

Some Allied Semicontinuous Functions in Topology

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ABSTRACT :In this paper, we define and study the new concepts of allied semicontinuous functions, namely, semi-sg-continuous, semi-gS-continuous, semi- α gs-continuous functions in topology. **Mathematics**

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I. INTRODUCTION

In 1963, Levine [16] introduced and studied semi-open sets and semi-continuous functions in topology. Later, in 1970 N. Biswas[5], S.G. Crossley et. al [7] and P.Das[9] have studied the concepts of semi-closed sets, semi-closure operators and semi-interior operators, respectively. For the first time, the concept of generalized closed (in brief, g-closed) sets was considered by Levine in 1970[17]. Later, in 1987, 1990 and 1995 respectively, Bhattacharya et. al [4], Arya et.al.[3] and J. Dontchev [12] have defined and studied the concepts of sg-closed sets, gs-closed sets, gsp-closed sets. In [6 & 25], P. Sundaram et.al have defined and studied the concepts of sg-continuous functions. In [25] P.Sundaram had defined and studied the concept of gs-continuous functions in topology. In 1995, Dontchev[12] has defined and investigated the notions of gsp-continuity and gsp-irresoluteness. The aim of this paper is to define and study the new classes of allied semicontinuous functions, namely, semi-sg-continuous functions, semi- α gs-continuous functions and semi-gS-continuous functions in topology. Also, we study their basic characterizations and decomposition theorems of all these functions with the other type of semicontinuous functions.

II. PRELIMINARIES

Throughout this paper (X, τ) and (Y, σ) (or Simply X and Y) respectively, denote topological spaces on which no separation axioms are assumed unless otherwise explicitly stated.

For a subset A of (X, τ) , the Closure of A, the Interior of A with respect to τ are denoted by $Cl(A)$ and $Int(A)$ respectively.

The following definitions and results are useful in this sequel.

DEFINITION 2.1: A subset A of a space X is called

- (i) semi-open-set [16] if $A \subset ClInt(A)$.
- (ii) semipre-open set [1] if $A \subset ClIntCl(A)$.
- (iii) α -open set [20] if $A \subset IntClInt(A)$.

The complement of semi-open (resp., semipre-open, α -open) set is called semi-closed [5] (resp. semipre-closed[1], α -closed [18]) set. The family of all semi-open (resp. semipre-open) sets of a space X is denoted by $SO(X)$ (resp. $SPO(X)$).

DEFINITION 2.2: Let A be a subset of space X, then the semi-interior [9] (resp. the semi-pre-interior [1&19], the α -interior [18]) of A is the union of all semi-open (resp. semi-pre-open, α -open) sets contained in A and is denoted by $sInt(A)$ (resp. $spInt(A)$, $\alpha Int(A)$).

DEFINITION 2.3: Let A be a subset of a space X, then the intersection of all semi-closed (resp. semipre-closed, α -closed) sets containing A is called the semi-closure[7] (resp. semi-pre-closure [1&19], α -closure[18]) of A and is denoted by $sCl(A)$ (resp. $spCl(A)$, $\alpha Cl(A)$).

DEFINITION 2.4: A subset A of a space X is called :

- (i) A generalised closed set (in brief, g-closed) [17] if $Cl(A) \subset U$ whenever $A \subset U$ and U is open.

- (ii) semi-generalised closed (in brief , sg-closed) [4] if $sCl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in X .
- (iii) generalized semi-closed (in brief, gs-closed) [3] if $sCl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .
- (iv) generalized semipre-closed (briefly,gsp-closed) [12] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .
- (v) α -generalized semiclosed (in brief, α gs-closed) [22] if $\alpha Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semiopen in X .

DEFINITION 2.5: A function $f : X \rightarrow Y$ is called :

- (i) semi-continuous [16] if the inverse image of each open set Y is semi-open in X .
- (ii) sg-continuous [6 & 25] if the inverse image of each open set of Y is sg-open in X .
- (iii) gs-continuous [25] if the inverse image of each open set of Y is gs-open in X .
- (iv) gsp-continuous [12] if the inverse image of each open set of Y is gsp-open in X .
- (v) α gs-continuous [23] if the inverse image of each open set of Y is α gs-open in X .

