Robinson and Gallagher [17] further developed a two-dimensional, field scale, finite element model based on density dependent fluid flow with water table and dynamic tidal boundary conditions. Jiao and Tang [18] derived an analytical solution to study the groundwater head fluctuations in the confined aquifer of a coastal aquifer-aquitard-aquifer system. Vanderbroght et al., [19] evaluated the effect of flow rate and flow regime on solute transport in two soils, sandy loam and loam. Cirpka [20] analyzed the transverse dispersion coefficient considering a spatially uniform flow field of a kinetically sorbing compound under sinusoidal temporal fluctuations. Kumar et al. [21] obtained analytical solutions for one-dimensional advection–diffusion equation with variable coefficients in a longitudinal finite initially solute free domain, for temporally and spatially dependent dispersion problems.

In almost all the studied have done so for by different investigators, they considered seepage velocity steady, un-steady, exponential or sinusoidal and boundary condition non-periodic. Deviating these in the present study, a mathematical model of contaminant transport in a homogeneous porous medium is analyzed analytically. To solve the advection-dispersion equation analytically we used Laplace transformation technique for a time-dependent periodic source of pollutant which entering from right end of the domain. The seepage flow is considered towards left to right direction i.e., $x = 0$ to $x = L$ which is the solute transport phenomena against the flow. Such a situation often occurs in practice when poor quality water is prevented from spreading by a flow of a freshwater. Authors basic attention is to discussed the behavior of concentration between $x = L$ to $x = L_1$. Analytical solutions can provide fast estimations of concentration distributions for solute movements and benchmark tools for evaluating numerical models.

II. MATHEMATICAL FORMULATION AND SOLUTION OF THE PROBLEM

Schematic representation of present problem is shown in Fig. (1). The porous domain is assumed to be horizontal, the seepage flow is along the x coordinate and the length of the porous domain is L. Initially the domain has already some pollution while a pulse type periodic injection of solute mass is entering from a source at right end of the boundary i.e., at position L. The concentration gradient is assumed zero at the sub domain $x = L_1$ of the present problem.

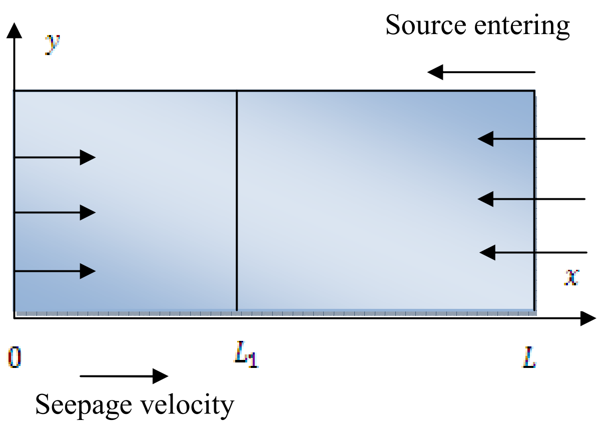

Fig. 1

One-dimensional advection-dispersion equation in homogeneous porous media with retardation factor can be written as

$$R\frac{\partial c}{\partial t} = \frac{\partial}{\partial x}\left(D(x,t)\frac{\partial c}{\partial x} - u(x,t)c\right) \quad (1)$$

This equation generally postulate that solutes move through porous media by advection, mechanical dispersion and molecular diffusion (induced by concentration gradients). In which c is the solute concentration in the liquid phase. The dispersion coefficient, D presumably includes the effects of both molecular diffusion and mixing in the axial direction, however the effect of molecular diffusion is excluded because that the mechanical dispersion mostly dominate the hydrodynamic dispersion process during solute transport. In Eq. (1), D and u may be constant or function of time or space and R is retardation factor accounting for equilibrium linear sorption processes. The retardation factor accounts of transport processes occurring both in liquid and in solid phases unlike contaminant transport. The dimension of D and u are L^2T^{-1} and LT^{-1} respectively. R is the dimension less quantity. The term on the left side of the equal sign indicate the retardation factor and change of concentration in time, the first two terms on the right side describe hydrodynamic dispersion and groundwater velocity. If both the parameters are independent to independent variables x and t, then these are called constant dispersion and uniform flow velocity respectively.

The coefficient of dispersion is considered directly proportional to seepage velocity (Marino [5], Yim and Mohsen [22]), i.e.

$$D(x,t) \propto u(x,t)$$

The dispersion coefficient varies in accordance with the seepage velocity. Let us write $u(x,t) = u_0 sin(mt)$, so that $D(x,t) = D_0 sin(mt)$, where m is a angular frequency whose dimension is inverse to the time variable t. Only positive value of seepage velocity is considered throughout the proposed problem.

Eq. (1) becomes,

$$R\frac{\partial c}{\partial t} = \frac{\partial}{\partial x}\left(D_0 sin(mt)\frac{\partial c}{\partial x} - u_0 sin(mt)\, c\right) \quad (2)$$

where D_0, u_0 are constants along the respective direction.

Let us introduce a new time variable using the following transformation (Crank [23]),

$$T = \int_0^t sin(mt)dt \quad \text{or} \quad mT = 1 - cos(mt) \quad (3)$$

Now differential equation (2) reduces into constant coefficients as

$$R\frac{\partial c}{\partial T} = \frac{\partial}{\partial x}\left(D_0\frac{\partial c}{\partial x} - u_0 c\right) \quad (4)$$

Initially the domain is not solute free. Let us assume it is linear increasing function of space variable. An input concentration of periodic nature is assumed at the $x = L$ of the domain. Under above assumptions, the initial and boundary conditions mathematically can be written as

$$c(x,t) = C_i exp(\alpha x),\ L_1 \le x \le L,\ t = 0 \quad (5)$$

$$c(x,t) = \begin{cases} C_0\{1 + cos(mt)\}, 0 < t \le t_0 \\ 0, t > t_0 \end{cases}, x = L \quad (6a)$$

$$\frac{\partial c}{\partial x} = 0,\ \ x = L_1,\ \ t \ge 0 \quad (6b)$$

where C_i is the resident concentration and α is a constant which is less than one and its dimension is inverse of space variable. The above conditions in terms of new time variable may be written as

$$c(x,T) = C_i exp(\alpha x),\ L_1 \le x \le L,\ T = 0 \quad (7)$$

$$c(x,T) = \begin{cases} C_0(2 - mT), 0 < T \le T_0 \\ 0, T > T_0 \end{cases}, T > 0, \; x = L \tag{8a}$$

$$\frac{\partial c}{\partial x} = 0, \quad x = L_1, \quad T \ge 0 \tag{8b}$$

Now introducing a new dependent variable by following transformation

$$c(x,T) = K(x,T) exp\left[\frac{u_0}{2D_0}x - \frac{u_0^2}{4RD_0}T\right] \tag{9}$$

