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Completely Prime Fuzzy,Prime Fuzzy Ideal Of A Po Ternary Semigroup

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ABSTRACT
In this Paper the terms [f), fuzzy left(right,lateral) identity, fuzzy left(right,lateral) zero of a po ternarysemigroup are introduced. It is proved that fuzzy left identity and fuzzy right identity and lateral zero of a poternary semigroup if it exists then they are same. And proved that fuzzy left zero ,fuzzy lateral zero ,fuzzy right zero of a po ternarysemigroup exists then they are equal.also proved that intersection of arbitrary family of fuzzy po ternary subsemigroups of a po ternary semigroup T is a fuzzy poternary subsemigroup of T. Furthermore, proved that intersection of all fuzzy po ternary semigroups of T containing f is ideal generated by fuzzy subset f. and also proved that f is a fuzzy ideal of a po ternary semigroup T ifff$oToT \subseteq f, ToTof \subseteq f, TofoT \subseteq f$ and (f] ⊆ f.And proved the union, intersection of arbitrary family of fuzzy ideals of po ternary semigroup T is fuzzy ideal of T.further proved that every completely prime fuzzy ideal is weakly completely prime fuzzy ideal. Also proved that if f is completely prime fuzzy ideal then $f_t$is ternary sub semigroup and completely prime ideal of T. also proved that f_t is completely prime ideal of T then f is weakly completely prime fuzzy and completely prime fuzzy ideal of T and proved that equivalent conditions of prime fuzzy ideal, f is prime fuzzy ideal iff 1-f is a fuzzy m-system ofT if $1 - f \neq \emptyset$. Finally proved that every maximal fuzzy ideal of T is a prime fuzzy ideal of T.

Mathematical subject classification (2010): 20M07; 20M11, 20M12
Keywords: Fuzzy po ternary semigroup, [f), fuzzy identity of a po ternary semigroup, fuzzy zero of a po ternary semigroup, fuzzy po ternarysemigroup generated by f, fuzzy ideal, completely prime fuzzy ideal, weakly completely prime fuzzy ideal, prime fuzzy ideal, m-system, Maximal ideal.

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I. INTRODUCTION:

The algebraic theory of semigroups was widely studied by Clifford[2,3]. The ideal theory in general semigroups was developed by Anjaneyulu[1]. Since then a series of researchers have been extending the concepts and results of abstract algebra. Padmalatha, A. Gangadhara Rao and A.Anjaneyulu[10] introduced posubsemigroup, posubsemigroup generated by a subset, two sided identity of a posemigroup, zero of a posemigroup, po ideal ,po ideal generated by a subset. On the other hand, P.M.Padmalatha , A.Gangadahara Rao, P.RamyaLatha [12] introduced completely prime, prime ideal of a posemigroupV.Sivaramireddy studied on ideals in partial ordered ternary semi groups [16].

The concept of a fuzzy set was introduced by Zadeh in 1965[6]. This idea opened up new thoughts and applications in a wide range of scientific fields. A. Rosenfeld applied the notion of fuzzy subset to several areas of mathematics, among other disciplines. N. Kuroki, J N Mordeson developed the fuzzy semigroups concept. N.Kehayopulu, M.Tsingelis introduced the notion of fuzzy subset of a posemigroups[7-9]. Motivated by the study of N.Kehayopulu, M.Tsingelis work in posemigroups we attempt in the paper to study the fuzzy identity and fuzzy zero of a posemigroup, operations on fuzzy posemigroups, results on completely prime fuzzy ideals and prime fuzzy ideals of posemigroup.

II. PRELIMINARIES:

DEFINITION 2.1: [5] A semigroupT with an ordered relation ≤ is said to be **po Ternary semigroup**ifT is a partially ordered set such that $a \le b \Rightarrow aa_1a_2 \le ba_1a_2, a_1aa_2 \le a_1ba_2, a_1a_2a \le a_1a_2b$ for all $a, b, a_1a_2 \in T$.

DEFINITION 2.2: A function f from T to the closed interval [0,1] is called a **fuzzy subset** of T. The po ternary semigroupT itself is a fuzzy subset of T such that T(x)=1, ∀x ∈ T. It is denoted by T or 1.

DEFINITION 2.3: Let A be a non-empty subset of T. We denote **f_A, the characteristic mapping** of A. i.e., The mapping of T into [0,1] defined by $f_A(x)=\begin{cases}1 & \text{if } x \in A\\ 0 & \text{if } x \notin A\end{cases}$ Then f_A is a fuzzy subset of T

DEFINITION 2.4:[5]**:** A fuzzy subset f of a po ternary semigroupT is called **fuzzy Ternary subsemigroup**of T if $f(xyz) \geq f(x) \wedge f(y) \wedge f(z) \forall x, y, z \in T$.

DEFINITION 2.5: Let T be a po ternary semigroup. For H⊆ T we define **(H]**={t∈T / t≤ h forsome h∈H}. For H={a} we write (a]= ({a}] = { t∈T / t ≤a}

DEFINITION 2.5: Let T be a po ternary semigroup. For H⊆ T we define [H)={t∈T / h ≤ t forsome h∈H}. For H={a} we write (a]= ({a}] = { t∈T / t ≤a}

DEFINITION 2.6: Letfbe a fuzzy subset of a po ternary semigroupT. We define (**f**]by $(f](x) = \vee_{x \leq y} f(y), \forall x \in T$.

NOTE 2.7: Clearlyf ⊆ (f].

Note 2.8: The set of all fuzzy subsets of T is denoted by F(T).

DEFINITION 2.9: Let (T, ≤) be a po ternary semigroup and f,g,h be fuzzy subsets of T. For x ∈ T the**product fogoh** is defined by (fogoh)(x) = $\begin{cases}\vee_{x \leq pqr} f(p) \wedge g(q) \wedge h(r) & \text{if } x \leq pqr \text{ exists}\\ 0 & \text{otherwise}\end{cases}$

DEFINITION 2.10:[11] A nonempty subset A of a po ternary semigroup T is said to be po left ternary ideal or po left ideal of T if i) b, c ∈ T, a ∈ A ⇒bca∈ A ii) a ∈ A and t ∈ T such that t ≤ a ⇒ t ∈ A.

