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Fuzzy Regular Subsemigroups of Partially Ordered Semigroups and Fuzzy Simple Partially Ordered Semigroups

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ABSTRACT
In this paper the terms, fuzzy regular subsemigroup,λ-cut, fuzzy left regular subsemigroup, fuzzy right regular subsemigroup, fuzzy Intraregularsubsemigroup, fuzzy completely regular subsemigroup, ideal generated by the ordered fuzzy pointa$_\lambda$, fuzzy semisimple, fuzzy left simple semigroup, fuzzy right simple semigroup, fuzzy simple semigroup, fuzzy globally idempotent, maximal fuzzy ideal in a posemigroup are introduced. It is proved that, if fis fuzzy regular subsemigroup of S thenfis fuzzy idempotent. It is proved that, if f is fuzzy completely regular then f is regular, left regular and right regular. It is proved that, If a_λ is fuzzy regular then a_λ is fuzzy semisimple. It is proved that, If an ordered fuzzy point a_λ of S is left (right) regular semigroup then a_λ is fuzzy semisimple. It is proved that, If a_λ is fuzzy intra regular semigroup then a_λ is fuzzy semisimple. It is also proved that,$f_{(Sa]}$ and $f_{(aS]}$is fuzzy left and fuzzy right ideals of S respectively. S is a fuzzy left (right) simple posemigroup if and only if $f_{(Sa]} = f_S = S$ ($f_{(aS]} = f_S = S$) $\forall a \in S$. It is proved that for any semigroup S the following are equivalent.a)S is a left(right) simple posemigroup b) S is a fuzzy left(right) simple semigroup. Finally we proved that if S is a posemigroup with unity e then the union of all proper fuzzy idealsofSistheuniquefuzzymaximalidealofS.

Mathematical Subject Classification:20m07, 20m11, 20m12
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I. INTRODUCTION

The algebraic theory of semigroups was studied by CLIFFORD [1,2], PETRICH [3] and LJAPIN[4].The ideal theory of semigroups was developed by ANJANEYULUA[5]. Many researchers have been extending the concepts and results of abstract algebra. As we know, in paper[6],P.M.Padmalatha and A.Gangadhara Raointroduced the concept of partially ordered semigroups(posemigroups), in that define regular element in posemigroup, regular posemigroup, left(right) regular element, intra regular element, completely regular elements in posemigroup, next proved that completely regular element implies regular, left(right) regular element in a posemigroup. Also studiedsemisimple element further proved that regular element implies semisimple element, also left(right) regular element implies semisimple element and intraregular implies semisimple. Next studied left(right) simple posemigroup and prove that (Sa]((aS]) is po left(right) ideal and its related properties.Also define proper po ideal, maximal po ideal and proven that if S is a posemigroup with unity 1 then the union of all proper po ideals of S is a unique maximal po ideal of S. Next proved the equivalent conditions (a) Principal po ideals of S form a chain (b) po ideals of S form a chain. Also define simple posemigroup and proved that every left(right) simple posemigroup is simple posemigroup. Finally studied semisimple element inposemigroup, related properties andsemisimpleposemigroup.

L A ZADEH[7] introduced the notion of fuzzy subset of a set in 1965.Since then, a series of research on fuzzy sets results fuzzy logic, fuzzy set theory, fuzzy algebra etc. A ROSENFELD [8] is the father of fuzzy abstract algebra. N Kuroki [9-12] developed fuzzy ideal theory of semigroups.N Kehayopulu and M Tsingelis introduced the notion of fuzzy ideals in partially ordered semigroups(posemigroups). In[13-16] they define fuzzy left(right) ideal in ordered groupoid and its properties. Next they characterized left regular and intra-regular ordered semigroups interms of semiprime left ideals, also they characterized left regular and intra-regular poe-semigroups(partially ordered semigroups having greatest element e)in terms of left ideal elements and fuzzy subsets. Also they characterized regular ordered semigroups in terms of fuzzy right(left) ideals and fuzzy quasi-ideals.In the paper[17], JizhongShen defined F-regular subsemigroup, F-weakly regular subsemigroup, F-completely regular subsemigroups and also proved that F-complete regularity implies F-weakly complete regularity. Also discussed image and inverse images of F-completely regularity(F-weakly completely regular) under surjective mapping.In paper[18], Xiang-Yun Xie and Jian Tang defined fuzzy left(right) ideal, level subset f_t and ordered fuzzy point. They characterized the fuzzy left(right) ideals of ordered semigroups generated in terms of ordered fuzzy points. Also they discussed fuzzy radicals of ordered semigroups. The main aim of this paper is applying the concept of fuzziness to posemigroups (partially ordered semigroups) by defining fuzzy regular posubsemigroup, fuzzy left(right) regular posubsemigroup, fuzzy intra-regular posubsemigroup, fuzzy completely regular posubsemigroup and some of the results. Also fuzzy left(right) po ideal and fuzzy maximal ideal in a posemigroup are defined and some of the results on them.