The set of Eqs. (4), (7) and (8) reduced into

$$R\frac{\partial K}{\partial T} = D_0 \frac{\partial^2 K}{\partial x^2} \tag{10}$$

$$K(x,T) = C_i exp(\alpha x - \beta x), L_1 \le x \le L, T = 0 \tag{11}$$

$$K(x,T) = \begin{cases} C_0(2 - mT)\, exp(-\beta x + \gamma^2 T), 0 < T \le T_0 \\ 0, T > T_0 \end{cases}, x = L \tag{12a}$$

$$\frac{\partial K}{\partial x} = -\frac{u_0}{2D_0}K \quad , \quad x = L_1, \quad T \ge 0 \tag{12b}$$

where $\beta = \frac{u_0}{2D_0}$, $\gamma^2 = \frac{u_0^2}{4RD_0}$.

Applying Laplace transformation on Eqs. (10) – (12), we have

$$Rp\bar{K} - C_i R\, exp(\alpha x - \beta x) = D_0 \frac{d^2 \bar{K}}{dx^2} \tag{13}$$

$$\bar{K}(x,p) = C_0\, exp(-\beta L) \left[\frac{1}{(p-\gamma^2)}\{2 - (2 - mT_0)exp(-(p-\gamma^2)T_0)\} - \frac{m}{(p-\gamma^2)^2}\{1 - exp(-(p-\gamma^2)T_0)\}\right], x = L \tag{14a}$$

$$\frac{d\bar{K}}{dx} = -\frac{u_0}{2D_0}\bar{K} \quad x = L_1 \tag{14b}$$

where $\bar{K}(x,p) = \int_0^\infty K(x,T)\, e^{-pT} dT$ and p is the Laplace transformation parameter.

Thus the general solution of Eq. (13) may be written as

$$\bar{K}(x,p) = C_1 cosh(Mx) + C_2 sinh(Mx) + \frac{C_i\, exp\,(\alpha - \beta)x}{\left\{p - \frac{D_0}{R}(\alpha-\beta)^2\right\}}$$

$$M = \sqrt{\frac{Rp}{D_0}}. \tag{15}$$

Using conditions (14a,b) on the above solution, we get

$$C_1 = C_0\, exp(-\beta L)\left[\frac{1}{(p-\gamma^2)}\{2 - (2 - mT_0)exp(-(p-\gamma^2)T_0)\} - \frac{m}{(p-\gamma^2)^2}\{1 - exp(-(p-\gamma^2)T_0)\}\right] \frac{McoshM(L_1) + \beta sinh\; M(L_1)}{McoshM(L_1-L) + \beta sinh\; M(L_1-L)}$$
$$- \frac{C_i\, exp((\alpha-\beta)L_1)sinh ML}{\left\{p - \frac{D_0}{R}(\alpha-\beta)^2\right\}}\left[\frac{\alpha}{McoshM(L_1-L) + \beta sinh\; M(L_1-L)}\right] + \frac{C_i\, exp((\alpha-\beta)L)}{\left\{p - \frac{D_0}{R}(\alpha-\beta)^2\right\}} \frac{McoshM(L_1) + \beta sinh\; M(L_1)}{McoshM(L_1-L) + \beta sinh\; M(L_1-L)} \tag{16}$$

and $C_2 = -C_0\, exp(-\beta L)\left[\frac{1}{(p-\gamma^2)}\{2 - (2 - mT_0)exp(-(p-\gamma^2)T_0)\} - \frac{m}{(p-\gamma^2)^2}\{1 - exp(-(p-\gamma^2)T_0)\}\right] \frac{MsinhM(L_1) + \beta cosh\; M(L_1)}{McoshM(L_1-L) + \beta sinh\; M(L_1-L)}$

$$+ \frac{C_i\, exp((\alpha-\beta)L_1)cosh ML}{\left\{p - \frac{D_0}{R}(\alpha-\beta)^2\right\}}\left[\frac{\alpha}{McoshM(L_1-L) + \beta sinh\; M(L_1-L)}\right] + \frac{C_i\, exp((\alpha-\beta)L)}{\left\{p - \frac{D_0}{R}(\alpha-\beta)^2\right\}} \frac{MsinhM(L_1) + \beta cosh\; M(L_1)}{McoshM(L_1-L) + \beta sinh\; M(L_1-L)} \tag{17}$$

Thus the solution in the Laplacian domain may be written as

$$\bar{K}(x,p) = C_0\, exp(-\beta L)\left[\frac{1}{(p-\gamma^2)}\{2 - (2 - mT_0)exp(-(p-\gamma^2)T_0)\} - \frac{m}{(p-\gamma^2)^2}\{1 - exp(-(p-\gamma^2)T_0)\}\right] \frac{McoshM(L_1-x) + \beta sinh\; M(L_1-x)}{McoshM(L_1-L) + \beta sinh\; M(L_1-L)}$$
$$- \frac{C_i\, exp((\alpha-\beta)L_1)}{\left\{p - \frac{D_0}{R}(\alpha-\beta)^2\right\}}\left[\frac{\alpha sinh M(x-L)}{McoshM(L_1-L) + \beta sinh\; M(L_1-L)}\right] - \frac{C_i\, exp((\alpha-\beta)L)}{\left\{p - \frac{D_0}{R}(\alpha-\beta)^2\right\}} \frac{McoshM(L_1-x) + \beta sinh\; M(L_1-x)}{McoshM(L_1-L) + \beta sinh\; M(L_1-L)} + \frac{C_i\, exp\,(\alpha-\beta)x}{\left\{p - \frac{D_0}{R}(\alpha-\beta)^2\right\}} \tag{18}$$