NOTE : A nonempty subset A of a po ternary semigroup T is a po left ternary ideal of T if and only if i) TTA ⊆ A ii) (A] ⊆ A.

DEFINITION 2.11:A nonempty subset A of a po ternary semigroup T is said to be po lateral ternary ideal or po lateral ideal of T if i) b, c ∈ T, a ∈ A ⇒bac∈ A ii) a ∈ A and t ∈ T such that t ≤ a ⇒ t ∈ A.

NOTE**2.12** : A nonempty subset A of a po ternary semigroup T is a po lateral ternary ideal of T if and only ifi) TAT UTTATT ⊆ A ii) (A] ⊆ A.

DEFINITION 2.13: A nonempty subset A of a po ternary semigroup T is said to be po right ternary ideal or po right ideal of T ifi) b, c ∈ T, a ∈ A ⇒abc∈ A ii) a ∈ A and t ∈ T such that t ≤ a ⇒ t ∈ A

NOTE**2.14** : A nonempty subset A of a po ternary semigroup T is a po right ternary ideal of T if and only if i) ATT ⊆ A ii) (A] ⊆ A.

DEFINITION 2.15: A nonempty subset A of a po ternary semigroup T is said to be po ternary ideal or po ideal of T if i) b, c ∈ T, a ∈ A ⇒bca∈A, bac∈A, abc∈ A ii) a ∈ A and t ∈ T such that t ≤ a ⇒ t ∈ A.

NOTE**2.16** : A nonempty subset A of a po ternary semigroup T is a po ternary ideal of T if and only ifi) TTA ⊆ A, TAT ⊆ A, ATT ⊆ A ii) (A] ⊆ A.

DEFINITION 2.17:[11]LetT be a po ternary semigroup. A fuzzy subset f of T is called a **fuzzypo left ideal** of T if (i) x ≤ y then f(x) ≥ f(y) (ii) f(xyz) ≥ f(z), ∀ x,y, z ∈ T

LEMMA 2.18: [10] LetT be a po ternary semigroup and f be a fuzzy subset of T. Then f is a fuzzypo left ideal of T if and only if f satisfies that (i) x ≤ y then f(x) ≥ f(y) ∀ x,y, z ∈T (ii) Tofof⊆ f.

DEFINITION 2.19: [11]LetT be a po ternary semigroup. A fuzzy subset f of T is called a f**uzzy poright ideal** of T if (i) x ≤ y then f(x) ≥ f(y) (ii) f(xyz) ≥ f(x), ∀ x,y,z ∈T.

LEMMA 2.20 [10] Let T be a po ternary Semigroup and f be a fuzzy subset of T. Then f is a fuzzy right ideal of T if and only if f satisfies that (i) x ≤ y then f(x) ≥ f(y) ∀ x,y, z ∈T (ii)fofoT⊆ f.

DEFINITION 2.21: [11]LetT be a po ternary semigroup. A fuzzy subset f of T is called a f**uzzy po lateral ideal** of T if (i) x ≤ y then f(x) ≥ f(y) (ii) f(xyz) ≥ f(y), ∀ x,y, z ∈T

LEMMA 2.22: [10] Let T be a po ternary Semigroup and f be a fuzzy subset of T. Then f is a fuzzy lateral ideal of T if and only if f satisfies that (i) x ≤ y then f(x) ≥ f(y) ∀ x,y, z ∈T (ii)foTof⊆f.

DEFINITION 2.23: [11]LetT be a po ternary semigroup. A fuzzy subset f of Tis called a **fuzzy ideal**of T if (i) x ≤ y then f(x) ≥ f(y) (ii) f(xyz) ≥ f(z), f(xyz) ≥ f(x), f(xyz)≥ f(y)∀ x,,y, z ∈T.

LEMMA 2.24 : [10] Let T be a po ternary semigroup and f be a fuzzy subset of T. Then f is a fuzzy ideal of T if and only if f satisfies that (i) x ≤ y then f(x) ≥ f(y) ∀ x,y, z ∈T (ii)fofoT⊆ f and Tofof⊆ f and foTof⊆ f.

Lemma 2.25:[7]Let T be a po ternarysemigroup and ∅ ≠ A ⊆ T. Then A is a left ideal of T if and only if the characteristic mapping $f_A$of A is a fuzzy left ideal of T.

Lemma 2.26:[7]LetT be a po ternary semigroup and ∅ ≠ A ⊆ T. Then A is a right ideal of Tif and only if the characteristic mapping $f_A$of A is a fuzzy right ideal of T

Lemma 2.27:[7]Let Tbe a poternarysemigroup and ∅ ≠ A ⊆ T Then A is an ideal of T if and only if the characteristic mapping $f_A$of A is a fuzzy ideal of T.

Proposition 2.28:[13]Let f,g,h be fuzzy subsets of T. Then the following statements are true.
a.f ⊆ (f], ∀f ∈ F(T)
b. If f ⊆ g then (f] ⊆ (g]

c. $(f]o(g] \subseteq (fog], \forall f, g \in F(T)$ d. $(f] = ((f]], \forall f \in F(T)$
e. For any fuzzy ideal f of T $f = (f]$
f. If f,g are fuzzy ideals of T, then fog ,f∪g are fuzzy ideals of T.
g. $fo(g \cup h] \subseteq (fog \cup foh]$ h. $(g \cup h]of \subseteq (gof \cup hof]$.
i. If a_λ is an ordered fuzzy point of T, then $a_\lambda = (a_\lambda]$.

