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Existence of Solution for Random Differential Inclusions In Banach Space

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ABSTRACT: In this paper, we discuss the existence result for second order random differential inclusions defined in a separable Banach Space using random fixed point theorem.
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I. INTRODUCTION

Some notations are as

$$P_{fc}(X)=\{A\subseteq X: nonempty, closed, (convex)\},$$

$$P_{(w)k(c)}(X)=\{A\subseteq X: nonempty, (w)compact, (convex)\}.$$

Let $F:\Omega\to P_f X$. We say that $F(\cdot)$ is measurable if any of the following equivalent conditions holds:

(i) for all $x\in X, \omega\to d_{F(\omega)}(X)=\inf_{z\in F(\omega)}\|x-z\|$ is measurable.

(ii) there exist $\{f_n(\cdot)\}_{n\geq 1}$ measurable selectors of $F(\cdot)$ such that $F(\omega)=cl\{f_n(\omega)\}_{n\geq 1}$ for all $\omega\in\Omega$ (Castaing's representation),

(iii) for all $U\subseteq X$ open, $\{\omega\in\Omega: F(\omega)\cap U\neq\phi\}\in\Sigma$.

For a measurable $F(\cdot)$, we denote by S_F^1 the set of all measurable selectors of $F(\cdot)$ contained in $L_X^1(\Omega)$ i.e. $S_F^1=\{f(\cdot)\in L_X^1(\Omega): f(\omega)\in F(\omega)\ \mu\, a.e.\}$.

II. EXISTENCE RESULTS

N.S.Papageorgoiu in [6] proved the following result about weak compactness in the Lebesgue Bochner space $L_X^1(\Omega)$. We apply result to prove our main result..Assume that (Ω, Σ, μ) is a σ-finite measure space and X a separable Banach space.

Theorem 2.1. If F:Ω→Pwkc(X) is integrably bounded, then S_F^1 is a non-empty, convex, w-compact subset of $L_X^1(\Omega)$.

Theorem 2.2. Let (Ω, ∈, μ) be a σ- finite measure space, Y a locally compact separable metric space and Z a metric space. Then f: Ω xY→Z is a Caratheodory function if and only if r(ω) (.) = f(ω) from Ω into C(Y,Z) is measurable, where C(Y,Z) is the space of continuous functions from Y into Z, endowed with the compact-open topology.

This result,was proved by the N.S.Papageorgoiu in [7, Theorem 3.3]. let[0, T] be a bounded interval with the Lebesgue measure dt and B(T) the σ-field of Borel sets. X is a seperable Banach space y(.) is the Hausdroff measure of noncompactness and $w:T\times R_+\to R_+$ is a Kamke function(Caratheodory function) s.t. forall $x\in X, w(t,x)\leq\varphi(t)$ a.e. for all $\varphi(\cdot)\in L_X^1, w(t,0)=0$ and $u(\cdot)\equiv 0$ is the only solution of $u(t)\leq\int_0^t(s,u(s))ds, u(0)=0$. We will assume that for all $t\in T, w(t,\cdot)$ is non-decreasing.

Theorem 2.3 . If F:T xX→Pk(X) is a multifunction s.t.

(1).For all x∈X,F (. , x) is a measurable and

$$|f(t,x)|\leq a(t)\|x\|+b(t)\text{ a.e., where } a(\cdot),b(\cdot)\in L_+^1$$

(2). For all t∈T, F(t, .) is Hausdorff continuous

(3). For all B⊆X bounded we have that $\gamma(F(t,B)) \le w(t,\gamma,(B))$ a.e.,

III. MAIN RESULT

Consider (Ω,Σ,μ) will Be a finite measure space, [0, T] a bounded closed interval in R_+ with the Lebesgue measure and $\lambda(.,)$ and X a seperable Banach space. We consider second order random differential inclusion

$$x''(\omega,t) \in F(\omega,t,x(\omega,t)) \qquad (3.1)$$

$$x(\omega,0), x'(\omega,0) \in G(\omega))$$

A solution of (3.1),we have X-valued stochastic process $x:\Omega\times T \to X$ which has absolutely continuous paths.