Taking Laplace inverse transformation of equation (18) by using complex inversion formulae which is discussed in detail in Appendix, the solution of advection-dispersion solute transport for periodic input condition in terms of $c(x,T)$ as,

$$c(x,T) = C_i F_1(x,T) + 2C_0 F_2(\eta,T) - C_0 F_3(x,T); \quad 0 < T \le T_0 \tag{19a}$$

$$c(x,T) = C_i F_1(x,T) + C_0[2F_2(x,T) - (2 - mT_0)F_2(x, T - T_0)] - C_0 F_3(x,T); T > T_0 \tag{19b}$$

where $F_1(x,T) = exp\{\alpha x - \gamma^2 T + \delta T\} - exp\{\alpha L_1 + (x - L_1)\beta - \gamma^2 T\}\left[exp\{\delta T\}\frac{\alpha sinh\{(x-L)(\alpha-\beta)\}}{H_6}\right.$

$$-2D_0\sum_1^\infty \omega_n^2 exp\left\{-\frac{D_0}{R}\omega_n^2 T\right\}\left\{\frac{\alpha\sin\{(x-L)\omega_n\}}{H_4}\right\}\Big]$$

$$-exp\{\alpha L+(x-L)\beta-\gamma^2 T\}$$

$$\left[exp\{\delta T\}E_3-2D_0\sum_1^\infty \omega_n^2 exp\left\{-\frac{D_0}{R}\omega_n^2 T\right\}E_2\right]$$

$$F_2(x,T)=exp\{(x-L)\beta\}[E_1-2D_0\, exp\{-\gamma^2 T\}$$

$$\sum_1^\infty \omega_n^2 exp\left\{-\frac{D_0}{R}\omega_n^2 T\right\}E_{21}\Big]$$

$$F_3(x,T)=exp\{(x-L)\beta\}$$

$$\Big[(-mT)E_1+\frac{mR}{u_0\beta}\Big\{\frac{cosh(L_1-L)\beta}{H_2}-\frac{cosh(L_1-x)\beta}{H_2}$$

$$+\beta(L_1-L)-\beta(L_1-x)E_1\}\Big]$$

$$+2D_0 exp\{(x-L)\beta-\gamma^2 T\}$$

$$\sum_1^\infty\left[\omega_n^2 exp\left\{-\frac{D_0}{R}\omega_n^2 T\right\}\left\{\frac{mR}{(\omega_n^2 D_0+R\gamma^2)}\right\}E_{21}\right]$$

$$H_1=cosh(L_1-x)\,\beta+sinh(L_1-x)\,\beta,$$

$$H_2=cosh(L_1-L)\,\beta+sinh(L_1-L)\,\beta,\ E_1=\frac{H_1}{H_2},$$

$$H_3=\omega_n\cos(L_1-x)\,\omega_n+\beta\sin(L_1-x)\,\omega_n,$$

$$H_4=(\omega_n^2 D_0+R\delta)\{\beta+(\beta^2+\omega_n^2)(L_1-L)\}$$

$$\sin(L_1-L)\,\omega_n,\ E_2=\frac{H_3}{H_4}$$

$$H_{41}=(\omega_n^2 D_0+R\gamma^2)\{\beta+(\beta^2+\omega_n^2)(L_1-L)\}$$

$$\sin(L_1-L)\,\omega_n,\ E_{21}=\frac{H_3}{H_{41}}$$

$$H_5=(\alpha-\beta)\cosh\{(L_1-x)(\alpha-\beta)\}$$

$$+\beta\sinh\{(L_1-x)(\alpha-\beta)\}$$

$$H_6=(\alpha-\beta)\cosh\{(L_1-L)(\alpha-\beta)\}$$

$$+\beta\sinh\{(L_1-L)(\alpha-\beta)\},\ E_3=\frac{H_5}{H_6}$$

$\delta=\frac{D_0(\alpha-\beta)^2}{R}$, $\beta=\frac{u_0}{2D_0}$, $\gamma^2=\frac{u_0^2}{4RD_0}$, ω_n is the positive root of the $[\omega_n \cot\omega_n(L_1-L)+\beta=0]$, and $T=\frac{1}{m}\{1-\cos(mt)\}$.

III. RESULTS AND DISCUSSION

The parameters governing the solute transport through porous domain vary significantly depending upon the nature of any particular site of the pollutant. Thus, to illustrate the significant factors arising from the use of this formulation, consideration will be given to the hypothetical case of porous domain. The numerical values of majority of the parameters used for model simulations presented here are taken directly from the literature or determined using existing empirical relationships. As an example, assumed the following value of parameters; $D_0=1.2$ (m^2/day), $u_0=0.48$ (m/day), $C_0=1.0$ (kgm^{-3}), $C_i=0.1$ (kgm^{-3}), $m=0.015$ (day^{-1}) and $\alpha=0.02$ (m^{-1}). All figures are drawn in a domain $1.2\le x(\mathrm{m})\le 2.2$ i.e., $L=2.2$ and $L_1=1.2$ where L_1 is a point lies between $0\le L_1\le L$. The roots of ω_n are taken from Carslaw and Jaeger [24], pp. 492. Only six roots are taken into account because other roots have no significant effect on the numerical values of the derived solution. Figs. (2, 3, 4) are drawn when the source pollution being entering in the domain while Figs. (5, 6, 7) shows the concentration profiles when source pollutant not entering in the domain. The time of elimination (pollutant is not entering in the domain) of source pollution is taken t_0 (day) $=5.0$.

Fig. (2) shows dimensionless concentration profiles in the domain $1.2\le x(\mathrm{m})\le 2.2$ for various times t (day) $=2.0$, 3.0 and 4.0 and retardation factor $R=1.25$. The Fig. (2) reveals that as the time increases the concentration values continuously increases. Fig. (3) is drawn for different retardation factors $R=1.25$, 1.45 and 1.65 with fixed time t (day) $=3.0$. It indicate that the effect of retardation factor on concentration profiles. It also indicate that as the retardation increases the concentration distribution in the domain continuously decreases at particular time. Fig. (4) shows the influence of angular frequency of flow on solute concentration which has taken $m=0.015$, 0.0165 and 0.0180 and fixed retardation factor $R=1.25$ at time t (day) $=3.0$. It is observed that the concentration values are higher for higher angular frequency and lower for lower angular frequency.

Figs. (5, 6, 7) are drawn for solution (19b). The trend of Figs. (5, 6, 7) are reverse of Figs. (2, 3, 4). Fig. (5) are drawn for various times t (day) $=7.0$, 8.0 and 9.0 and fixed retardation factor which shows the concentration values are lower for higher time. Fig. (6) shows the concentration profile for different retardation factors $R=1.25$ and 1.65 and fixed time t (day) $=9.0$. It indicates that the concentration values are higher for higher retardation factor in the domain. Fig. (7) is drawn for different angular frequency $m=0.015$, 0.0165 and 0.0180 with fixed retardation factor $R=1.25$ at time t (day) $=9.0$. It is observed that, the concentration value is lower for higher angular frequency.

IV. CONCLUSION

An analytical solution for the transport of non-reactive solute in homogeneous porous media has been developed and some of the main features of the obtained results have been illustrated. The basic assumption in the present study is dispersion coefficient directly proportional to seepage flow (which is of periodic nature). The governing transport equation is solved analytically, implying Laplace transform technique. The effect of periodic source concentration on temporal distribution of solute concentration is illustrated in different graphs. The value of the period of the solution plays an important role in the discussion of periodic solutions.