Definition 2.29:[13]Let T be a po ternary semigroup, $a \in T$ and $\lambda \in (0,1]$. An ordered fuzzy point **a_λ** ,$a_\lambda : T \to [0,1]$ defined by $a_\lambda(x) = \begin{cases} \lambda \text{ if } x \in (a] \\ 0 \text{ if } x \notin (a] \end{cases}$
clearly a_λ is a fuzzy subset of T. For every fuzzy subset f of T, we also denote $a_\lambda \subseteq f$ by $a_\lambda \in f$

Definition 2.30:[5] Let f be a fuzzy subset of X. Let $t \in [0,1]$. Define $f_t = \{x \in X / f(x) \geq t\}$. We call f_t a t-cut or a level set.

Definition 2.31:[12]A po (left/right/lateral) ideal of A of a po ternary semigroup T is said to be completely prime (left/right lateral) ideal of T provided $x, y, z \in T$ and $xyz \in A$ implies either $x \in A$ or $y \in A$ or $z \in A$..

III. FUZZY IDENTITY AND FUZZY ZERO OF A PO TERNARY SEMIGROUP:

Definition 3.1: Let f be a fuzzy subset of a po ternary semigroup TWe define [f) by $[f)(x) = \vee_{x \geq y} f(y), \forall x \in T$
where $y \in T$.

Proposition 3.2: Let f,g,h be fuzzy subsets of T Then the following statements are true.
a) $f \subseteq [f), \forall f \in F(T)$ b) If $f \subseteq g$ than $[f) \subseteq [g)$

Proof: a) $\forall x \in T$, Since $[f)(x) = \vee_{x \geq y} f(y)$
Since $x \geq x \Rightarrow [f)(x) = \vee_{x \geq x} f(x) \geq f(x)$
Therefore $f \subseteq [f)$

b) Let $f \subseteq g$ then $\forall x \in T$, $f(x) \leq g(x)$
Thus $[f)(x) = \vee_{x \geq y} f(y) \leq \vee_{x \geq y} g(y) = [g)(x)$
Therefore $[f) \subseteq [g)$.

DEFINITION 3.3: A ordered fuzzy point a_λ of a po ternary semigroup T is said to be fuzzy left identity of T if $a_\lambda ofof = f$ and $f \subseteq a_\lambda$, $\forall f \in F(T)$, $a \in T$ and $\lambda \in (0,1]$.

DEFINITION 3.4: A ordered fuzzy point a_λ of a po ternary semigroup T is said to be fuzzy right identity of T if $fofoa_\lambda = f$ and $f \subseteq a_\lambda$., $\forall f \in F(T)$, $a \in T$ and $\lambda \in (0,1]$.

DEFINITION3.5 :A ordered fuzzy point a_λ of a po ternary semigroup T is said to be fuzzy lateral identity of T if $foa_\lambda of = f$ and $f \subseteq a_\lambda$., $\forall f \in F(T)$, $a \in T$ and $\lambda \in (0,1]$.

DEFINITION3.6 : Afuzzy subset f of a po ternary semigroup T with identity is said to be fuzzy left identity of T if $fof_1 of_2 = f$ and $f_1 \subseteq f$, $f_2 \subseteq f$.$\forall f_1, f_2 \in F(T)$.

DEFINITION3.7 :A fuzzy subset f of a po ternary semigroupT with identity is said to be fuzzy lateral identity of T if $f_1 of \; of_2 = f$ and $f_1 \subseteq f$, $f_2 \subseteq f$. $\forall f_1, f_2 \in F(T)$.

DEFINITION3. 8 :A fuzz subset f of a po ternary semigroup T with identity is said to be fuzzy right identity of T if $f_1 of_2 of = f$ and $f_1 \subseteq f$, $f_2 \subseteq f$. $\forall f_1, f_2 \in F(T)$.

DEFINITION3.9: A *orderd fuzzy point* a_λ of a po ternary semi group T is said to be fuzzy zero of T if $a_\lambda ofof = foa_\lambda of = fofo\, a_\lambda = a_\lambda$ *and* $f \subseteq a_\lambda$, $\forall f \in F(T)$

THEOREM 3.10: If a_λ is a fuzzy po left zero , b_λ is a fuzzy po right zero and , c_λ po lateral zero of a poternarysemigroupTthen $a_\lambda = b_\lambda = c_\lambda$ where $\lambda \in [0,1]$.

Proof: since a_λ be fuzzy po left zero of T
$$a_\lambda ohog = a_\lambda \forall f, g, h \in F(T) \text{ and } a_\lambda \subseteq f$$
$$\Rightarrow a_\lambda oc_\lambda ob_\lambda = a_\lambda \text{ and } a_\lambda \subseteq f \quad \forall f \in F(T)$$
Since b_λ is a fuzzy po right zero of T $\Rightarrow fohob_\lambda = b\lambda$, $a\lambda oc\lambda ob\lambda = b\lambda$ *and* $b\lambda \subseteq g \forall g \in FT$

Since c_λ is a fuzzy po lateral zero of $\Rightarrow$ $foc_\lambda og$,$a_\lambda oc_\lambda ob_\lambda = c_\lambda$ *and* $c_\lambda \subseteq h, \forall h \in F(T)$
Therefore $a_\lambda oc_\lambda ob_\lambda = a_\lambda = b_\lambda = c_\lambda$.

THEOREM 3.11 :Any fuzzy po ternary semi group has atmost one fuzzy zero element

Proof:let $a_\lambda, b_\lambda c_\lambda$ .be any three fuzzy zeros of a po ternary semigroup T $\Rightarrow a_\lambda, b_\lambda, c_\lambda$ be considered as fuzzy left,lateral and fuzzy right zeros of T respectively.
By Theorem 3.10 $a_\lambda = b_\lambda = c_\lambda$
Therefore Fuzzy po ternary semigroup has at most one fuzzy po zero element.

Note 3.12: The zero fuzzy zero element of a po ternarysemigroup usually denotd by '0'.