II. PRELIMINARIES

Definition 2.1: [6] A semigroup(S,.) with an ordered relation ≤ is said to be posemigroupif S is a partially ordered set such that $a \leq b \Rightarrow ax \leq bx, xa \leq xb$ for all $a, b, x \in S$.

Definition 2.2:[19] A function f from posemigroupS to the closed interval [0,1] is called a fuzzy subset of S.The posemigroup S itself is a fuzzy subset of S such that$S(x) = 1, \forall x \in S$. It is denoted by S or 1.

Definition 2.3:[19] Let A be a non-empty subset of S. We denotef_A, the characteristic mapping of A. i.e., The mapping of S into [0,1] defined by $f_A(x)=\begin{cases}1 & \text{if} x \in A\\ 0 & \text{if} x \notin A\end{cases}$.Then f_A is a fuzzy subset of S.

Definition 2.4:[19] Let f and g be two fuzzy subsets of posemigroup S. Then the inclusion relation$f\subseteq g$ is defined by $f(x)\leq g(x)$, ∀ x∈S and $f\cup g$,$f\cap g$are defined by
$(f \cup g)(x)=\max\{f(x), g(x)\}=f(x) \vee g(x)$,
$(f \cap g)(x) = \min \{f(x), g(x)\} = f(x) \wedge g(x), \forall x \in S$

Definition 2.5:[18]Let (S, ≤) be a posemigroup and f,g be two fuzzy subsets of S. The product fogis defined by
$(fog)(x) = \begin{cases} \vee_{x\leq yz} f(y) \wedge g(z) & \text{if} x \leq yz \text{ exists}\\ 0 & \text{otherwise}\end{cases}$

Definition 2.6:[18]Let S be a posemigroup. For H⊆S
we define (H]={t∈S / t≤ h forsome h∈H}. For H={a} we write (a]= ({a}] = { t∈S / t ≤a}

Definition 2.7: A fuzzy subset f of a posemigroup S is called fuzzy subsemigroupof S if $f(xy) \geq f(x) \wedge f(y), \forall x, y \in S$.

Proposition 2.8:[19]A fuzzy subset f of a posemigroup S is fuzzy subsemigroup of S ⟺ fof ⊆ f.

Definition 2.9: A fuzzy subset f of a posemigroup S is called fuzzy posubsemigroup of S if (i) x ≤ y then f(x) ≥ f(y) (ii) f(xy) ≥ f(x) ∧f(y) , ∀x, y ∈S.

Definition 2.10:[18]Let S be a posemigroup. A fuzzy subset f of S is called a fuzzy left idealof S if (i) x ≤ y then f(x) ≥ f(y) (ii) f(xy) ≥ f(y), ∀ x,y∈S.

Lemma 2.11: [18]Let S be a posemigroup and f be a fuzzy subset of S. Then f is a fuzzy left ideal of S if and only if f satisfies that (i) x ≤ y then f(x) ≥ f(y) ∀ x,y∈S (ii) Sof ⊆ f.

Definition2.12:[18]Let S be a posemigroup. A fuzzy subset f of S is called a fuzzy right idealof Sif (i) x ≤ y then f(x) ≥ f(y) (ii) f(xy) ≥ f(x), ∀ x,y∈S.

Lemma 2.13:[18]Let S be a posemigroup and f be a fuzzy subset of S. Then f is a fuzzy right ideal of S if and only if f satisfies that (i) x ≤ y then f(x) ≥ f(y) ∀ x,y∈S (ii) foS⊆ f.

Definition 2.14:[18]Let S be a posemigroup. A fuzzy subset f of S is called a fuzzy idealof S if (i) x ≤ y then f(x) ≥ f(y) (ii) f(xy) ≥ f(y) and f(xy) ≥ f(x), ∀ x,y∈S.

Lemma 2.15:[18]Let S be a posemigroup and f be a fuzzy subset of S. Then f is a fuzzy ideal of S if and only if f satisfies that (i) x ≤ y then f(x) ≥ f(y) ∀ x,y∈S (ii) foS⊆ f and Sof⊆ f.

Definition 2.16: Let S be a posemigroup. A fuzzy ideal f of S is called idempotentif $f^2 = f \circ f = f$.

III. FUZZY REGULAR SUBSEMIGROUP OF A PO SEMIGROUP

Definition 3.1:[6]An element a of a posemigroup S is said to be regularif there exist $x \in S$such thata $\leq axa$

Definition 3.2:[6]A posemigroup S is said to be regular semigroupprovided every element is regular.

Definition 3.3: Let S be a posemigroupand$x \in S$. Define$R_x = \{x'/x' \in s, x \leq xx'x\}$. Let f be a fuzzy subsemigroup of S if $\forall x \in S$ there exist $x' \in R_x$ such that $f(x) \leq f(x')$ provided $f(x) \neq 0$ then f is called fuzzy regular subsemigroup of S.

Theorem 3.4:[17]Let A be a non-empty subset of a posemigroup S . A is regular subsemigroup of S if and only if f_A, the characteristic function of A is a fuzzy regular subsemigroup of S.

