Prediction of Shear Strength of PPFRC Moderate Deep Beams Using Strut-and-Tie Models

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ABSTRACT

Strut-and-tie is a system of forces' distribution in the form of "load-path" connectivity from the applied load point to the support point. Strut-and-Tie Method (STM) has been developed based on simple truss model. STM models represent the load carrying mechanism of a structural member by approximating the flow of internal forces by means of struts representing the flow of compressive stresses and ties representing the flow of tensile stresses. IS 456: 2000, along with other various codes of different countries, classifies the beam into three categories; namely normal beam, moderate deep beam, and deep beam, according to their span to depth ratios. The aim was to provide a systematic and comprehensive study on the shear strength of Polypropylene Fiber Reinforced Concrete (PPFRC) moderate deep beams without web reinforcement and to compare experimental result of shear strength with theoretical result by STM of PPFRC. Experimental results of ultimate shear strength are compared with the theoretical results calculated from formula of STM given by various source such as ACI 318-08, Nielsen (1984), and Schlaich et al. (1987). We found that the experimental value and theoretical value by STM are within 15% variation range for all types of beams.

Keywords – **Strut-and-tie, Polypropylene Fiber Reinforce Concrete, Moderate deep beam, Ultimate shear strength, stress-strain**

1. INTRODUCTION

Strut-and-Tie Method (STM) has been developed based on simple truss model. The simple truss model is only rational for the design of cracked reinforced concrete beams. The design based on the simple truss model is limited to certain parts of structure. However, STM as an extension to simple truss model is applicable to analyze and design the whole of a reinforced concrete member experiences three effects; shear, flexural, and axial effect. The original strut-and-tie model has been developed as a Lower Bound solution of plastic theory where equilibrium of a system is considered together with the yield criterion. A statical or geometrical discontinuity such as point loads or frame corners, corbels, holes and other openings, the theory is not applicable.

Since 2002, strut-and-tie method has been included as an alternative design method in North America [ACI 318-2008][2]. This report describes the development of strut-and-tie theory in analysis of fibrous reinforced concrete element. A shear behavior of moderate deep beams is included as an example, and compared theoretical values with experimental values. The scope of this study covers the development of the strut-and-tie method as an analytical procedure based on Appendix A of ACI 318-2008[2] and other sources.

The concept of incorporating strong thin fiber to strengthen brittle matrices is not new. The concept is more than 4500 years old. Potentially, the addition of fibers causes substantial changes in properties of both fresh and hardened concrete. Due to low effectiveness, poor alkaline resistance high cost, use of other fibers such as nylon, rayon, carbon etc. has been almost ruled out after initial investigation. The use of strong and stiff fibers in concrete improves the post cracking performance of concrete considering reserved strength. After micro cracking, fibers spanning the cracks, control crack propagation and control the rate of widening of cracks under tensile loading. This role of fiber imparts ductility of concrete and delays its failure. The process of fiber pull out absorbs lot of energy and hence the toughness of concrete and its impact resistance are considerably increased. Remarkable improvements in elastic modulus, tensile strength, crack resistance, crack control, durability, fatigue resistance, impact resistance, abrasion resistance etc., resulted in FRC material, which arrived as a boon to overcome the drawbacks of steel-reinforced concrete.

2. RESEARCH SIGNIFICATION

The scope of this study covers the development of the strut-and-tie method as a design procedure and to compare ultimate shear strength of fiber reinforced concrete moderate deep beams without web reinforcement (Stirrups) using strut-and-tie method with experimental results.

Table 1 Formulas given by Different Sources

Sources	Strut compressive capacity
ACI 318-08	***Without Longitudinal Reinforcement*** $0.85\beta_s f'_c A_{cs}$ Prismatic: β_s= 1.0 Bottle-Shaped w/reinf. satisfying crack control: β_s= 0.75 Bottle-Shaped not satisfying crack control: $\beta_s = 0.60\lambda$ λ =1.0 for normal weight concrete λ =0.85 for sand-lightweight concrete λ =0.75 for all lightweight concrete Strut in tension members: β_s= 0.40 All other cases: β_s= 0.60 ***With Longitudinal Reinforcement*** $f_{cu}A_c + f'_s A'_s$
Schlaich et al. (1987)	$0.85f'_c$ "for an undisturbed and uniaxial state of compressive stress" (prismatic) $0.68f'_c$ "if tensile strains in the cross direction or transverse tensile reinforcement may cause cracking parallel to the strut with normal crack width" $0.51f'_c$ "as above for skew cracking or skew reinforcement" $0.34f'_c$ "for skew cracks with extraordinary crack width. Such cracks must be expected, if modeling of the struts departs significantly from the theory of elasticity's flow of internal forces"
Nielsen (1984)	$(0.8 - \frac{f'c}{200}) f'_c A_{cs}$