IV. OPERATIONS ON FUZZY PO TERNARY SEMIGROUPS:

Definition 4.1: Let $\{f_i\}_{i \in I}$ be family of fuzzy subsets of an ordered ternary semigroup T where I is an index set. Define intersection and union $\underset{i \in I}{\cap} f_i$ and $\underset{i \in I}{\cup} f_i$ as follows.
$$(\underset{i \in I}{\cap} f_i) = \underset{i \in I}{\wedge} f_i(x) = \min\{f_1(x), f_2(x), \ldots \ldots \ldots\}, \forall x \in T.$$
$$\in T$$
$$(\underset{i \in I}{\cup} f_i)(x) = \underset{i \in I}{\vee} f_i(x) = \max\{f_1(x), f_2(x), \ldots \ldots \ldots\}, \forall x \in T$$

Definition 4.2: A fuzzy subset f of a po ternary semigroup T is called fuzzy po ternary subsemigroupof T if (i) $x \leq y \Rightarrow f(x) \geq$

$f(y)$(ii)$f(xyz) \geq f(x) \wedge f(y) \wedge f(z), \forall x, y, z \in T$.

Theorem 4.3: The intersection of two fuzzy po ternary subsemigrouops of a po ternary semigroup Tis a fuzzy po ternary subsemigroup of T.

Proof: Let f_1, f_2be two fuzzy po ternary sub semigroups of a po ternary semigroup T.

a) Consider $(f_1 \cap f_2)(xyz) = f_1(xyz) \wedge f_2(xyz) \geq f_1(x) \wedge f_1(y) \wedge f_2(x) \wedge f_2(y)) \wedge f_1(z) \wedge f_2(z) \geq f_1(x) \wedge f_2(x) \wedge f_1(z) \wedge f_1(y) \wedge f_2(y) \wedge f_2(z)$

$$\geq (f_1 \cap f_2)(x) \wedge (f_1 \cap f_2)(y) \wedge (f_1 \cap f_2)(z), \forall x, y, z \in T$$

b) Let $x \leq y$

Consider $(f_1 \cap f_2)(x) = f_1(x) \wedge f_2(x) \geq f_1(y) \wedge f_2(y) = (f_1 \cap f_2)(y)$

$\Rightarrow f_1 \cap f_2$is a fuzzy po ternary semigroup of T

Theorem 4.4: The intersection of arbitrary family of fuzzy po ternary sub semigroups of a po ternary semigroup T is a fuzzy po ternary sub semigroupof T.

Proof: Let $f_1, f_2, f_3 \ldots \ldots$be the family of fuzzy po ternary sub semigroups of a po ternary semigroup T

a) Consider $(\bigcap_{i \in I} f_i)(xyz) = f_1(xyz) \wedge f_2(xyz) \ldots \ldots \ldots$

$\geq f_1(x) \wedge f_1(y) \wedge f_2(x) \wedge f_2(y) \wedge f_1(z) \wedge f_2(z) \ldots \ldots \ldots$

$\geq f_1(x) \wedge f_2(x) \wedge f_1(z) \wedge f_1(y) \wedge f_2(y) \wedge f_2(z)$

$\geq (\bigcap_{i \in I} f_i)(x) \wedge (\bigcap_{i \in I} f_i)(y) \wedge (\bigcap_{i \in I} f_i)(z)$

b) Let $x \leq y$,

Consider $(\bigcap_{i \in I} f_i)(x) = f_1(x) \wedge f_2(x) \ldots \ldots \ldots \geq f_1(y) \wedge f_2(y) \wedge \ldots \ldots \ldots (\bigcap_{i \in I} f_i)(y)$

$\Rightarrow$ The intersection of arbitrary family of fuzzy po ternary sub semigroups of a poternarysemigroup T is a fuzzy po ternary sub semigroup of T.

Definition 4.5: Let$T$ be a po ternary semigroup and f be a fuzzy subset of T. The smallest fuzzy po ternary semigroup of Tcontaining f is called a fuzzy po ternary semigroup of T generated by f.It is denoted by (f).

Theorem 4.6: Let Tbe a po ternary semigroup and f is a fuzzy subset of T.Then (f) =The intersection of all fuzzy po ternary semigroups of T containing f.

Proof: Let $\Delta = \{g/g \text{ is a fuzzy po semigroup of } T \text{ and } f \subseteq g\}$ since T itself is a fuzzy po ternary semigroup and $f \subseteq T$

$$\Rightarrow T \in \Delta \Rightarrow \Delta \neq \emptyset$$

Let $F^* = \bigcap_{g \in \Delta} g_1 \Rightarrow F^* \neq \emptyset$ by Theorem 4.3, F^* is a fuzzy poternarysemigroupof T.

Since$F^* \subseteq g_1, \forall g_1 \in \Delta$, F^* is the smallest fuzzy po ternary semigroup of Tcontaining f.

Therefore$F^* = (f)$.

V. FUZZY IDEALS OF PARTIALLY ORDERED TERNARY SEMIGROUPS:

Theorem 5.1: A fuzzy subset f of a po ternary semigroup T is a fuzzy left ideal of T iffiT$oTof \subseteq f$ (ii)$(f] \subseteq f$

Proof: Le t f be fuzzy left ideal.

(i) Consider $(ToTof)(x) = \bigvee_{x \leq pqr} [T(p) \wedge T(q) \wedge f(r)] = \bigvee_{x \leq pqr} f(r) \leq f(x) \Rightarrow ToTof \subseteq f$

(ii) Consider $(f](x) = \bigvee_{x \leq y} f(y), \forall x \in T$

$\Rightarrow (f](x) = \bigvee_{x \leq y} f(y) \leq f(x)$ since $x \leq y \Rightarrow f(x) \geq f(y), \forall x, y, z \in T$

Therefore $(f] \subseteq f$

Conversely, suppose that (i) $ToTof \subseteq f$(ii)$(f] \subseteq f$

a) Consider $f(xyz) = f(a)$ where $a = xyz$

$\geq (ToTof)(a) = \bigvee_{a \leq pqr} T(p) \wedge T(q) \wedge f(r) \geq T(x) \wedge T(y) \wedge f(z) = 1 \wedge f(z) = f(z)$

$$\Rightarrow f(xyz) \geq f(z), \forall x, y, z \in T$$

b) Let $x \leq y \Rightarrow f(x) \geq (f](x) = \bigvee_{x \leq y} f(y) \geq f(y)$since $(f] \subseteq f$

$\Rightarrow f(x) \geq f(y)$

Therefore f is a fuzzy left ideal of T.