Proof: Suppose A is regular subsemigroup of S and $x, y \in A \Rightarrow xy \in A$

Therefore$f_A(xy) \geq f_A(x) \wedge f_A(y)$.
Let$x \in A \Rightarrow f_A(x) = 1$. Then from the regularity of A there exists $x' \in R_x$ such that $x' \in A$ and $f_A(x') = 1. \Rightarrow f_A(x) \leq f_A(x')$. Therefore f_A is a fuzzy regular subsemigroup of S. Conversely suppose that f_A is a fuzzy regular subsemigroup of S.
Let x,y∈A $\Rightarrow f_A(x) = f_A(y) = 1$ and f_A is fuzzy subsemigroup of S. $\Rightarrow f_A(xy) \geq f_A(x) \wedge f_A(y) = 1 \Rightarrow f_A(xy) = 1 \Rightarrow xy \in A \Rightarrow A$ is a subsemigroup of S. Let $x \in A \Rightarrow f_A(x) = 1$.Since f_A is a fuzzy regular, there exists $x' \in R_x$ such that $f_A(x) \leq f_A(x'). \Rightarrow f_A(x') \geq 1 \Rightarrow x' \in A$
Therefore A is regular subsemigroup of S.

Definition 3.5:[18]Let f be a fuzzy subset of a posemigroup S. Let $\lambda \in [0,1]$. Define $f_\lambda = \{x \in S / f(x) \geq \lambda\}$ is the λ-cut of f**.**

Theorem 3.6:[17]Let S be a posemigroup. f is fuzzy subsemigroup of S if and only if $\forall\lambda \in [0,1]$, f_λ is a subsemigroup of S.

Proof: Assume that f is a fuzzy subsemigroup of S. Let $a, b \in f_\lambda \Rightarrow f(a) \geq \lambda, f(b) \geq \lambda$ Since $f(ab) \geq f(a) \wedge f(b) \geq \lambda \wedge \lambda = \lambda \Rightarrow f(ab) = \lambda \Rightarrow ab \in f_\lambda$
$\Rightarrow f_\lambda$ issubsemigroupofS.Conversely suppose that f_λ is a subsemigroup of S. Suppose there exists one x_1, y_1such that$f(x_1y_1) < f(x_1) \wedge f(y_1) < f(x_1) \Rightarrow f(x_1y_1) < f(x_1)$
Define $\lambda = \frac{1}{2}[f(x_1) - f(x_1y_1)]$ then $\lambda \in (0,1]$and $0 \leq f(x_1y_1) < \lambda \leq 1$, $0 < \lambda < f(x_1) \leq 1$
sothat$x_1 \in f_\lambda$,similarly$y_1 \in f_\lambda \Rightarrow x_1y_1 \in f_\lambda \Rightarrow f(x_1y_1) \geq \lambda$
But$f(x_1y_1) < \lambda$whichisacontradiction.
Therefore $f(xy) \geq f(x) \wedge f(y), \forall x, y \in S \Rightarrow f_\lambda$ is subsemigroup of S.

Theorem 3.7: Let f be a fuzzy subset of a posemigroup S. If f is a fuzzy regular subsemigroup of S if and only if $\forall\lambda \in (0,1]$, f_λ is a regular subsemigroup of S provided $f_\lambda \neq \emptyset$.

Proof: Assume that f is fuzzy regular subsemigroup of S. From theorem 3.6, f_λ is subsemigroup of S. Let $x \in f_\lambda$ since f is fuzzy regular $\exists x' \in R_x$ such that $f(x) \leq f(x') \Rightarrow f(x') \geq \lambda \Rightarrow x' \in f_\lambda$.Therefore $\forall x \in f_\lambda \exists x' \in f_\lambda$ such that $x \leq xx'x \Rightarrow f_\lambda$ is a regular subsemigroup of S. Conversely, suppose that f_λ is a regular subsemigroup of S provided $f_\lambda \neq \emptyset$ Assume that f is not fuzzy regular ⇒there exists $x \in S$such that f(x)≠ 0 and $\forall x' \in R_x$, $f(x) > f(x')$. Set$\lambda = f(x)$, clearly $x \in f_\lambda$ and $\forall x' \in R_x \Rightarrow \lambda = f(x) > f(x') \Rightarrow x' \notin f_\lambda$
which is acontradiction.Sincef_λ is regular. Therefore f is fuzzy regular subsemigroup of S.

Theorem 3.8: Let S be a posemigroup. If f is fuzzy regular subsemigroup of S then $fof = f$

Proof: Let f be a fuzzy subset of S. From [5] If f is a fuzzy subsemigroup of S if and only if $fof \subseteq f$. Let $x \in S$ if f(x)=0 then $(fof)(x) \leq f(x) \Rightarrow (fof)(x) = f(x) = 0$
if $f(x) \neq 0$ then $\exists x' \in R_x$ such that $f(x) \leq f(x')$,since f is fuzzy regular
Now$(fof)(x) = {}_{x \leq yz}^{\vee}[f(y) \wedge f(z)] = {}_{x \leq xx'x}^{\vee}[f(xx') \wedge f(x)]$
$\geq f(xx') \wedge f(x)$
$\geq f(x) \wedge f(x') \wedge f(x) = f(x), \forall x \in S$
$\Rightarrow f \subseteq fof$. Therefore $fof = f$ if f is fuzzy regular.