DEFINITION 2.6: A function $f : X \rightarrow Y$ is called :

- (i) sg-irresolute [6] if the inverse image of every sg-open set of Y is sg-open in X .
- (ii) gs-irresolute [25] if the inverse image of every gs-open set of Y is gs-open in X .
- (iii) gsp-irresolute [12] if the inverse image of every gsp-open set of Y is gsp-open in X .
- (iv) α gs-irresolute [23] if the inverse image of every α gs-open set of Y is α gs-open in X .
- (v) Irresolute [8] if the inverse image of every semiopen set of Y is semiopen in X .

DEFINITION 2.7: A function $f : X \rightarrow Y$ is called :

- (i) contra-semi-continuous [13] if the inverse image of each open set of Y is semiclosed in X .
- (ii) strongly continuous [2] if the inverse image of each subset of Y is clopen in X .

DEFINITION 2.8: A space X is called :

- (i) semi-g-regular [14] if for each sg-closed set A and each point $x \notin A$, there exist disjoint semi-open sets $U, V \subseteq X$ such that $A \subseteq U$ and $x \in V$.
- (ii) α gs-regular [24] if for each α gs-closed set A and each point $x \notin A$, there exist disjoint semi-open sets $U, V \subseteq X$ such that $A \subseteq U$ and $x \in V$.
- (iii) semi-connected [10] if X cannot be written as the disjoint union of two non-empty semiopen sets.

- (iv) sg-connected [6] if X cannot be written as the disjoint union of two non-empty sg-open sets.

DEFINITION 2.9[15]: Let X be a topological space and $A \subseteq X$. Then the semi-generalized kernel of A is denoted by $sg\text{-ker}(A)$ and is defined to be the set

$$sg\text{-ker}(A) = \bigcap \{G \in SGO(X) / A \subseteq G\}$$

THEOREM 2.10[15]: Let X be a topological space and x in X then $y \in sg\text{-ker}(\{x\})$ iff $x \in sgCl(\{x\})$.

DEFINITION 2.11[8] : A function $f : X \rightarrow Y$ is called pre-semi-open if the image of each semi-open set of X is semi-open in Y .

III. PROPERTIES OF SEMI-SG-CONTINUOUS FUNCTIONS

In section we define and study the following

DEFINITION 3.1: A function $f : X \rightarrow Y$ is said to be semi-sg-continuous if the inverse image of every sg-open set in Y is semi-open set in X .

Clearly, every semi-sg-continuous function is sg-irresolute.

DEFINITION 3.2: A function $f : X \rightarrow Y$ is called strongly sg-continuous if the inverse image of every sg-open set in Y is open in X .

THEOREM 3.3: Let $f : X \rightarrow Y$ be a single valued function, where X and Y are topological spaces, then the following statements are equivalent:

- (i) The function f is semi-sg-continuous.
- (ii) For each point p in X and each sg-open set V in Y with $f(p) \in V$, there is a semi-open set U in X such that $p \in U, f(U) \subseteq V$.

PROOF: (i) \rightarrow (ii). Let $f(p) \in V$ and $V \subseteq Y$ an sg-open set then $p \in f^{-1}(V)$ is semi-open set in X , since f is with semi-sg-continuous. Let $U = f^{-1}(V)$, then $p \in U$ and $f(U) \subseteq V$.

(ii) \rightarrow (i). Let V be an sg-open set and $p \in f^{-1}(V)$ then $f(p) \in V$, there exists a semi-open set in U_p in X such that $p \in U_p$ and $f(U_p) \subseteq V$. Then $p \in U_p \subseteq f^{-1}(V)$ and $f^{-1}(V) = \bigcup U_p \in SO(X)$. This implies that f is semi-sg-continuous

We define the following

DEFINITION 3.4: A function $f : X \rightarrow Y$ is called sg-gsp-continuous if the inverse image of every sg-open set in Y is gsp-open set in X .

Clearly , every sg-irresolute function is sg-gsp-continuous.

DEFINITION 3.5: A function $f : X \rightarrow Y$ is called semi-gsp-continuous if the inverse image of every gsp-open set of Y is semi-open in X .

DEFINITION 3.6: A function $f : X \rightarrow Y$ is called gsp-sg-continuous if the inverse image of every gsp-open set of Y is sg-open in X .

THEOREM 3.7: Let $f : X \rightarrow Y$ is semi-sg-continuous surjection and X is semi-connected then Y is sg-connected.

PROOF: Suppose Y is not sg-connected. Let $Y = A \cup B$ where A and B are disjoint non-empty sg-open sets in Y . Since f is semi-sg-continuous surjection

$X = f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty and semi-open sets in X . This contradicts the fact that X is semi-connected. Hence Y is sg-connected.

We, define the following.