Theorem 5.2: A fuzzy subset f of a po ternary semigroup Tis a fuzzy lateral ideal of Tiff (iTo$f o\ T \subseteq f$

(ii)$(f] \subseteq f$

Proof: Proof follows from Theorem 5.1.

Theorem 5.3: A fuzzy subset f of a po ternary semigroup Tis a fuzzy **right** ideal of Tiff (ifo$ToT \subseteq f$ (ii)$(f] \subseteq f$

Corollary 5.4: A fuzzy subset f of a poternary semigroup T is a fuzzy ideal of T iff (i)$foToT \subseteq f, ToTof \subseteq f$

,TofoT (ii)$(f] \subseteq f$

Corollary 5.5: Let f be a fuzzy subset of a po ternary semigroup T. If f is fuzzy left (right ,lateral) ideal of Tthen$f = (f]$.

Theorem 5.6: The intersection of any two fuzzy left ideals of a po ternary semigroupT is a fuzzy left ideal of T

Proof: Let f, g be any two fuzzy ideals of T

$\Rightarrow f, g$ are fuzzy left ideals.

a) Consider $(f \cap g)(xyz) = f(xyz) \wedge g(xyz) \geq f(z) \wedge g(z) = (f \cap g)(z)$

$$\Rightarrow (f \cap g)(xyz) \geq (f \cap g)(z)$$

b) Let $x \leq y \Rightarrow f(x) \geq f(y)$ and $g(x) \geq g(y)$
Consider $(f \cap g)(x) = f(x) \wedge g(x) \geq f(y) \wedge g(y) = (f \cap g)(y)$
$\Rightarrow (f \cap g)$ is a fuzzy left ideal of T

Theorem 5.7: The intersection of arbitrary family of fuzzy left(right ,lateral) ideals of a po ternary semigroupT is a fuzzy left(right , lateral) ideal of T.

Proof: Let $\{f_\alpha\}_{\alpha \in \Delta}$ be a family of fuzzy left ideals of T and $f = \bigcap_{\alpha \in \Delta} f_\alpha$.
Let$x, y, z \in T$. Consider $f(xyz) = \bigcap_{\alpha \in \Delta} f_\alpha(xyz) \geq \bigcap_{\alpha \in \Delta} f_\alpha(z) = f(z)$
$\Rightarrow f(xyz) \geq f(z)$
Let$x \leq y$. Consider$f(x) = \bigcap_{\alpha \in \Delta} f_\alpha(x) \geq \bigcap_{\alpha \in \Delta} f_\alpha(y) = f(y)$
$\Rightarrow f(x) \geq f(y)$
Therefore the intersection of arbitrary family of fuzzy left ideals of a po ternarysemigroup is a fuzzy left ideal of$T$.
Similarly the intersection of arbitrary family of fuzzy left (right , lateral) ideals of a poternary semigroup is a fuzzy left (right , lateral) ideal of T.

Theorem 5.8: The union of any two fuzzy left (right , lateral) ideals of a po ternary semigroup$T$ is a fuzzy left(right , lateral) ideal of T

Proof: Let f, g be any 2 fuzzy left ideals of a poternary semigroup T

(i) Let$x, y, z \in T$. Consider $(f \cup g)(xyz) = f(xyz) \vee g(xyz) \geq f(z) \vee g(z) = (f \cup g)(z)$

$\Rightarrow (f \cup g)(xyz) \geq (f \cup g)(z)$
(ii) Let $x \leq y \Rightarrow f(x) \geq f(y)$ and $g(x) \geq g(y)$

Consider $(f \cup g)(x) = f(x) \vee g(x) \geq f(y) \vee g(y) = (f \cup g)(y)$

$\Rightarrow (f \cup g)(x) \geq (f \cup g)(y)$
Therefore the union of any two fuzzy left ideals is a fuzzy left ideal of T.

Similarly, the union of any two fuzzy right(right , lateral) ideals of a po ternarysemigroup$T$ is a fuzzy right left ,
lateral ideal of T.

Theorem 5.9: The union of arbitrary family of fuzzy left (right , lateral) ideals of a po ternary semigroup$t$ is a fuzzy left(right , lateral) ideal o fT

Proof: Let $\{f_\alpha\}_{\alpha \in \Delta}$ be a family of fuzzy left ideal of T and$f = \bigcup_{\alpha \in \Delta} f_\alpha$.
Let$x, y, z \in T$. Consider $f(xyz) = \bigcup_{\alpha \in \Delta} f_\alpha(xyz) \geq \bigcup_{\alpha \in \Delta} f_\alpha(z) = f(z)$

$\Rightarrow f(xyz) \geq f(z)$
Let $x \leq y$. Consider $f(x) = \bigcup_{\alpha \in \Delta} f_\alpha(x) \geq \bigcup_{\alpha \in \Delta} f_\alpha(y) = f(y)$

Therefore the union of arbitrary family of fuzzy left ideals of a po ternary semigroup is a fuzzy left ideal of T.
Similarly the union of arbitrary family of fuzzy right(left,lateral) ideals of a po ternarysemigroup is a fuzzy left (right , lateral) ideal of T.

VI. COMPLETELY PRIME FUZZY IDEAL AND PRIME FUZZY IDEALS:

Definition 6.1: A po (left/right/lateral) ideal of A of a po ternary semigroup T is said to be completely prime (left/right/ lateral) ideal of T provided $x, y, z \in T$ and $xyz \in A$ implies either $x \in A$ or $y \in A$ or $z \in A$.