Theorem: 3.1. If F: Ω x T x X→P_{kc}X is a multifunction s.t.

(1) For all x ∈ X, $F(\cdot,\cdot,x)$ is jointly measurable and $|F(\omega,t,x| \le a(\omega,t)\lambda$ a.e.with $a(\cdot,\cdot)$, $b(\cdot,\cdot)$,measurable and for all ω∈Ω, $a(\omega,\cdot)$, $b(\omega,\cdot) \in L^1_+$,

(2) For all $((\omega,t) \in \Omega\times T$, $F(\omega,t,\cdot)$ is Hausdorff continuous

(3) For all B⊆X bounded we have that

$$\gamma(F(\omega,t,B)) \le w(\omega,t,\gamma,B)\lambda \text{ a.e.}$$

Where $w(\cdot,\cdot,\cdot)$ is a Kamke function jointly measurable in $(\omega,t) \in \Omega\times T$ and if $G:\Omega \to P_{fc}(X)$ is measurable then

$$x''(\omega,t) \in F(\omega,t,x(\omega,t))$$

$$x(\omega,0), x'(\omega,0) \in G(\omega))$$

admits random solution.

Proof: Consider the $R:\Omega \to 2^{C_c(T)}$ multifunction defined by

$R(\omega)=\{x(\cdot)\in C_c(T): x(t) \in F(\omega,t,x(t))\ a.e.x(0),x'(0) \in G(\omega)\}$

From Theorem 2.3 we know that for all $\omega \in \Omega, R(\omega) \ne \phi$. we claim that it is also closed.

Fix ω∈Ω and consider $\{x_n(\cdot)\}_{n\ge1} \subseteq R(\omega), x_n(\cdot) \to C_c(T)x(\cdot)$. So $\sup_{n\ge1}\|x_n(\cdot)\|_\infty \le M < \infty$ for all and all we have that and But from the dominated coverage theorem we have that $\sup_{n\ge1}\|x_n(\bullet)\|_\infty \le M < \infty$.

For all t', t∈T and all n≥1. we have that $x(0), x'(0) \in G(\omega)$ and

$$x_n(t) \in x_0 + x_1(t')t + (t-s)\int_{t'}^{t} F(\omega,s,x_n(s))ds$$

$$x(t) \in x_0 + x_1(t')t + t\int_{t'}^{t} F(\omega,s,x(s))ds - s\lim_{n\to\infty}\int_{t'}^{t} F(\omega,s,x_n(s))ds.$$

Using the dominated convergence theorem ,we have

$$\int_{t'}^{t} F(\omega,s,x(s))ds$$

Thus for all t', t∈T we get that

$$x(t) \in x_0 + x_1(t')t + (t-s)\int_{t'}^{t} F(\omega,s,x(s))ds.$$

$$x''(t) \in F(\omega,s,x(t)) \text{ a.e. } x(0), x'(0) \in G(\omega)$$

then the problem $x''(\omega,t) \in F(\omega,t,x(\omega,t))$, $x(\omega,0) = x_0, x'(\omega,0) = x_1$ admits random solution.

Hence we have that for all $\omega \in \Omega, R(\omega) \in P_f(X)$. Rewrite $R(\cdot)$ as follows.

$$R(\omega) = \{x(\cdot) \in C_c(T): x(t) \in x_0 + x_1(t')t + (t-s)\int_{t'}^{t} F(\omega,s,x(s))ds \text{ for all } t', t \in T\}$$

Note that by Theorem 2.1, for all Hence thus we have for all $\omega \in \Omega$, $\int_{t'}^{t} F(\omega,s,x(s))ds \in P_{wkc}(X)$.