Corollary 3.9: Let S be a posemigroup and f is fuzzy ideal of S. If f is fuzzy regular subsemigroup of S then f is fuzzy idempotent**.**

Definition 3.10:[18] Let f be a fuzzy subset of a posemigroup S. We define (f]by $(f](x) = {}_{x \leq y}^{\vee} f(y), \forall x \in S$.

Note 3.11: Clearly$f \subseteq (f]$.

Definition 3.12: [13]Let S be a posemigroup, $a \in S$ and $\lambda \in [0,1]$. An ordered fuzzy pointa_λ,$a_\lambda: S \to [0,1]$ defined by $a_\lambda(x) = \begin{cases} \lambda \text{if} x \in (a] \\ 0 \text{ if} x \notin (a] \end{cases}$

clearlya_λ is a fuzzy subset of S.

Lemma 3.13:[18]If a_λ is an ordered fuzzy point of S then a_λ=$(a_\lambda]$.

Note 3.14: Let f be a fuzzy subset of an posemigroup S then $(f] = {}_{y_\lambda \in (f]}^{\cup} y_\lambda$.

Definition 3.15: Let f be a fuzzy subset of a posemigroup S. Then$\forall x \in S$, the fuzzy subsetxfx of S is defined by$\forall y \in S$, $(xfx)(y) = \begin{cases} {}_{y \leq xsx}^{\vee f(s)} \text{if} \exists y \leq xsx \\ 0 \text{ otherwise} \end{cases}$

Theorem 3.16: If f is a fuzzy subsemigroup of a posemigroup S. Then f is fuzzy regular if and only if$\forall x \in S$,$(xfx)(x) \geq f(x)$provided$f(x) \neq 0$

Proof: Suppose f is fuzzy regular.Consider $(xfx)(x) = {}_{x \leq xsx}^{\vee} f(s) = {}_{x \leq xx'x}^{\vee} f(x') \geq f(x') \geq f(x)$, since f is fuzzy regular $\Rightarrow (xfx)(x) \geq f(x) \forall x \in S$.conversely assume that$\forall x \in S$, $(xfx)(x) \geq f(x)$. Since
$f(x) \leq (xfx)(x) =$
${}_{x \leq xx'x}^{\vee} f(x') = {}_{x \leq xx'x}^{\max} f(x')$provided$f(x) \neq 0$That is $\forall x \in S \exists$ at least one x' such that $x \leq xx'x$and$f(x) \leq f(x')$since$f(x) \neq 0$
$\Rightarrow$ f is fuzzy regular.

Corollary 3.17: If an ordered fuzzy point a_λ of a po semigroup S is regular if and only if$\forall x \in S$, $(xa_\lambda x)(x) \geq a_\lambda(x)$provided$a_\lambda(x) \neq 0$.

Proof: Proof follows from Theorem 3.14.

Definition 3.18: Let S be a posemigroupand $x \in S$, define $LR_x = \{x' \in S / x \leq x^2x'\}$. Let f be a fuzzy subsemigroup of S. For every $x \in S \exists x' \in LR_x \ni f(x) \leq f(x')$ provided f(x)≠ 0 then f is called fuzzy left regular subsemigroupof S.

Definition 3.19: Let S be a posemigroupand $x \in S$, define $RR_x = \{x' \in S / x \leq x'x^2\}$. Let f be a fuzzy subsemigroup of S. For every $x \in S \exists x' \in RR_x \ni f(x) \leq f(x')$ provided $f(x) \neq 0$ then f is called fuzzy right regular subsemigroupof S.

Definition 3.20: Let S be a posemigroupand $x \in S$, define $IR_x = \{(x_1, x_2) \in S \times S / x \leq x_1 x^2 x_2\}$. Let f be a fuzzy subsemigroup of S. For every $x \in S \exists (x_1, x_2) \in IR_x \ni f(x) \leq f(x_1) \wedge f(x_2)$ provided $f(x) \neq 0$ then f is called fuzzy Intraregularsubsemigroupof S.

Definition 3.21: Let S be a posemigroupand $x \in S$, define $R_x = \{x' \in S / x \leq xx'x\}$ and $C_x = \{x' \in S / xx' = x'x\}$.Let f be a fuzzy subsemigroup of S and for every $x \in S \exists x' \in R_x \cap C_x \ni f(x) \leq f(x')$ provided $f(x) \neq 0$ then f is calledfuzzy completely regular subsemigroupof S.

Theorem3.22: Let S be a posemigroupand f is a fuzzy subsemigroup of S. If f is fuzzy completely regular then f is regular, left regular and right regular.

Proof: Suppose that f is completely regular. Then for every $x \in S \exists x' \in R_x \cap C_x \ni f(x) \leq f(x')$ provided $f(x) \neq 0$. Since $x' \in R_x \Rightarrow x \leq xx'x$ and $(x) \leq f(x')$, $f(x) \neq 0 \Rightarrow$ f is fuzzy regular.