ACKNOWLEDGEMENT

This work is a part of Post Doctoral Fellowship of the author Dilip Kumar Jaiswal and gratefully acknowledges the financial assistance in the form of UGC-Dr. D. S. Kothari Post Doctoral Fellowship.

APPENDIX: DERIVATION OF LAPLACE INVERSION TRANSFORM

The procedure used to invert the Laplace transform is to evaluate the closed contour and used the residue theorem. Branch point must be excluded from inside the contour and the original solution. $c(x,T)$, can be obtained by finding the solution to

$$c(x,T) = \frac{1}{2\pi i}\int_{\Gamma-i\infty}^{\Gamma+i\infty} \bar{K}(x,p)\, exp(pT)\, dp \quad \text{(A-1)}$$

or

$$c(x,T) = \sum_i Res\,(i)$$

$$\frac{1}{2\pi i}\int_{\Omega} \bar{K}(x,p)\, exp(pT)\, dp \quad \text{(A-2)}$$

where *Res* (*i*) are the residue at the poles which lie to the left of the line $p = \Gamma$ and outside of the contour Ω. From Eq. (18), the poles of the expressions

$\frac{1}{(p-\gamma^2)}$, $\frac{1}{(p-\gamma^2)^2}$, $\frac{1}{\left[p-\frac{D_0}{R}(\alpha-\beta)^2\right]}$ and $\frac{M\cosh M(L_1-x)+\beta\sinh M(L_1-x)}{M\cosh M(L_1-L)+\beta\sinh M(L_1-L)}$

are $p = \gamma^2$, $p = \gamma^2$ pole of order 2, $p = \frac{D_0}{R}(\alpha-\beta)^2$ $p = -\frac{D_0}{R}\omega_n^2$ where ω_n is the positive root of the $[\omega_n \cot\omega_n (L_1 - L) + \beta = 0]$, respectively.

Therefore the residues of expressions with these poles may get as follows:

$$Res\ (\text{at } p=\gamma^2) = \lim_{p=\gamma^2} \frac{1}{(p-\gamma^2)} exp(pT) \quad \text{(A-3)}$$

$$Res\ (\text{at } p=\gamma^2 \text{ of order } 2) = \lim_{p=\gamma^2} \frac{1}{1!}\frac{d}{dp}\frac{1}{(p-\gamma^2)^2} exp(pT) \quad \text{(A-4)}$$

$$Res\ \left(\text{at } p = \frac{D_0}{R}(\alpha-\beta)^2\right) = \lim_{p=\frac{D_0}{R}(\alpha-\beta)^2} \frac{1}{\left(p-\frac{D_0}{R}(\alpha-\beta)^2\right)} exp(pT) \quad \text{(A-5)}$$

$$Res\ \left(\text{at } p = -\frac{D_0}{R}\omega_n^2\right) = \lim_{p=p=-\frac{D_0}{R}\omega_n^2} \frac{M\cosh M(L_1-x)+\beta\sinh M(L_1-x)}{M\cosh M(L_1-L)+\beta\sinh M(L_1-L)} exp(pT) \quad \text{(A-6)}$$

The inverse Laplace transform of equation (18) is equal to the sum of the all residues as,

$$L^{-1}\left\{\frac{\{2-(2-mT_0)exp(-(p-\gamma^2)T_0)\}}{(p-\gamma^2)} \frac{M\cosh M(L_1-x)+\beta\sinh M(L_1-x)}{M\cosh M(L_1-L)+\beta\sinh M(L_1-L)}\right\}$$
$$-L^{-1}\left\{\frac{m\{1-exp(-(p-\gamma^2)T_0)\}}{(p-\gamma^2)^2} \frac{M\cosh M(L_1-x)+\beta\sinh M(L_1-x)}{M\cosh M(L_1-L)+\beta\sinh M(L_1-L)}\right\}$$
$$-L^{-1}\left\{\frac{C_i\, exp\{(\alpha-\beta)L_1\}}{\left\{p-\frac{D_0}{R}(\alpha-\beta)^2\right\}} \left[\frac{\alpha\sinh M(x-L)}{M\cosh M(L_1-L)+\beta\sinh M(L_1-L)}\right]\right\}$$
$$-L^{-1}\left\{\frac{C_i\, exp((\alpha-\beta)L)}{\left\{p-\frac{D_0}{R}(\alpha-\beta)^2\right\}} \frac{M\cosh M(L_1-x)+\beta\sinh M(L_1-x)}{M\cosh M(L_1-L)+\beta\sinh M(L_1-L)}\right\}$$
$$+L^{-1}\left\{\frac{C_i\, exp\,(\alpha-\beta)x}{\left\{p-\frac{D_0}{R}(\alpha-\beta)^2\right\}}\right\} = \sum Residues \quad \text{(A-8)}$$

The solution of advection-dispersion solute transport for periodic input condition in terms of $c(x,T)$ is defined in Eqs. (19a,b).

REFERENCES

[1] C. E., Jacob, *Flow of Groundwater, In Engineering Hydraulics*, Edited By H. Rouse (John Wiley 1950), New York, 321-386.

[2] J. G., Ferris, Cyclic fluctuations of water level as a basis for determining aquifer transmissibility, *IAHS Publ.*, *33*, 1951, 148–155.

[3] P.A., Carr, and G., Van Der Kamp. Determining Aquifer Characteristics by The Tidal Methods. *Water Resources Research*, *5(5)*, 1969, 1023-1031.

[4] C.S., Fang, S.N., Wang, and W., Harrison, Groundwater flow in a sandy tidal beach: two-

dimensional finite element analysis, *Water Resources Research*, *8*, 1972, 121-128.

[5] M.A., Marino, Flow against dispersion in non-adsorbing porous media, *Journal of Hydrology*, *37 (1-2),* 1978, 149-158.

[6] A.N.S., Al-Niami, and K.R., Rushton, Analysis of flow against dispersion in porous media, *Journal of Hydrology*, *33*, 1977, 87-97.

[7] N., Kumar, Unsteady flow against dispersion in finite porous media with unsteady velocity distribution. *Nordic Hydrol*., *14*, 1983, 167-178.

[8] L. W., Gelhar, A. L., Gutjahr, and R. L., Naff, Stochastic Analysis of Macro-dispersion in a Stratified Aquifer, *Water Resources Research*, *15(6),* 1979, 1387-1397.

[9] G., Matheron, G., deMarsily, Is transport in porous media always diffusive? A counter example, *Water Resources Research*, *16*, 1980, 901–917.