Table 2 Notations for Table 1

Notations
A'_s = area of compression steel (in^2) A_c = area of concrete in the strut (in^2) A_{cs} = area of concrete in the strut (in^2) A_{si} = total area of surface reinforcement at spacing si (in^2) f'_c = concrete compressive strength (ksi) f_{cu} = effective concrete compressive strength (ksi) α_i = the angle between the reinforcement and the axis of the strut (DEG.)

3. EXPERIMENTAL PROGRAMME

Testing was carried out on 12 PPFRC (FT) simply supported moderate deep beams. These beams were tested in simply supported conditions under one point loading at a center of effective span from support.

3.1 Test specimen

Twelve Polypropylene Fiber Reinforced Cement Concrete moderate deep beams, simply supported on effective span of 1200 mm were tested under one point loading. Length of the beams and width of the web were kept constant (1300 mm and 150 mm respectively). The beams were divided into four series having depths of 300 mm, 400 mm, 500 mm and 600 mm respectively. Each series comprised of three beams. i.e. beam notation "D60" denotes the beam having overall depth D of 60 cm.

3.2 Test materials

The cement used was ordinary Portland cement of grade 53. ordinary river sand having fineness modules of 2.8 and maximum size of 4.75 mm, and crushed basalt gravel having a maximum size of 20 mm were used as a fine and coarse aggregate respectively. The concrete mix of 1:1.5:3 (cement: fine aggregate: coarse aggregate) by weight with water cement ratio of 0.45 was used for all beams. Fibrillated form of Polypropylene Fibers having melting point 165 Celsius, tensile strength 670 N/mm2 was homogeneously mixed with cement. Fiber content/Beam = 0.75% (By volume) of concrete .Longitudinal tension reinforcement consists of High yield strength deformed bars (415 N/mm2) used, Vertical Shear Reinforcement (stirrups) are not provided. Six cubes (150mm) and eight cylinders (four cylinders for compressive strength and four cylinders for splitting strength, 150mm diameter and 300mm height) were cast as control specimens from each mix. All specimens were cured at least for 28 days.

3.3 Testing

All the beams were tested under two point concentrated loadings positioned at one third spans. All the beams were simply supported with an effective span of 1200 mm. Beams were centered on platform and leveled horizontally and vertically by adjusting the bearing plates. Load was applied gradually.

Crack propagations were traced by pencil and their tips were marked corresponding to the load readings

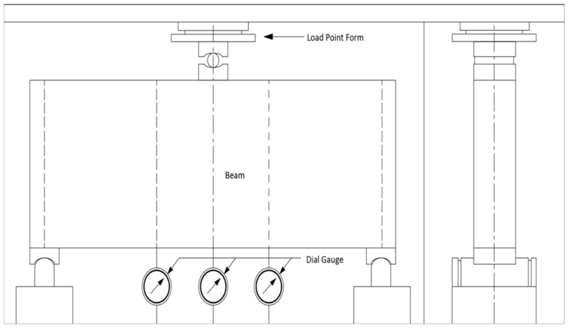

Fig.1 Test setup

4. DISCUSSION OF TEST RESULTS

Experimental results are compared with theoretical results. Theoretical results are calculated from formula of STM which are given by various sources such as ACI 318-08[2], Nielsen (1984)[3], and Schlaich et al. (1987)[5,6]. The results of 12 PPFRC (FT) compared with theoretical results.