Hence $x_0 + x_1(t')t + (t-s)\int_{t'}^{t} F(\omega,s,x(s))ds \in P_{fc}(X)$.

Thus we have for all $\omega \in \Omega$,

$$R(\omega) = \{x(\cdot) \in C_c(T): d_{x(t)\in x_0+x_1(t')t+(t-s)\int_{t'}^{t} F(\omega,s,x(s))ds}(x(t)) = 0 \text{ for all } t', t \in T\}$$

Set

$$\varphi(\omega,t',t,x(\cdot)) = d_{x(t)\in x_0+x_1(t')t+(t-s)\int_{t'}^{t} F(\omega,s,x(s))ds}(x(t))$$

For all x*∈X* we have that

$$\sigma_{x(t)\in x_0+x_1(t')t+(t-s)\int_{t'}^{t} F(\omega,s,x(s))ds}(x^*) = (x^*, x(t')) + \sigma_{\int_{t'}^{t} F(\omega,s,x(s))ds}(x^*)$$

$$= (x^*, x(t')) + \int_{t'}^{t} \sigma_{F(\omega,s,x(s))ds}(x^*)$$

observe that $(\omega,s) \to \sigma_{F(\omega,s,x(s))ds}(x^*)$ is measurable and for all $\omega \in \Omega$, $\omega \to \int_{t'}^{t} \sigma_{F(\omega,s,x(s))ds}(x^*) \in L^1$. Hence deduce that $\omega \to \int_{t'}^{t} \sigma_{F(\omega,s,x(s))ds}(x^*)$ is

Measurable.

$$\Rightarrow \omega \to \sigma_{x(t')+\int_{t'}^{t} \sigma F(\omega,s,x(s))ds}(x^*)$$

is measurable and thus by Theorem III.37 of Castaing-Valadier [2] implies that

$$\omega \to x(t') + \int_{t'}^{t} F(\omega,s,x(s))ds$$

is $\hat{\Sigma}$ – measurable for all $t \in T$ (where $\hat{\Sigma}$ is the completion of Σ with respect to $\mu(\cdot)$)

$\Rightarrow \omega \to \varphi(\omega,t',t,x(\cdot))$ is $\hat{\Sigma}$ – measurable.

Next ,we claim that for all $\omega \in \Omega$, $(t',t,x(\cdot)) \to \varphi(\omega,t',t,x(\cdot))$ is continuous on $T\times T\times C_c(T)$. So let $(t'_n, t, x_n(\cdot)) \to (t', t, x(\cdot))$ in $T \times C_c(T)$

We have

$$|\varphi(\omega,t'_n,t,x_n(\cdot)) - \varphi(\omega,t',t,x(\cdot))|$$

$$= \left|d_{x_n(t'_n)+\int_{t'_n}^{t} F(\omega,s,x_n(s))ds}(x_n(t)) - d_{x(t')+\int_{t'}^{t} F(\omega,s,x(s))ds}(x(t))\right|$$

$$\le \|(x_n(t)) - (x(t))\|$$

$$+h[x_n(t_n^{'})+\int_{t_n^{'}}^{t_n} F(\omega,s,x_n(s)ds, x(t')+\int_{t'}^{t} F(\omega,s,x(s)ds]$$

$$\leq \|(x_n(t))-(x(t))\|+\|(x_n(t^{'}))-(x(t^{'}))\|$$

$$+h\left[\int_T \chi_{(t_n^{'},t_n))}(S)F(\omega,s,x_n(s))ds, \int_T \chi_{(t^{'},t))}(S)F(\omega,s,x(s))ds\right]$$

$$\leq \|(x_n(t))-(x(t))\|++h\left[\int_T \chi_{(t_n^{'},t_n))}(S)F(\omega,s,x_n(s))ds, \chi_{(t^{'},t))}(S)F(\omega,s,x(s))ds\right]$$