[10] F.J., Molz, O., Guven, J.G., Melville, and J.F., Keely, Performance and analysis of aquifer tracer tests withimplications for contaminant transport modeling-a project summary, *Groundwater*, *25(3),* 1987, 337-341.

[11] D. J., Goode, and L.F., Konikow, Apparent Dispersion in Transient Groundwater Flow, *Water Resources Research, 26(10),* 1990, 2339–2351.

[12] M., Koch, G., Zhang, Numerical simulation of the effects of variable density in a contaminant plume. *Ground Water*, *30(5),* 1992, 731–742.

[13] D., Zhang, and S.P., Neuman, Effect of Local Dispersion on Solute Transport in Randomly Heterogeneous Media, *Water Resources Research*, *32(9),* 1996, 2715-2723.

[14] S., Attinger, M., Dentz, H., Kinzelbach, and , W. Kinzelbach, Temporal behaviour of a solute cloud in a chemicallyheterogeneous porous medium, *J. Fluid Mech*., *386*, 1999, 77-104.

[15] M., Dentz, H., Kinzelbach, S., Attinger, and W., Kinzelbach, Temporal behavior of a solute cloud in aheterogeneous porous medium: 1. Point-like injection, *Water Resources Research*, *36(12),* 2000a, 3591-3604.

[16] M., Dentz, H., Kinzelbach, S., Attinger, and W., Kinzelbach, Temporal behavior of a solute cloud in aheterogeneous porous medium: 2. Spatially extended injection, *Water Resources Research*, *36(12),* 2000b, 3605-3614.

[17] M.A., Robinson, D.L., Gallagher, A model of groundwater discharge from an unconfined coastal aquifer. *Ground Water*, *37 (1),* 1999, 80–87.

[18] J.J., Jiao and Z. Tang, An analytical solution of groundwater response to tidal fluctuation in a leaky confined aquifer, *Water Resources Research*, *35(3),* 1999, 747- 751.

[19] J., Vanderbroght, A., Timmerman, and J., Feyen, Solute Transport for Steady State and Transient Flowsin Soils with and without Macropores, *Soil Science Society of American Journal*, *64*, 2000, 1305-1317.

[20] O. A., Cirpka, Effects Of Sorption on Transverse Mixing in Transient Flows, *J. Cont. Hydrol.*, *78*, 2005, 207-229.

[21] A., Kumar, D. K. Jaiswal and N., Kumar, Analytical solutions of one-dimensional advection–diffusion equation with variable coefficients in a finite domain, *J. Earth Syst. Sci., 118(5),* 2009, 539–549

[22] C.S., Yim, and M.F.N. Mohsen, Simulation of tidal effects on contaminant transport in porous media. *Ground Water*, *30(1),* 1992, 78-86.

[23] J., Crank, *The Mathematics of Diffusion*, 2nd ed. (Oxford Univ. Press London 1975).

[24] H.S., Carslaw, and J.C., Jaeger, *Conduction of Heat in Solids*, 2nd ed. (Clarendon Press, Oxford, Great Britain 1986).

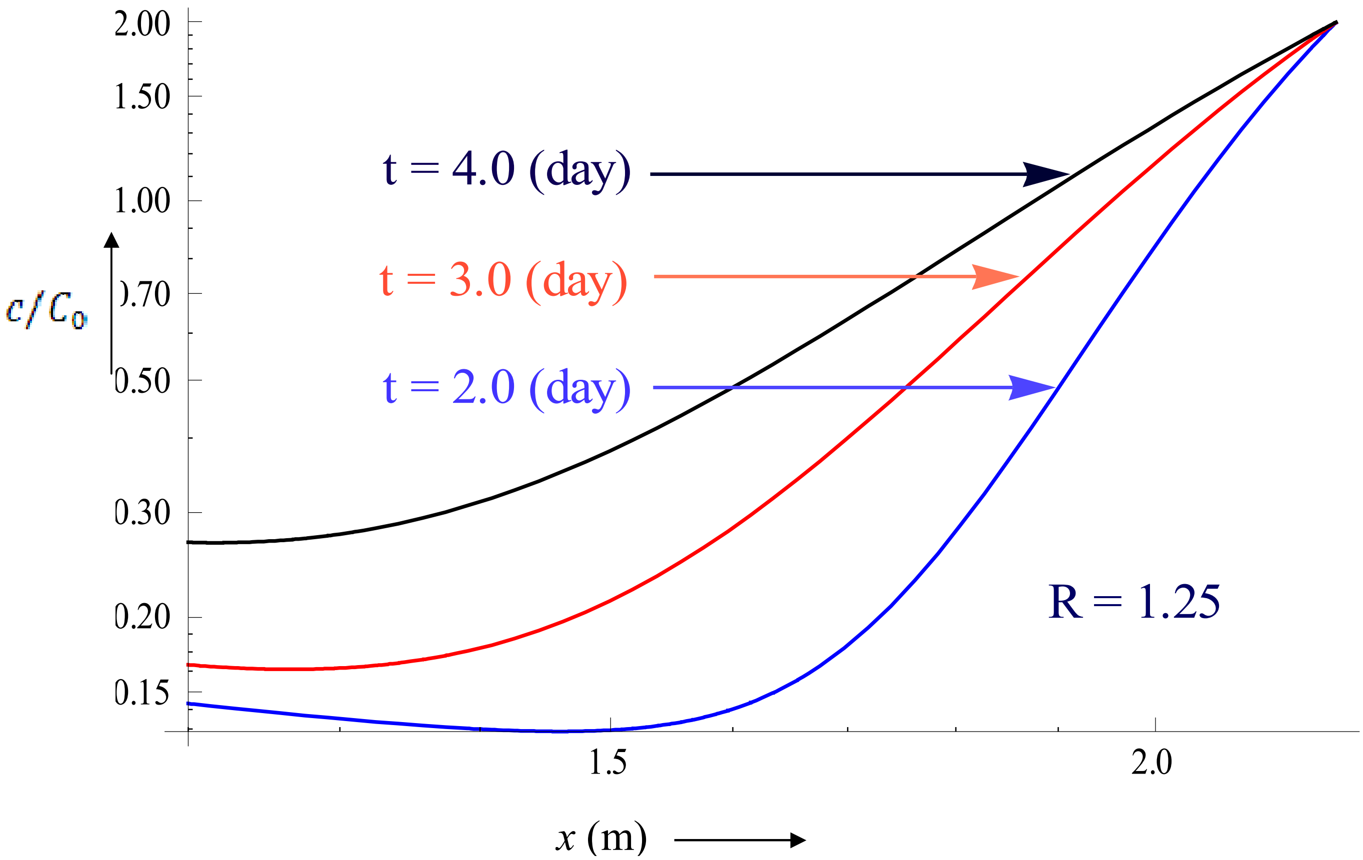

Fig. 2 Concentration profile of solute in presence of input source at different time for fixed retardation factor and angular frequency.