Now for every $x \in S \exists x' \in R_x \cap C_x \ni f(x) \leq f(x')$ provided $f(x) \neq 0$

$\Rightarrow x \leq xx'x$ and $xx' = x'x$. Consider $x \leq xx'x = xxx' = x^2x' \Rightarrow x \leq x^2x'$ and $f(x) \leq f(x') \Rightarrow$ f is fuzzy left regular. Also $x \leq xx'x = x'xx = x'x^2$. Therefore f is fuzzy right regular.

Definition 3.23: Let $a \in S$, $\lambda \in (0,1]$. Define ideal generated by the ordered fuzzy point a_λ of Sby

$$< a_\lambda > (\mathbf{x}) = \begin{cases} \lambda \text{ if } x \in (a) = (a \cup aS \cup Sa \cup SaS] = (s'as'] \\ 0 \quad \text{otherwise} \end{cases}$$

Definition 3.24: An ordered fuzzy element $a_\lambda$of a po semigroup S is said to be fuzzy semisimpleif $a_\lambda \subseteq < a_\lambda >^2$.

Note 3.25: Clearly $a_\lambda \subseteq < a_\lambda >$

Theorem 3.26: Let S be a po Semi group and a_λ is a fuzzy subsemigroup of S. If a_λ is fuzzy regular then a_λ is fuzzy semisimple.

Proof: Suppose a_λ isfuzzy regular. Consider

$< a_\lambda >^2(x) = \vee_{x \leq yz} [< a_\lambda > (y) \wedge < a_\lambda > (z)]$
$\geq < a_\lambda > (y) \wedge < a_\lambda > (z)$
$\geq < a_\lambda > (xx') \wedge < a_\lambda > (x)$
$\geq a_\lambda(xx') \wedge a_\lambda(x) \geq a_\lambda(x) \wedge a_\lambda(x') \wedge a_\lambda(x)$
$\geq a_\lambda(x) \wedge a_\lambda(x) \wedge a_\lambda(x) = a_\lambda(x) \forall x \in S \Rightarrow$
$a_\lambda \subseteq < a_\lambda >^2$. Therefore a_λ is fuzzy semisimple.

Theorem 3.27: Let S be a po Semi group. If an ordered fuzzy point $a_\lambda$of S is left(right) regular semigroup then a_λ is fuzzy semisimple.

Proof: Suppose $a_\lambda$is fuzzy left regular. Then $\forall$ x∈S $\exists x' \in LR_x$ such that $a_\lambda(x) \leq a_\lambda(x')$ provided $a_\lambda(x) \neq 0$.Consider

$< a_\lambda >^2 (x) = \vee_{x \leq yz} [< a_\lambda > (y) \wedge < a_\lambda > (z)]$
$\geq < a_\lambda > (y) \wedge < a_\lambda > (z)$
$\geq < a_\lambda > (x^2) \wedge < a_\lambda > (x')$
$\geq a_\lambda(x^2) \wedge a_\lambda(x')$
$\geq a_\lambda(x) \wedge a_\lambda(x) \wedge a_\lambda(x')$
$\geq a_\lambda(x) \wedge a_\lambda(x) = a_\lambda(x)$
$\Rightarrow a_\lambda \subseteq < a_\lambda >^2$

Therefore a_λ is fuzzy semisimple.Similarly if a_λ is fuzzy right regular semigroup then a_λ is fuzzy semisimple.

Theorem 3.28: Let a_λ be an ordered fuzzy point of a posemigroup S. If a_λ is fuzzy intra regular semigroup then a_λ is fuzzy semisimple**.**

Proof: Let $a_\lambda$is fuzzy intra regular. $\Rightarrow \forall$ x∈S $\exists$ $(x_1, x_2) \in IR_x \ni a_\lambda(x) \leq a_\lambda(x_1) \wedge a_\lambda(x_2)$ provided $a_\lambda(x) \neq 0$.

Consider$< a_\lambda >^2(x) = (< a_\lambda > o < a_\lambda >)(x)$
$= \vee_{x \leq yz} [< a_\lambda > (y) \wedge < a_\lambda > (z)]$
$\geq < a_\lambda > (y) \wedge < a_\lambda > (z)$
$\geq < a_\lambda > (x_1 x^2) \wedge < a_\lambda > (x_2)$
$\geq a_\lambda(x_1 x^2) \wedge a_\lambda(x_2)$
$\geq a_\lambda(x_1) \wedge a_\lambda(x^2) \wedge a_\lambda(x_2)$
$\geq a_\lambda(x) \wedge a_\lambda(x^2)$
$\geq a_\lambda(x) \wedge a_\lambda(x) \wedge a_\lambda(x)$
$= a_\lambda(x), \forall$x∈S
$\Rightarrow a_\lambda \subseteq < a_\lambda >^2$Therefore a_λ is fuzzy semisimple.

IV. FUZZY SIMPLE PARTIALLY ORDERED SEMIGROUPS:

Definition4.1: A posemigroup S is said to be a left simple posemigroupif S is its only poleft ideal.