DEFINITION 3.8 : A space X is called gsp-connected if X cannot be written as the disjoint union of two non-empty gsp-open sets.

THEOREM 3.9: Let $f: X \rightarrow Y$ is gsp-sg-continuous surjection and X is sg-connected then Y is gsp-connected.

PROOF: Obvious.

Easy proofs of the following are omitted.

THEOREM 3.10: Let $f: X \rightarrow Y$ is pre-sg-continuous, onto and X is sg-connected then Y is semi-connected.

THEOREM 3.11: Let $f: X \rightarrow Y$ is strongly-sg-continuous, onto and X is connected then Y is sg-connected.

THEOREM 3.12: Let $f: X \rightarrow Y$ is sg-gsp-continuous surjection and X is gsp-connected then Y is sg-connected.

We define the following

DEFINITION 3.13: A function $f: X \rightarrow Y$ is called contra-semi-sg-continuous if the inverse image of each sg-open of Y is semi-closed in X .

DEFINITION 3.14: A function $f: X \rightarrow Y$ is called contra-strongly-sg-continuous if the inverse image of each sg-open of Y is closed in X .

The routine proof of the following is omitted.

THEOREM 3.15 : The following are equivalent for a function $f: X \rightarrow Y$

- (i) f is contra-semi-sg-continuous..
- (ii) For each sg-closed subset F of Y , $f^{-1}(F) \in SO(X)$
- (iii) For each $p \in X$ and each sg-closed subset F of Y containing $f(p)$, there exists semi-open set U in X containing point p such that $f(U) \subset F$

THEOREM 3.16 : If a function $f: X \rightarrow Y$ is contra-semi-sgs-continuous and Y is semi-g-regular, then f is semi-sg-continuous.

We define the following

DEFINITION 3.17: A function $f: X \rightarrow Y$ is said to be perfectly semi-sg-continuous if the inverse image of each sg-open set of Y is both semi-clopen in X .

REMARK 3.18 : Let $f: X \rightarrow Y$ be a function , then

- (i) If f is perfectly semi-sg-continuous function is semi-sg-continuous function.

- (ii) If f is perfectly semi-sg-continuous function is contra semi-sg-continuous function.

THEOREM 3.19 : The following are equivalent for a function $f: X \rightarrow Y$ is

- (i) f is perfectly semi-sg-continuous.
- (ii) The inverse image of every sg-open set in Y is both semi-open and semi-closed in X .
- (iii) The inverse image of every sg-closed set in Y is both semi-open and semi-closed in X .

PROOF: Obvious.

THEOREM 3.20: The following are equivalent for a function $f: X \rightarrow Y$

- (i) f is contra-semi-sg-continuous..
- (ii) The inverse image each sg-closed set in Y is semi-open.

PROOF: Easy.

We , state the following.

LEMMA 3.21 : Suppose that $SF(X)$ is closed under arbitrary intersections then the following are equivalent for a function $f: X \rightarrow Y$.

- (i) f is contra-semi-sg-continuous.
- (ii) the inverse image of each sg-closed set of Y is semi-open in X .
- (iii) For each x in X and each sg-closed set B in Y with $f(x) \in B$, there exists a semi-open set A in X such that $x \in A$ and $f(A) \subset B$.
- (iv) $f(sCl(A)) \subset sg\text{-ker}(f(A))$ for every subset A of X .
- (v) $sCl(f^{-1}(B)) \subset f^{-1}(sg\text{-ker}(B))$ for every subset B of Y .

Easy proofs of the followings are omitted

THEOREM 3.22 : If $f: X \rightarrow Y$ is a contra-semi-sg-continuous from a semi-connected space X onto any space Y , then Y is not a discrete space.

THEOREM 3.23 : If $f: X \rightarrow Y$ is a contra-semi-sg-continuous surjection and X is semi-connected space, then Y is connected.

Next, we state some decomposition on semi-sg-continuous functions :

THEOREM 3.24 : If $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be two functions then

- (i) $g \circ f: X \rightarrow Z$ is semi-sg-continuous function, if g is sg-irresolute and f is semi-sg-continuous.
- (ii) $g \circ f: X \rightarrow Z$ is semi-sg-continuous function, if g is sg-gsp-continuous and f is semi-sg-continuous.
- (iii) $g \circ f: X \rightarrow Z$ is semi-sg-continuous function, if g is strongly-gsp-continuous and f is semi-continuous.