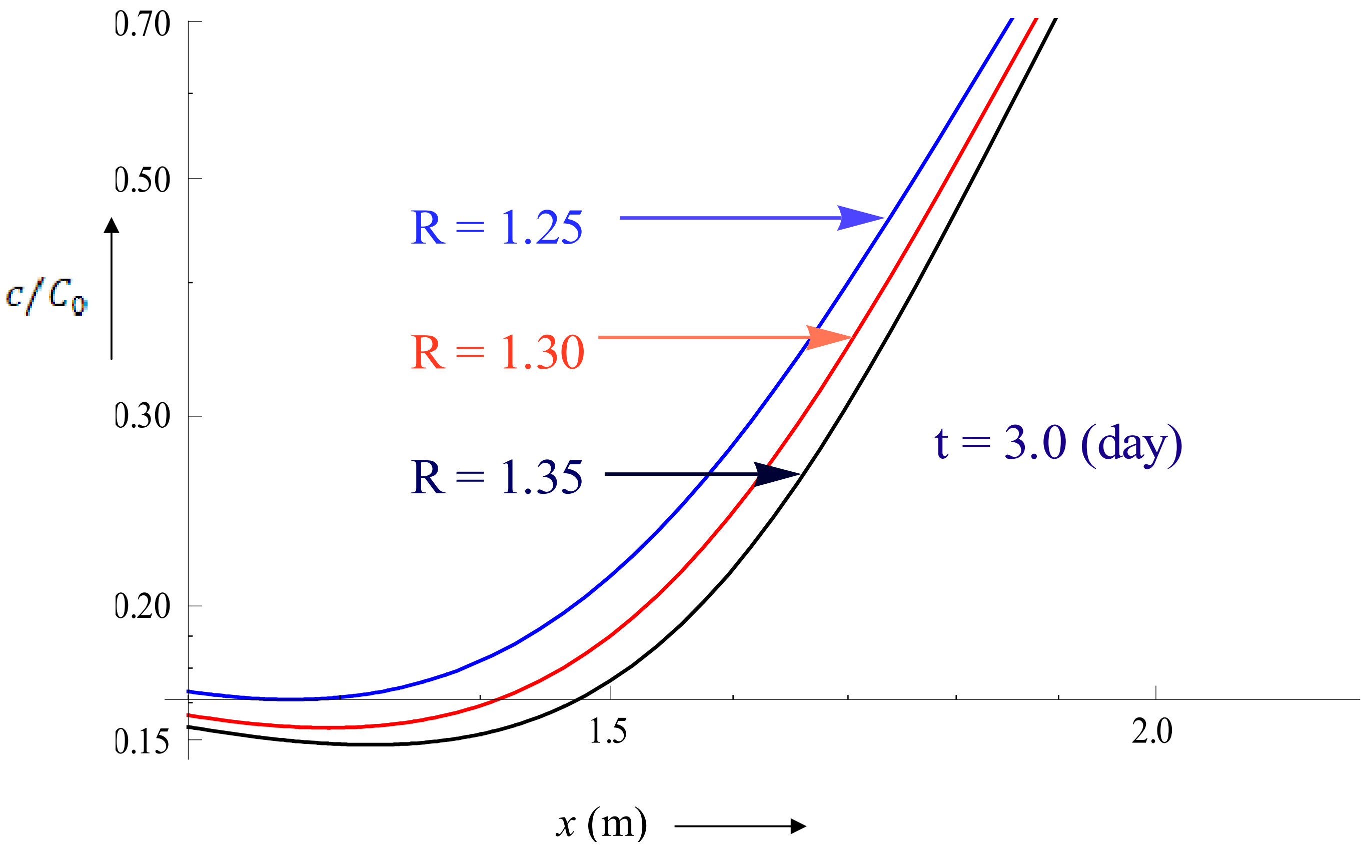

Fig. 3 Concentration profile of solute in presence of input source at fixed time for different retardation factor and one angular frequency.