Definition 6.2: Let T be an po ternary semigroup, $a \in T$and$\lambda \in [0,1]$. An ordered fuzzy point a_λ of T defined by$a_\lambda(x) = \begin{cases} \lambda \text{ if } x \in (a] \\ 0 \text{ if } x \notin (a] \end{cases}$
clearly $a_\lambda$is a fuzzy subset of T. For every fuzzy subset f of T, we also denote $a_\lambda \subseteq f$ by $a_\lambda \in f$

Definition 6.3: A fuzzy ideal f of a po ternary semigroup T is called completely prime fuzzy idealif∀ three ordered fuzzy points $x_t, y_r, z_s \in$ T ($\forall t, r, s \in (0,1]$) such that $x_t o\, y_r\, o\, z_s \subseteq f$ then $x_t \subseteq f$ or $y_r \subseteq f$ or $z_s \subseteq f$.

Definition 6.4: A fuzzy ideal f of a po ternary semigroup T is called weakly completely prime fuzzy ideal if∀three ordered fuzzy points $x_t, y_t, z_t$of T ($\forall t \in (0,1]$) such that $x_t o\, y_t\, o\, z_t \subseteq f$ then $x_t \subseteq f$ or $y_t \subseteq f$ or $z_t \subseteq f$

Theorem 6.5: Let f be a fuzzy ideal of po ternary semigroup T. If f is completely prime fuzzy then f is weakly completely prime fuzzy ideal.

Proof: Let fbe completely prime fuzzy ideal of T. Let x_t, y_t, z_t be any three fuzzy points of T such that $x_t\, o\, y_t\, o\, z_t \subseteq f$, $t \in (0,1]$. ⇒ Either $x_t \subseteq f$ or $y_t \subseteq f$ $or z_t \subseteq f$sincefis completely prime fuzzy ideal.
⇒f is weakly completely prime fuzzy ideal.

Theorem 6.7: Let f be fuzzy ideal of a po ternarysemigroupT. If $f_t$is completely prime ideal of a po ternarysemigroup T, $\forall t \in (0,1]$ then f is weakly completely prime fuzzy ideal of T.

Proof: Assume that f_t is completely prime ideal of T.
Let for any three fuzzy points x_t, y_t, z_t $of\ T \ni$ $x_t o y_t o\, z_t \subseteq f$ for $t \in (0,1]$
Let $f(abc) = l$ $where$ $l \in (0,1] \Rightarrow abc \in f_l$ since f_l is completely prime
⇒ either $a \in f_l or\ b \in f_l\ or\ c \in f_l \Rightarrow f(a) \geq l$ or $f(b) \geq l$ or $f(c) \geq l$
Now $\forall t, x_t o y_t o\, z_t \subseteq f \Rightarrow (x_t o y_t o\, z_t)(abc) \leq f(abc) = l$
$\Rightarrow (x_t o y_t o\, z_t)(abc) = \vee\ [x_t(a) \wedge y_t(b) \wedge$ $zt(c) \leq t, \forall a,b,c \in$T

$\Rightarrow x_t(a) \wedge y_t(b) \wedge z_t(c)$
$\leq \vee (x_t(a) \wedge y_t(b) \wedge z_t(c) \leq t$
$\leq f(abc) = l$
$\Rightarrow x_t(a) \wedge y_t(b) \wedge z_t(c) \leq l \quad ,\forall a, b, c \in T$
$\Rightarrow x_t(a) \leq l$ or $y_t(b) \leq l$ or $z_t(c) \leq l$

$\Rightarrow x_t(a) \leq l \leq f(a)$ or $y_t(b) \leq l \leq f(b) or\, z_t(c) \leq l \leq f(c) \Rightarrow x_t \subseteq f$ or $y_t \subseteq f, z_t \subseteq f \ \forall a, b, c \in T.$
Therefore f is weakly completely prime fuzzy ideal of T.

Theorem 6.8: Let f be a fuzzy ideal of po ternary semigroupT If $f_t$is completely prime ideal of T then f is
completely prime fuzzy ideal ofT.

Proof: Assume that $f_t$is completely prime ideal of T let $a, b, c \in T \Rightarrow abc \in T.$ Now $(x_t o y_r\, o\, z_s)(abc) \leq f(abc)$
Let $f(abc) = min\{t, r, s\} = u(say) \Rightarrow abc \in f_u$
Since f_u is completely prime ideal of T.
⇒Either $a \in f_u$ or $b \in f_u or\, c \in f_u \Rightarrow f(a) \geq u$ or $f(b) \geq u$ or $f(c) \geq u$
clearly $x_t(a) \wedge y_r(b) \wedge z_s(c) \leq \vee [x_t(a) \wedge y_r(b) \wedge z_s(c)]$

$= (x_t o y_r o\, z_s)(abc) \leq t \wedge r \wedge s$
$= u$
$\Rightarrow x_t(a) \wedge y_r(b) \wedge z_s(c) \leq u$
$\Rightarrow x_t(a) \leq u \leq f(a)$ or $y_r(b) \leq u \leq f(b)\ or\ z_s(c) \leq f(c) \leq u$
$\Rightarrow x_t \subseteq f$ or $y_r \subseteq f,\ z_s \subseteq f$
Therefore f is completely prime fuzzy ideal of T.

Definition 6.9: Let T be a po ternary semigroup. A fuzzy ideal f of T is said to be fuzzyprime if $\forall\ three$ fuzzy idealsg, h and $i$of T$gohoi \subseteq f$ then either $g \subseteq f$or $h \subseteq f\ or\ i \subseteq f$.