THEOREM 3.25 : If $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be two functions then

- (i) $g \circ f: X \rightarrow Z$ is semi-continuous function, if f is semi-sg-continuous and g is sg-continuous.

- (ii) $\text{gof} : X \rightarrow Z$ is semi-continuous function, if f is semi-strongly-sg-continuous and g is sg-continuous.
- (iii) $\text{gof} : X \rightarrow Z$ is continuous function, if f is strongly-sg-continuous and g is sg-continuous.

We define the following.

DEFINITION 3.26 : A function $f : X \rightarrow Y$ is called strongly-semi-open if the image of each semi-open set of X is open in Y .

LEMMA 3.27: If $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ be two functions such that $\text{gof} : X \rightarrow Z$ is semi-sg-continuous, then

- (i) If $f : X \rightarrow Y$ is strongly-semi-open surjective, then g is strongly-sg-continuous.
- (ii) If $f : X \rightarrow Y$ is surjective, pre-semi-open, then g is semi-sg-continuous.

Easy proof of the theorem is omitted.

LEMMA 3.28 : Let $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ be two functions. Then,

- (i) If f is irresolute and g is semi-sg-continuous then $\text{gof} : X \rightarrow Z$ semi-sg-continuous.
- (ii) If f is irresolute and g is contra-semi-sg-continuous then $\text{gof} : X \rightarrow Z$ contra-semi-sg-continuous.
- (iii) If f is contra-semi-continuous and g is sg-continuous then gof is contra-semi-continuous.
- (iv) If f is strongly-semi-continuous and g is contra-semi-sg-continuous then gof is contra-strongly-sg-continuous.

PROOF: Obvious.

IV. PROPERTIES OF SEMI-GS-CONTINUOUS FUNCTIONS

DEFINITION 4.1 : A function $f : X \rightarrow Y$ is said to be semi-gs-continuous if the inverse image of every gs-open set in Y is semi-open set in X .

Clearly, every semi-gs-continuous function is gs-irresolute.

The routine proof of the following is omitted.

THEOREM 4.2: Let $f : X \rightarrow Y$ be a single valued function, where X and Y are topological spaces, then the following statements are equivalent:

- (i) The function f is semi-gs-continuous.
- (ii) For each point p in X and each gs-open set V in Y with $f(p) \in V$, there is

a semi-open set U in X such that $p \in U, f(U) \subseteq V$.

We define the following.

Definition 4.3: A function $f : X \rightarrow Y$ is said to be gs-gsp-continuous if the inverse image of every gs-open set in Y is gsp-open set in X .

Clearly, every gs-irresolute is gs-gsp-continuous.

Definition 4.4: A function $f : X \rightarrow Y$ is called strongly-gs-continuous if the inverse image of every gs-open set in Y is open in X .

We, prove the following.

THEOREM 4.5: Let $f : X \rightarrow Y$ is semi-gs-continuous surjection and X is semi-connected then Y is gs-connected.

PROOF: Suppose Y is not gs-connected. Let $Y = A \cup B$ where A and B are disjoint non-empty gs-open sets in Y . Since f is semi-gs-continuous surjection

$X = f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty and semi-

open sets in X . This contradicts the fact that X is semi-connected. Hence Y is gs-connected.

We define the following.

Definition 4.6 : A function $f : X \rightarrow Y$ is called gsp-gs-continuous if the inverse image of every gsp-open set in Y is gs-open in X .

The routine proof of the following is omitted.

THEOREM 4.7 : Let $f : X \rightarrow Y$ is gsp-gs-continuous surjection and X is gs-connected then Y is gsp-connected.

We, define the following.

Definition 4.8 : A function $f : X \rightarrow Y$ is called pre-gs-continuous if the inverse image of every semi-open set of Y is gs-open in X .

Easy proofs are omitted of the following.

THEOREM 4.9: Let $f : X \rightarrow Y$ is pre-gs-continuous, onto and X is gs-connected then Y is semi-connected.

THEOREM 4.10: Let $f : X \rightarrow Y$ is strongly-gs-continuous, onto and X is connected then Y is gs-connected.

THEOREM 4.11: Let $f : X \rightarrow Y$ is gsp-gs-continuous surjection and X is gsp-connected then Y is gs-connected.