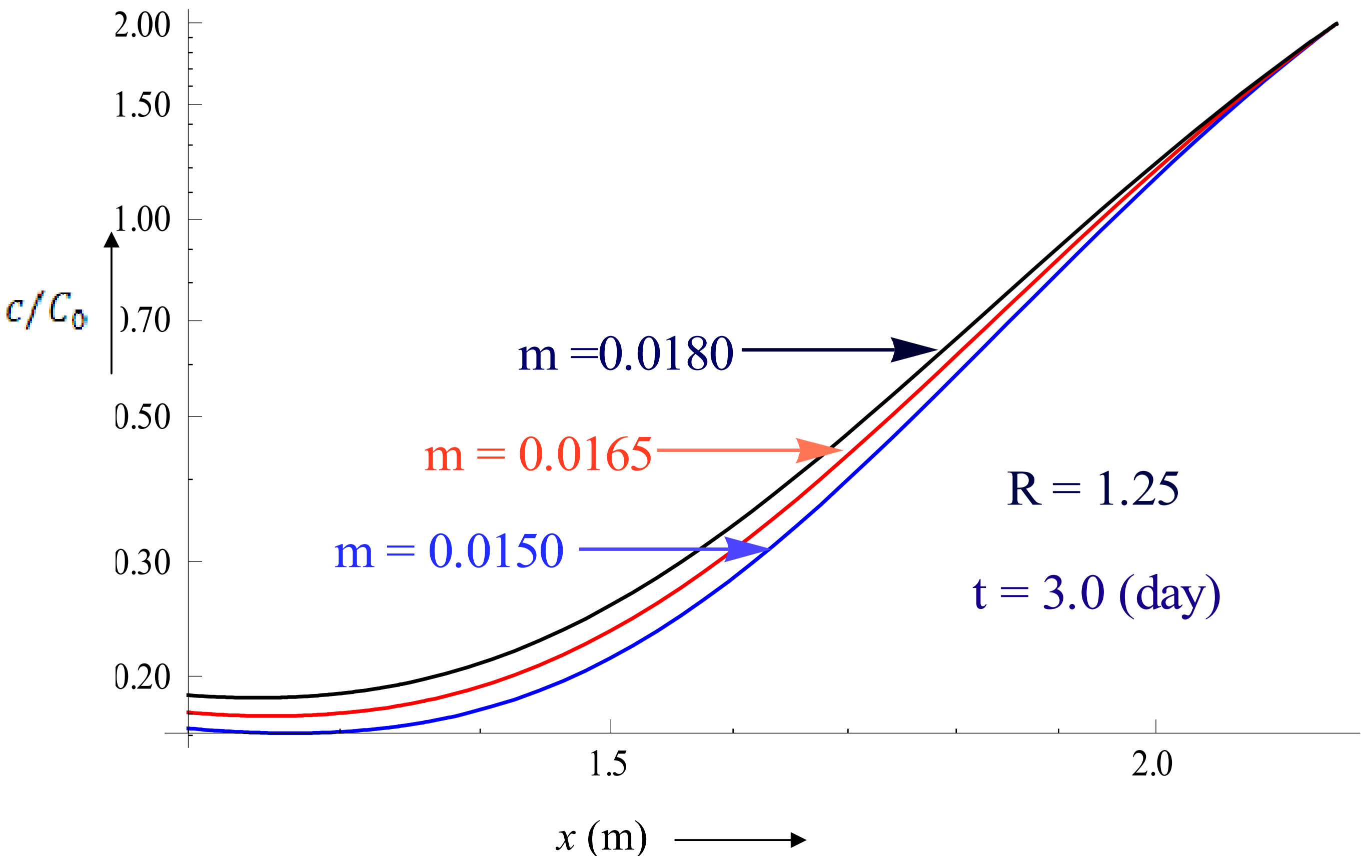

Fig. 4 Concentration profile of solute in presence of input source at fixed time for different angular frequency and one retardation factor.

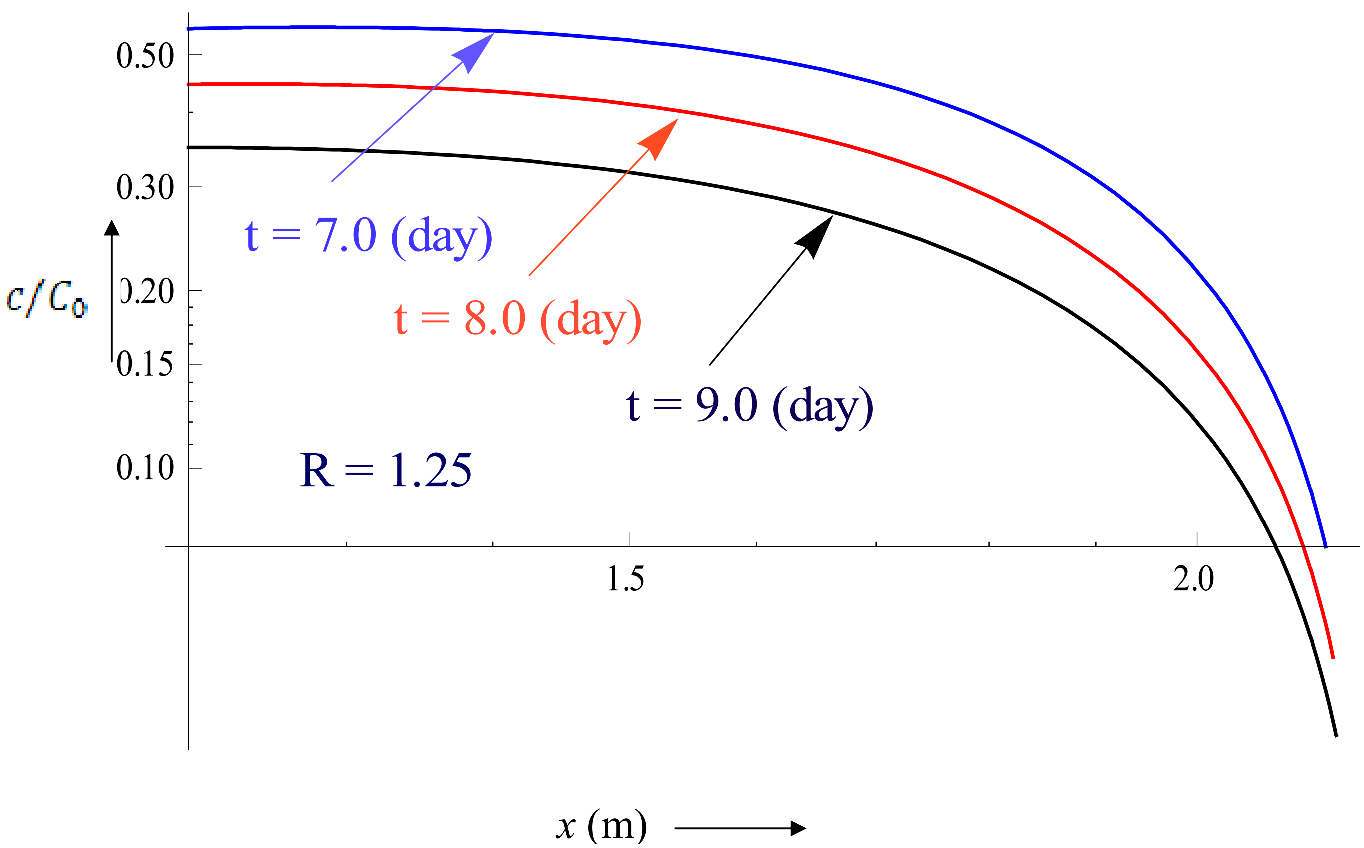

Fig. 5 Concentration profile of solute in absence of input source at different time for fixed retardation factor and angular frequency.