Definition 6.10: The fuzzy ideal generated by a_λ, denoted by (a_λ) is

$$\forall x \in T, (a_\lambda)(x) = \begin{cases} \lambda\ if\ x \in (a) \\ 0\ if\ x \notin (a) \end{cases}$$

where (a) is an ideal of T generated by a,

$$i.e\ (a) = (a \cup aaT \cup aTa \cup Taa \cup TaTaT] = (T^1 a T^1\, a\, T^1]$$

Definition 611: Let $a_\lambda$be ordered fuzzy point of T ,$\lambda \neq 0$

$$\forall\ x \in T, (Toa_\lambda oT)(x) = \begin{cases} \lambda\ if\ x \in (TaT] \\ 0\ if\ x \notin (TaT] \end{cases}$$ and

$Toa_\lambda oT$ is a fuzzy ideal of T

Theorem 6.12: Let f be a fuzzy ideal of a poternary semigroupT Then the following are equivalent
(a) f is fuzzy prime ideal
(b) $\forall three$ ordered fuzzy points $x_r, y_t, z_s\ of\ T\ if < x_r > o < y_t > o < z_s > \subseteq f$then $x_r \subseteq f$ or $y_t \subseteq f\ or\, z_s \subseteq f.\quad r, t, s \in (0,1]$
(c) $\forall three$ ordered fuzzy points $x_r, y_t, z_s\ of\ T$ if $To\, x_r o\, To\, y_t\ o\ T\ oz_s \subseteq f$
$\Rightarrow\ x_r \subseteq for\, y_t \subseteq f\ or\ z_s \subseteq f \quad (rts > 0)$

Proof: $(\boldsymbol{a}) \Rightarrow (\boldsymbol{b})$:
Let f be a fuzzy prime ideal of T.
Let x_r, y_t, z_sbe any three ordered fuzzy points of T $\ni t, r, s \in (0,1] \& < x_r > o < y_t > o < z_s > \subseteq f$
Since $< x_r >, < y_t >< z_s >$ are fuzzy ideals of T
Either $x_r \subseteq f\ or\ y_t \subseteq f\ or\, z_s \subseteq f$
$(\boldsymbol{b}) \Rightarrow (\boldsymbol{c})$:
Assume that $\forall\ three$ ordered fuzzy points $x_r, y_t, z_s\ of\ T\ if < x_r > o < y_t > o < z_s > \subseteq f$ then either $x_r \subseteq f$ or $y_t \subseteq f\ \ or\ \ z_s \subseteq f$.
Suppose $To\, x_r o\, To\, y_t\ o\ T\ oz_s \subseteq f$
$\Rightarrow To\, x_r o\, To\, y_t\ o\ T\ oz_s \subseteq f$since(T$ox_r oT)o(Toy_t oT)\ \ o\ (Toz_s\ \ oT) \leq r \wedge t \wedge s$
and also $< x_r > o < y_t > o < z_s > \subseteq r \wedge t \wedge s \implies x_r \subseteq f\ or\ y_t \subseteq for\ \ z_s \subseteq f$
$(\boldsymbol{c}) \Rightarrow (\boldsymbol{a})$:
Assume that for any three ordered fuzzy points x_r, y_t, z_s of $Tand\ Tox_r oToy_t oToz_s oT \subseteq f$
then $x_r \subseteq f\ or\ y_t \subseteq f\ or\, z_s \subseteq f(rts > 0)$
Let g,h ,i be three fuzzy ideals of T $\ni gohoi \subseteq f$
if possible suppose that $g \nsubseteq f$,$h \nsubseteq f\ andi \nsubseteq f$ then $\exists\, x, y, z \in T \ni g(x) > f(x)$and $h(y) > f(y)$, i(z)> $f(z)$,
Let $r = g(x), t = h(y), s = i(z)$then $r, t, s > 0, (rts > 0), x_r \subseteq f, y_t \subseteq g\, , z_s \subseteq h$ then $Tox_r oToy_t oToz_s oT$
$\subseteq (Tox_r oT)o(Toy_t oT)o(\,Toz_s\ \ oT)$
$\leq (x_r)o(y_t)o(z_s) \subseteq gohoi \subseteq f$
therefore either $x_r \subseteq f$ or $y_t \subseteq f$ or$z_s \subseteq f$
say $x_r \subseteq f\ then\ f(x) \geq t = g(x)\ \Rightarrow f \supseteq g$ a contradiction
⇒ either $g \subseteq f\ or\ h \subseteq for\ i \subseteq f$
therefore f is a fuzzy prime ideal of T.

Theorem 6.13: Let f be a fuzzy ideal of a po ternarysemigroupT. f is fuzzy prime iff fuzzy ideals $f_1, f_2, f_3, \ldots\ldots f_n, n \in N$, if $f_1 o f_2 o \ldots\ldots o f_n \subseteq f \Rightarrow f_i \subseteq f$ for some $i = 1,2,3, \ldots\ldots, n$

Proof: Let f be fuzzy prime, assume that $f_1 o f_2 o \ldots\ldots o f_n \subseteq$
f where$f_1, f_2, f_3, \ldots\ldots f_n$are fuzzy ideals if n=1 then clearly $f_1 \subseteq f$
if n=3 then $f_1 o f_2 o f_3 \subseteq f$ since f is fuzzy prime, either $f_1 \subseteq f\ or\ f_2 \subseteq for\, f_3 \subseteq f$
by induction on n, $f_1 o f_2 o \ldots\ldots o f_n \subseteq f \Rightarrow f_i \subseteq f\ forsome\ i = 1,2,3, \ldots\ldots, n$
conversely suppose if $f_1 o f_2 o \ldots\ldots o f_n \subseteq f \Rightarrow f_i \subseteq f\ forsome\ i = 1,2,3, \ldots\ldots, n$

since $n \in N\ take\ n = 3$ then clearly f is fuzzy prime.

Theorem 6.14: Let f be a fuzzy ideal of a po ternarysemigroupT. Then if f is completely prime fuzzy then f is a fuzzy prime ideal of T and a weakly completely prime fuzzy ideal.