Definition4.2: Let S be a posemigroup. S is called fuzzy left simple semigroupif every fuzzy left ideal of S is a constant function.

Definition 4.3: Let S be a posemigroup and f be a fuzzy subset of S. Define

$$f_{(Sa]}(x) = \begin{cases} 1 \; if x \in (Sa] \\ 0 \; otherwise \end{cases}$$

Theorem 4.4: Let S be a posemigroup. Then $f_{(Sa]}$ is a fuzzy left ideal of S, for every $a \in S$

Proof: (i) Let $x, y \in S$ and $x \leq y$

If $y \in (Sa]$ since $x \leq y \Rightarrow x \in (Sa]$ then $f_{(Sa]}(x) = 1 = f_{(Sa]}(y)$

If $y \notin (Sa]$then $f_{(Sa]}(y) = 0 \leq f_{(Sa]}(x)$.

By summarizing the above $f_{(Sa]}(x) \geq f_{(Sa]}(y)$

(ii) If $y \notin (Sa]$then $f_{(Sa]}(y) = 0 \leq f_{(Sa]}(xy)$.

If $y \in (Sa]$ then $f_{(Sa]}(y) = 1$

Since $y \in (Sa]$ and (Sa] is a left ideal of S from [18]

then$xy \in (Sa]$, $\forall x \in S$.

$\Rightarrow f_{(Sa]}(xy) = 1 = f_{(Sa]}(y)$Therefore$f_{(Sa]}(xy) \geq f_{(Sa]}(y)$.

From (i) and (ii) $f_{(Sa]}$ is a fuzzy left ideal of S.

Theorem 4.5: For a posemigroup S, the following are equivalent.

a) S is a left simple posemigroup

b) S is a fuzzy left simple semigroup.

Proof:(a)⇒(b):
Suppose S is a left simple posemigroup.
Let f be any fuzzy left ideal of S. Then it follows from [5] that there exist elements $x, y \in S$
such that b=xa and a=yb.
Since f is a fuzzy left ideal of S, $f(a) = f(yb) \geq f(b) = f(xa) \geq f(a)$
⇒f(a) = f(b) $\forall a, b \in$ S ⇒ f is a constant fuzzy ideal.
Therefore S is a fuzzy left simple po semigroup.
(b)⇒(a):
Assume that S is a fuzzy left simple posemigroup.
Let A be any po left ideal of S. Then from [9] C_A is a fuzzy left ideal of S.
⇒ $C_A$is a constant function.
Let$x \in S$Since$A \neq \emptyset$,

$$C_A(x) = 1 \Rightarrow x \in A \Rightarrow S \subseteq A$$

Therefore$A = S$.
Hence S is a left simple posemigroup.
Theorem 4.6: Let S be a posemigroup. S is a fuzzy left simple semigroup if and only if

$$f_{(Sa]} = f_S = S, \quad \forall a \in S$$

Proof: Assume that S is a fuzzy left simple posemigroup. By Theorem 4.5 S is a left simple posemigroup. Then by [18], (Sa]=S. Therefore$f_{(Sa]} = f_S = S$.
Conversly assume that$f_{(Sa]} = f_S = S \Rightarrow f_{(Sa]}(x) = f_S(x)$
⇒(Sa]=S. Then from [18] S is a left simple posemigroup. Then by Theorem 4.5 S is a fuzzy left simple posemigroup.
Definition4.7: A posemigroup S is said to be a right simple posemigroupif S is its only po right ideal.
Definition4.8: Let S be a posemigroup. S is called fuzzy right simple Semigroupif every fuzzy right ideal of S is a constant function.
Definition4.9: Let S be a posemigroup and f be a fuzzy subset of S. Define

$$f_{(aS]}(x) = \begin{cases} 1 \; if x \in (aS] \\ 0 \; otherwise \end{cases}$$

Definition4.10: A posemigroup S is said to be a fuzzy simple semigroupif every fuzzy ideal of S is a constant function.
Theorem 4.11: Let S be a posemigroup.Then $f_{(aS]}$ is a fuzzy right ideal of S for every $a \in S$
Proof: (i) Let $x, y \in S$ and $x \leq y$
If $y \in (aS]$ since $x \leq y \Rightarrow x \in (aS]$ then $f_{(aS]}(x) = 1 = f_{(aS]}(y)$
If $y \notin (aS]$then $f_{(aS]}(y) = 0 \leq f_{(aS]}(x)$.
By summarizing the above $f_{(aS]}(x) \geq f_{(aS]}(y)$
(ii) If $x \notin (aS]$then $f_{(aS]}(x) = 0 \leq f_{(aS]}(xy)$.
If $x \in (aS]$ then $f_{(aS]}(x) = 1$
Since $x \in (aS]$ and (aS] is a po right ideal of S, from Theorem [18]
Then $xy \in (aS]$, $\forall y \in S$.⇒ $f_{(aS]}(xy) = 1 = f_{(aS]}(x)$
Therefore$f_{(aS]}(xy) \geq f_{(aS]}(x)$.
From (i) and (ii),$f_{(aS]}$ is a fuzzy right ideal of S.
Theorem 4.12: For a posemigroup S, the following are equivalent.
a) S is a right simple posemigroup
b) S is a fuzzy right simple semigroup**.**
Proof:(a)⇒(b):
Suppose S is a right simple posemigroup.
Let f be any fuzzy right ideal of S. Then it follows from[5] that there exist elements $x, y \in S$
such that ax=b and a=by.
Since f is a fuzzy right ideal of S, $f(a) = f(by) \geq f(b) = f(ax) \geq f(a)$
⇒f(a) = f(b) $\forall a, b \in$ S ⇒ f is a constant fuzzy ideal.
Therefore S is a fuzzy right simple po semigroup.
(b)⇒(a):
Assume that S is a fuzzy right simple posemigroup.
Let A be any po right ideal of S. Then from [22] C_A is a fuzzy right ideal of S.
⇒ $C_A$is a constant function.
Let $x \in S$Since $A \neq \emptyset$,