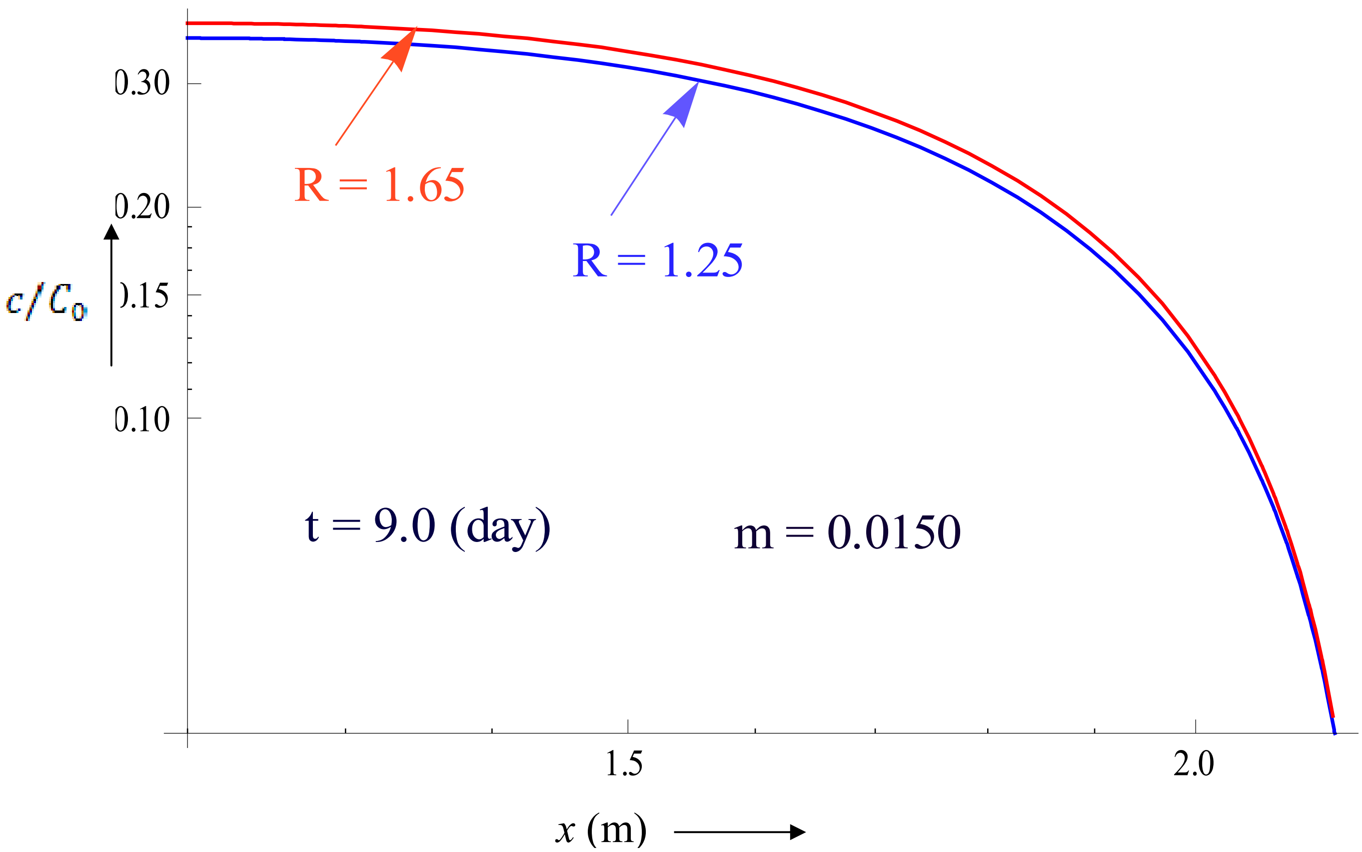

Fig. 6 Concentration profile of solute in absence of input source at fixed time for different retardation factor and one angular frequency.

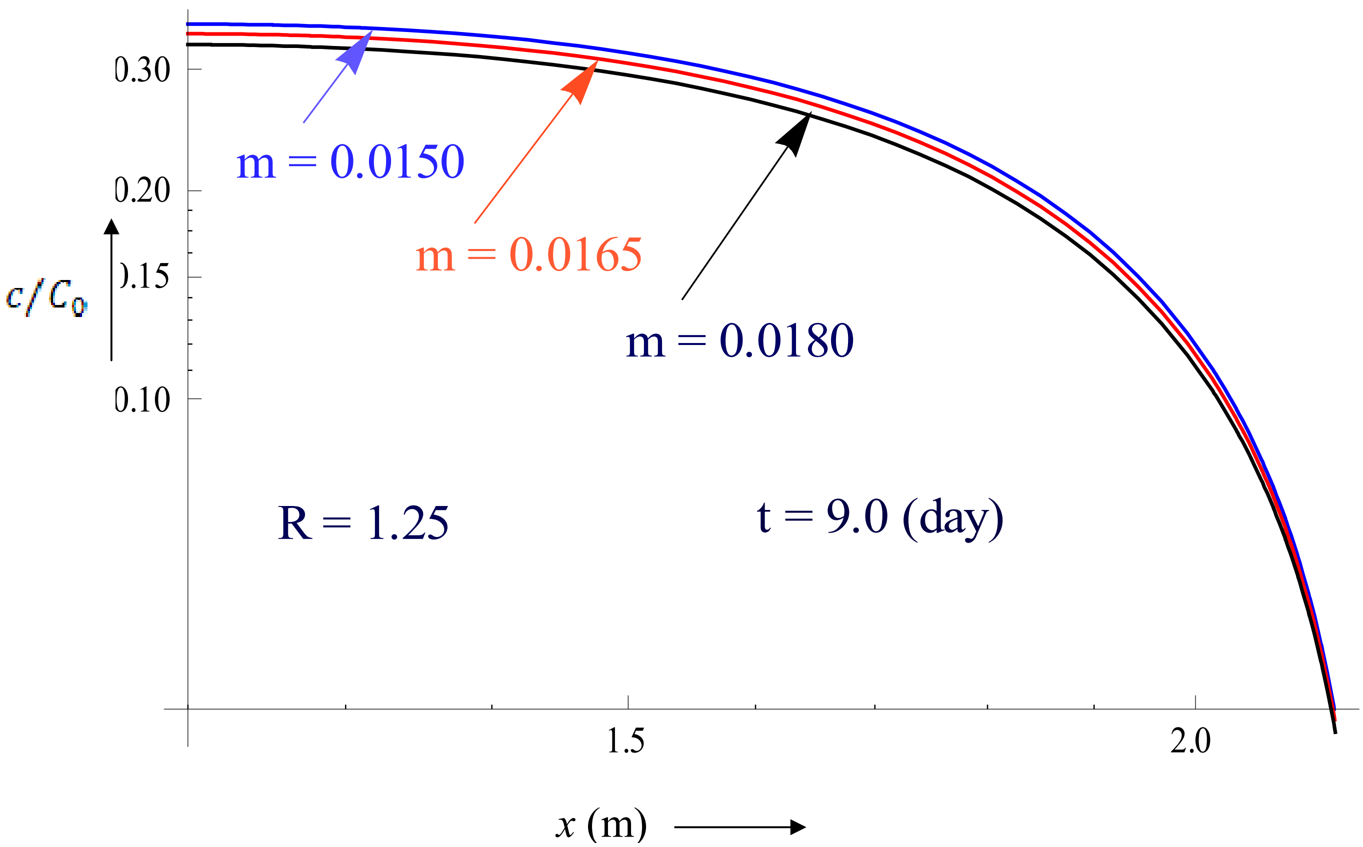

Fig. 7 Concentration profile of solute in absence of input source at fixed time for different angular frequency and one retardation factor.