Theorem 6.15: LetT be a commutative po ternarysemigroup and f is a fuzzy po ideal of T. Then f is completely prime fuzzy if and only if f is fuzzy prime

Definition 6.16: Let f be a fuzzy subset of a po ternary semi group T f is said to be fuzzy m-systemof T provided if $f(x) > t_1, f(y) > t_2\ f(z) > t_3 \Rightarrow \exists\ c, s, t \in T \ni f(c) > t_1 \vee t_2 \vee t_3\ and\ c\ \leq xsytz$

Theorem 6.17: A fuzzy ideal fof a po ternary semigroup T is a prime idealiff$1 - f$ is a fuzzy m-system of T provided $1 - f \neq \emptyset$.

Proof: Suppose that f is a fuzzy prime ideal of T.
Let $\forall t, s, r \in [0,1], a, b, c \in T$ if $(1 - f)(a) > t, (1 - f)(b) > s\ (1 - f)(c) > r$
$\Rightarrow f(a) < 1 - t \Rightarrow a_{1-t} \nsubseteq f$ then $a_{1-t} \nsubseteq f$,$b_{1-s} \nsubseteq f$, $c_{1-r} \nsubseteq f$

$\Rightarrow a_{1-t} o b_{1-s} o c_{1-r} \nsubseteq f$ Now
$a_{1-t} o f_{\{x\}} o b_{1-s} o c_{1-r} = (axbyc)_{(1-t)\wedge(1-s)\wedge(1-r)} \nsubseteq f$

$$\Longrightarrow f(axbyc) < (1 - t) \wedge (1 - s) \wedge (1 - r) = 1 - (t \vee s \vee r)$$

$$(1 - f)(axbyc) > t \vee s \vee r$$

Therefore $1 - f$is a fuzzy m-system of T.
conversely suppose $1 - f$ is a fuzzy m-system of T
Let g,h,i be three fuzzy ideals of T $\ni gohoi \subseteq f$
Suppose $g \nsubseteq f$, $h \nsubseteq f, i \nsubseteq f$
$\Rightarrow \exists$ an ordered fuzzy points $x_\lambda \in g, y_\mu \in h, z_\eta \in i\ and\ x_\lambda \notin f\ and\ y_\mu \notin f, z_\eta\ \notin f$
$\Rightarrow f(x) < \lambda, f(y) < \mu\ and\ f(z) < \eta$
$\Rightarrow (1 - f)(x) > 1 - \lambda, (1 - f)(y) > 1 - \mu, (1 - f)(z) > 1 - \eta$
$\Rightarrow \exists c, t, s \in T \ni (1 - f)(c) > (1 - \lambda) \vee (\ 1 - \mu) \vee 1 - \eta = 1 - (\lambda \wedge \mu \wedge \eta)\ and\ c \leq xtysz$
$\Rightarrow f(c) < \lambda \wedge\ \mu \wedge \eta$ since $c \leq xtysz \Rightarrow f(c) \geq f(xtysz) \Rightarrow f(xtysz) <\ \lambda \wedge \mu \wedge \eta$
but $x_\lambda \subseteq g, y_\mu \subseteq h\ and z_\eta \subseteq i\ \Rightarrow x_\lambda o y_\mu o z_\eta \subseteq gohoi \subseteq f$

$$\Rightarrow x_\lambda o y_\mu o z_\eta \subseteq f \Rightarrow (x_\lambda o y_\mu o z_\eta)(t) \leq f(t), \forall t \in T \Rightarrow\ \lambda \wedge\ \mu \wedge \eta \leq f$$

But $xtysz \in S\ and\ f(xtysz) <\ \lambda \wedge\ \mu \wedge \eta$
$\Rightarrow \lambda \wedge\ \mu \wedge \eta > f(xtysz)$which is contradiction
Therefore either $g \subseteq f\ or\ h \subseteq f$ ori $\subseteq$ f
$\Rightarrow$ f is fuzzy prime.

Definition 6.18: The po ternarysemigroup T itself is a fuzzy subset of T $\ni \mathrm{T}(\mathrm{x}) = 1, \forall \mathrm{x} \in \mathrm{T}$. It is denoted by 1 or T

Definition 6.19: A fuzzy ideal f of a po ternarysemigroup T is called maximal if there doesn't exist any proper fuzzy ideal g ofT $\ni f \subset g$.

Theorem 6.20: Let T be a poternarysemigroup. Every maximal fuzzy ideal f of T is a prime fuzzy ideal of T if $f^3 = f$.

Proof: Let f be a fuzzy maximal ideal of T. Let g, h, i be tthree fuzzy ideals of $T \ni gohoi \subseteq f$
Suppose if possible $g \nsubseteq f$, $h \nsubseteq f$andi $\nsubseteq f$
$\Rightarrow g \cup f$ is a fuzzy ideal of T and $f \subset f \cup g \subseteq T = 1$ since f is maximal, $g \cup f = T = 1$. Similarly if $h \nsubseteq f$ then $h \cup f = T = 1$
Now $T = ToToT = (g \cup f) \circ (h \cup f) o (i \cup f) = (gohoi) \cup f$
$=f \cup f$
$\Rightarrow T \subseteq f$ it is a contradiction
Therefore either $g \subseteq f$ or $h \subseteq f$
$\Rightarrow$ f is fuzzy prime ideal of T

Definition 6.21: Let T be a poternarysemigroup. T is called fuzzy semi simple if $\forall t \in (0,1]$ if $a_t \in (< a_t >^3]$.

Theorem 6.22: Let T be a po ternarysemigroup and f is maximal fuzzy ideal of T such that $f = f^3$ then f is fuzzy semisimple.

Proof: Let f be maximal fuzzy ideal such that $f = f^3$
$\Rightarrow$ f is fuzzy prime ideal.
If $a_t \in 1 - f$ then $< a_t > \nsubseteq f \Rightarrow < a_t >^3 \nsubseteq f$ since f is fuzzy prime.
Now $1 = f \cup < a_t > = f \cup < a_t >^3$since f is fuzzy maximal.
Therefore $a_t \in < a_t >^3$. $\Rightarrow$ T is fuzzy semi simple.

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