$$C_A(x) = 1 \Rightarrow x \in A \Rightarrow S \subseteq A$$

ThereforeA=S.
Hence S is a right simple posemigroup.
Theorem4.13: Let S be a posemigroup. S is a fuzzy right simplesemigroup if and only if $f_{(aS]} = f_S = S, \ \forall a \in S$
Proof: Assume that S is a fuzzy right simplesemigroup. By Theorem 4.12, S is a right simple posemigroup. Then from [6], (aS]=S. Therefore$f_{(aS]} = f_S = S$.
Conversly assume that$f_{(aS]} = f_S = S \Rightarrow f_{(aS]}(x) = f_S(x)$
⇒(aS]=S. Then from [6], S is a right simple posemigroup. Then by Theorem 4.12, S is a fuzzy right simple semigroup.
Definition4.14: Let f,g be two fuzzy subsets of S, (fog]is defined by

$$(fog](x) = \vee_{x \leq yz} (fog)(yz), \forall x, y, z \in S$$

Definition 4.15: A fuzzy ideal f of a posemigroup S is said to befuzzyglobally idempotentif $(f^n] = (f]$, ∀n
Definition 4.16: Let S be a posemigroup. S is said to be fuzzy globally idempotentif
$(S^n] = S$, ∀n
Theorem 4.17: Let S be a posemigroup with unity e and f be a fuzzy ideal of S with$f(e) = 1$ then $f = S = f_S$
Proof: Let $x \in S$. Consider$f(x) = f(xe)$ ≥f(e)=1

⇒ $f(x) \geq 1 \Rightarrow f(x) = 1, \forall$ x∈S. Therefore$f = f_S = S$.
Definition 4.18: A non-zero fuzzy ideal f of a posemigroup S is called a proper fuzzy idealif $f \neq C_S = S$.
Definition 4.19: A fuzzy ideal f of a posemigroup

S is called maximalif there doesn't exist any proper fuzzy ideal g of S $\ni f \subset g$.

Theorem 4.20: If $\{f_i\}$ is a fuzzy ideals of a po semigroup S then the arbitrary union of fuzzy ideals is fuzzy ideal of S.

Proof:let $\{f_i\}$ is a fuzzy ideals of a po semigroup S. Let $x, y \in S$ such that $x \leq y$.

Consider$\cup f_i(x) =$
$max\{f_1(x), f_2(x), f_3(x), \dots \dots \dots \dots\}$
$= f_1(x) \vee f_2(x) \vee f_3(x) \vee \dots \dots$
$\geq f_1(y) \vee f_2(y) \vee f_3(y) \dots \dots \dots$
since each f_i is a fuzzy ideal.
$= \max\{f_1(y), f_2(y) \dots \dots\} = \cup f_i(y)$
Therefore $\cup f_i(x) \geq \cup f_i(y)$ if $x \leq y$
Consider$\cup f_i(xy) = f_1(xy) \vee f_2(xy) \vee f_3(xy) \vee ..$
$\geq f_1(y) \vee f_2(y) \vee f_3(y) \vee \dots \dots \dots$
since each f_i is a fuzzy left ideal.
$= \cup f_i(y)$
Therefore $\cup f_i(xy) \geq \cup f_i(y)$, Similarly $\cup f_i(xy) \geq \cup f_i(x)$.
Thus the arbitrary union of fuzzy ideals is a fuzzy ideal of S.

Theorem4.21: Let S be a posemigroup with unity e then the union of all proper fuzzy ideals of S is the unique fuzzy maximal ideal of S.

Proof: Let f_M be the union of all proper fuzzy ideals of S.
$\Rightarrow f_M$ is a fuzzy ideal of S by theorem 4.20.
If f_M is not proper then $f_M = C_S \Rightarrow f_M(x) = 1, \forall x \in S$
$\Rightarrow f_i(x) = 1$ for some fuzzy ideal f_i since $\cup f_i = f_M$
$\Rightarrow f_i = f_S$ but f_i is proper.
Therefore f_M is a proper fuzzy ideal of S.
Since f_M contains all proper fuzzy ideals of S.
$\Rightarrow f_M$ is maximal fuzzy ideal of S.
If g_M is any other maximal fuzzy ideals of S then$g_M \subseteq f_M \subseteq C_S$.
Therefore$f_M = g_M$. Hence f_M is the unique fuzzy maximal ideal of S.