$$+h\left[\chi_{(t_n^{'},t_n))}(S)F(\omega,s,x_n(s))ds, \chi_{(t^{'},t))}(S)F(\omega,s,x(s))ds\right]+\|(x_n(t_n^{'}))-(x(t^{'}))\|$$

$$=\|(x_n(t_n^{'}))-(x(t'))\|+\left[\begin{array}{l}\int_T \left|\chi_{(t_n^{'},t_n))}(S)F(\omega,s,x_n(s))ds, \chi_{(t^{'},t))}\right| |F(\omega,s,x_n(s))| \\ ++\chi_{(t^{'},t))}(S)hF(\omega,s,x_n(s)),(F(\omega,s,x(s)))]+\|(x_n(t_n^{'}))-(x(t^{'}))\|\end{array}\right]$$

Note that for all $n \geq 1$,

$$|F(\omega,s,x_n(s))| \leq a(\omega,s)M+b(\omega,s)=\varphi(\omega,s)\mu\times\lambda \text{ , a.e.}$$

Where

$$M=\sup_{n\geq 1}\|x_n(\cdot)\|_\infty < \infty$$

Thus, we can write that

$$|\varphi(\omega,t_n^{'},t_n,x_n(\cdot))|-|\varphi(\omega,t_n^{'},t_n,x(\cdot))|$$

$$\leq \|(x_n(t))-(x(t))\|+\int_T \left|\chi_{(t_n^{'},t_n))}(S)-\chi_{(t^{'},t))}(S)\right|\psi(\omega,s)ds$$

$$+\int_T \chi_{(t_n^{'},t_n))}(S)h(F(\omega,s,x_n(S))),F(\omega,s,x(s)))ds+\|(x_n(t_n^{'}))-(x(t')\|.$$

Passing to the limit as, we get

$$\int_T \left|\chi_{(t_n^{'},t_n))}(S)-\chi_{(t^{'},t))}(S)\right|\psi(\omega,s)ds \to 0$$

And

$$\int_T \chi_{(t^{'},t))}(S)h(F(\omega,s,x_n(S)),F(\omega,s,x(s)))ds \to 0$$

Since $F(t,\omega,\cdot)$ is h-continuous. We have

$$\lim_{n\to\infty}|\varphi(\omega,t_n^{'},t_n,x_n(\cdot))|-|\varphi(\omega,t^{'},t,x(\cdot))|=0$$

$$\Rightarrow (t^{'},t,x(\,)) \to \varphi(\omega,t^{'},t,x(\cdot)) \text{ is continuous}$$

$$\Rightarrow \varphi(\cdot,\cdot,\cdot) \text{ is } \hat{\Sigma}\times B(T)\times B(T)\times B(C_x(T)\text{- measurable.}$$

Set $u(\omega,x(\cdot))=\sup_{t,t'\in D}\varphi(\omega,t^{'},t,x(\cdot))$,where D is a dense subset of T. Clearly $(\omega,x(\cdot))=u(\omega,x(\cdot))$ is $\hat{\Sigma}\times B(C_x(T)$- measurable.

Now

$$R(\omega)=\{x(\cdot)\in Cx(T): u(\omega,x(\cdot))=0\}$$

$$\Rightarrow G, R\in\hat{\Sigma}\times B(C_x(T).$$

Hence applying Anumann's selection theorem we can find $\hat{r}:\Omega\to C_x(T)(\hat{\Sigma},B(C_x(T)))$ measurable s.t. for all $\omega\in\Omega, \hat{r}(\omega)\in R(\omega)$. Let $\hat{r}: \Omega\to C_x(T)$ be$(\Sigma,B(C_x(T)))$ is measurable, s.t. $\hat{r}(\omega)(\cdot)=r(\omega)(\cdot)$u a.e. Set $x(\omega)(t)\to r(\omega)(t)$. Then Theorem 2.2,we have t $x(\cdot,\cdot)$ is a stochastic process with continuous paths. By definition of $R(\omega)x(\cdot,\cdot)$ admits a random solution of (3.1).

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