Table 3 Comparison of $V_{u(exp)}$ and $V_{u(th)}$ for PPFRC (FT)

	Experimental Result	ACI 318-08	Nielsen (1984)	Schlaich et al. (1987)
D30	3.80	3.385	3.621	3.611
	3.60	3.292	3.540	3.512
	3.00	2.771	3.064	2.955
D40	5.15	5.080	5.649	5.418
	6.30	5.861	6.378	6.252
	6.45	5.869	6.385	6.260
D50	9.05	7.813	8.509	8.334
	9.55	8.279	8.929	8.831
	9.45	8.245	8.898	8.795
D60	15.15	13.235	13.869	14.118
	12.75	11.212	12.128	11.959
	14.75	13.151	13.799	14.028

*Unit of V_u is **Tons (UK).**

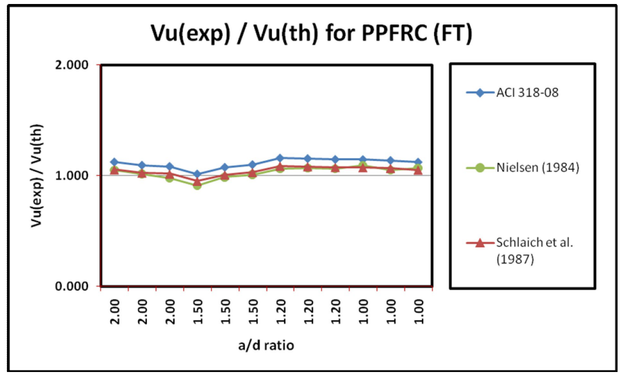

Fig. 2 Graphical presentation of $V_{u(exp)}/V_{u(th)}$ ratio For PPFRC (FT) beams

Table 4 Ratio of ($V_{u(exp)}/V_{u(th)}$) for PPFRC (FT)

	a/h	l/h	ACI 318-08	Nielsen (1984)	Schlaich et al. (1987)
D30	1.33	4	1.123	1.049	1.052
	1.33	4	1.094	1.017	1.025
	1.33	4	1.083	0.979	1.015
D40	1	3	1.014	0.912	0.951
	1	3	1.075	0.988	1.008
	1	3	1.099	1.010	1.030
D50	0.80	2.4	1.158	1.064	1.086
	0.80	2.4	1.154	1.070	1.081
	0.80	2.4	1.146	1.062	1.074
D60	0.66	2	1.145	1.092	1.073
	0.66	2	1.137	1.051	1.066
	0.66	2	1.122	1.069	1.051

5. CONCLUSION

I. Experimental results of ultimate shear strength are compared with the theoretical results calculated from formula of STM given by various source such as ACI 318-08[2], Nielsen (1984)[4], and Schlaich et al. (1987)[6,7]. The Table 3 & Table 4 indicates indirect verification of experimental results with theoretical results. The theoretical results by STM are within ± 15% variation for all types of beam.

II. The average ratio of (V_{exp}/V_{th}) for beams of D30 series is 1.100 for ACI 318-08[2], 1.015 for Nielsen (1984)[3], and 1.031 for Schlaich et al. (1987)[5,6].

III. The average ratio of (V_{exp}/V_{th}) for beams of D40 series is 1.063 for ACI 318-08[2], 0.970 for Nielsen (1984)[3], and 0.099 for Schlaich et al. (1987)[5,6].

IV. The average ratio of (V_{exp}/V_{th}) for beams of D50 series is 1.153 for ACI 318-08[2], 1.065 for Nielsen (1984)[3], and 1.080 for Schlaich et al. (1987)[5,6].

V. The beams of series of D60 average of ratio of (V_{exp}/V_{th}) is 1.135 for ACI 318-08[2], 1.071 for Nielsen (1984)[3], and 1.063 for Schlaich et al. (1987)[5,6].

VI. This shows that Schlaich et al. (1987)[5,6] and Nielsen (1984)[3] predict conservative shear strength, but nearly accurate results for all beams.

VII. The ACI 318-08[2] gives very conservative results for all beams.

NOMENCLATURE:

ACI : American Concrete Institute
STM : Strut-and-Tie Models or Strut-and-Tie Method
PPFRC : Polypropylene Fiber Reinforce Concrete
FRC : Fiber Reinforce Concrete
V_u : Ultimate Shear Strength
FT : Fibrillated Type

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