Theorem 4.22: If S is a fuzzy left(right) simple semigroup then S is a fuzzy simple semigroup.
Proof: Suppose S is a fuzzy left(right) simple semigroup.
Let f be a fuzzy ideal of S $\Rightarrow$ f is a fuzzy left and fuzzy right ideal of S.
$\Rightarrow$f is a constant function $\Rightarrow$S is a fuzzy simple semigroup.

Theorem 4.23: Let a_λ be a fuzzy ordered element of a posemigroup S. If a_λ is semisimple and idempotent then$a_\lambda \subseteq < a_\lambda >^n, \forall n$.

Proof: If a_λ is semisimple and idempotent .
Let a $\in$ S and n is a natural number.
$\Rightarrow \mathbf{a_\lambda} \subseteq < \mathbf{a_\lambda} >^{\mathbf{2}}$is true for n=2 since $\mathbf{a_\lambda}$is fuzzy semisimple.
Assume that the statement is true for n-1. That is $\mathbf{a_\lambda} \subseteq < \mathbf{a_\lambda} >^{\mathbf{n-1}}$
Consider $< a_\lambda >^n = < a_\lambda >^{n-1}$ o $< a_\lambda >$
$\supseteq a_\lambda o a_\lambda = {a_\lambda}^2 = a_\lambda$since a_λ is idempotent.
Therefore $< a_\lambda >^n \supseteq a_\lambda$, $\forall n$

V. CONCLUSION

The purpose of this paper ischaracterize fuzzy completely regular and fuzzy regular, establish the relation between fuzzy regular and fuzzy idempotent. Also fuzzy left (right) simple semigroup
left(right)simpleposemigroup.

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REFERENCES:

[1]. Clifford A.H and Preston G.B., The algebraic theory of semigroupsvol – I (American Math. Society, Province (1961)).
[2]. Clifford A.H and Preston G.B., The algebraic theory of semigroupsvol – II (American Math. Society, Province (1967)).
[3]. Petrch.M., Introduction to semigroups ,Merril publishing company, Columbus, Ohio(1973).
[4]. LJAPIN E. S., Semigroups, American Mathematical Society,Providence, Rhode Island (1974).
[5]. ANJANEYULU A., Structure and ideal theory of semigroups – Thesis, ANU (1980).
[6]. P.M.Padmalatha and A.Gangadhara Rao, Simple partially ordered semigroups, Global Journal of Pure and Applied Mathematics, Volume 10,, Number 3(2014).
[7]. L.A. Zadeh, Fuzzy sets, Inform. Control 8 (1965) 338–353.
[8]. A. Rosenfeld, Fuzzy groups, J. Math. Anal. Appl. 35 (1971) 512–517
[9]. N. Kuroki, Fuzzy bi-ideals in semigroups, Math. Univ. Paul. 28 (1979) 17–21.
[10]. N. Kuroki, On fuzzy semigroups, Inform. Sci. 53 (1991) 203–236.
[11]. N. Kuroki, Fuzzy generalized bi-ideals in semigroups, Inform. Sci. 66 (1992) 235–543.
[12]. N. Kuroki, Fuzzy semiprime ideals in semigroups, Fuzzy Sets Syst. 8 (1982) 71–79.
[13]. N.Kehayopulu, M.Tsingelis, Fuzzy sets in ordered groupoids, Semigroup Forum 65(2002),128-132
N.Kehayopulu, M.Tsingelis, Fuzzy ideals in ordered semigroups, Quasigroups and related systems 15(2007),279-289.
[14]. N.Kehayopulu, M.Tsingelis, Left regular and Intra-regular ordered semigroups in

terms of fuzzy subsets, Quasigroups and related systems 14(2006),169-178.
[15]. N.Kehayopulu, M.Tsingelis, Regular ordered semigroupsinterms of fuzzy subsets, Information Sciences 176(2006),3675-3693.
[16]. JizhongShen, On fuzzy Regular Subsemigroups of a semigroup, Information Sciences 51,111-120(1990)
[17]. Xiang-Yun Xie,, Jian Tang, Fuzzy radicals and prime fuzzy ideals of ordered semigroups, Information Sciences178 (2008), 4357–4374.
[18]. J. N. Mordeson, D. S. Malik, N. Kuroki, Fuzzy Semigroups(Springer-VerlagBerlin Heidelberg, New York 2003)
[19]. T.M.G. Ahsanullah and F.A. AI-Thukair, Conditions on a semigroup to be a fuzzyneighborhood group, Fuzzy Sets and Systems, 55(1993), 333-340.
[20]. Wang Xue-Ping,Liu Wan-Jin, Fuzzy regular subsemigroups in semigroups. Information Sciences 68,225-231(1993)
[21]. W.M.Wu, Normal fuzzy subsemigroups, Fuzzy Math1(1).21-30(1981)

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