Next, we give some decompositions on semi-gs-continuity :

Theorem 4.12: If $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ be two functions then

- (i) $\text{gof} : X \rightarrow Z$ is semi-gs-continuous function, if g is gs-irresolute and f is semi-sg-continuous.
- (ii) $\text{gof} : X \rightarrow Z$ is semi-gs-continuous function, if g is gs-gsp-continuous and f is semi-sg-continuous.
- (iii) $\text{gof} : X \rightarrow Z$ is semi-gs-continuous function, if g is strongly-gs-continuous and f is semi-continuous.
- (iv) $\text{gof} : X \rightarrow Z$ is gs-gsp-continuous function, if g is gs-gsp-continuous and f is gs-irresolute.

Theorem 4.13: If $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ be two functions then

- (i) gof is semi-continuous, if f is semi-sg-continuous and g is gs-continuous.
- (ii) gof is irresolute, if f is semi-gs-continuous and if g pre-gs-continuous.

(iii) $g \circ f$ is semi-continuous, if f is irresolute and g is semi-continuous.

LEMMA 4.14: Let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be two functions such that $g \circ f: X \rightarrow Z$ is semi-gs-continuous. Then,

- (i) If f is strongly-semi-open surjective, then g is strongly-gs-continuous.
- (ii) If f is surjective pre-semi-open, then g is semi-gs-continuous.

Easy proof of the theorem is omitted.

We, state the following :

LEMMA 4.15: If $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be two functions. Then,

- (i) If f is irresolute and g is semi-gs-continuous then $g \circ f: X \rightarrow Z$ semi-gs-continuous.
- (ii) If f is irresolute and g is contra-semi-gs-continuous then $g \circ f$ is contra-semi-gs-continuous.
- (iii) If f is contra-semi-gs-continuous and g is gs-continuous then $g \circ f$ is contra-semi-continuous.
- (iv) If f is strongly-semi-continuous and g is contra-semi-gs-continuous then $g \circ f: X \rightarrow Z$ is contra-strongly-gs-continuous.

V. PROPERTIES OF SEMI- α GS-CONTINUOUS FUNCTIONS

DEFINITION 5.1 : A function $f: X \rightarrow Y$ is said to be semi- α gs-continuous if the inverse image of every α gs-open set in Y is semi-open in X .

Clearly, every semi- α gs-continuous function is α gs-irresolute.

Proof of the following is similar to Th.3.3.

THEOREM 5.2: Let $f: X \rightarrow Y$ be a single valued function, where X and Y are topological spaces, then the following statements are equivalent:

- (i) The function f is semi- α gs-continuous.
- (ii) For each point p in X and each α gs-open set V in Y with $f(p) \in V$, there is

a semi-open set U in X such that $p \in U$, $f(U) \subseteq V$.

We, define the following.

DEFINITION 5.3: A function $f: X \rightarrow Y$ is said to be α gs-gsp-continuous if the inverse image of every α gs-open set of Y is gsp-open set in X .

THEOREM 5.4 Let $f: X \rightarrow Y$ is α gs-irresolute then f is α gs-gsp-continuous.

PROOF: Let V be a α gs-closed subset of Y . Then $f^{-1}(V)$ is α gs-closed set in X , as f is α gs-irresolute. We have, every α gs-closed set is gsp-closed set, then $f^{-1}(V)$ is gsp-closed set in X . Therefore, f is α gs-gsp-continuous.

DEFINITION 5.5: A function $f: X \rightarrow Y$ is called strongly- α gs-continuous if the inverse image of every α gs-open set in Y is open in X .

We recall the definition.

DEFINITION 5.6[23]: A topological space X is said to be α gs-connected if X cannot be written as the disjoint union of two non-empty α gs-open sets.

THEOREM 5.7: Let $f: X \rightarrow Y$ is semi- α gs-continuous surjection and X is semi-connected then Y is α gs-connected.

Proof is similar to Th.4.5.

DEFINITION 5.8 : A function $f: X \rightarrow Y$ is called gsp- α gs-continuous if the inverse image of every gsp-open set in Y is α gs-open in X .

DEFINITION 5.9 : A function $f: X \rightarrow Y$ is called pre- α gs-continuous if the inverse image of every semi-open set of Y is α gs-open in X .

The routine proofs of the followings are omitted.

THEOREM 5.10 : If $f: X \rightarrow Y$ is gsp- α gs-continuous, surjection and X is α gs-connected then Y is gsp-connected.

THEOREM 5.11: Let $f: X \rightarrow Y$ is pre- α gs-continuous, onto and X is α gs-connected then Y is semi-connected.

THEOREM 5.12: Let $f: X \rightarrow Y$ is strongly- α gs-continuous, onto and X is connected then Y is α gs-connected.

We define the following

DEFINITION 5.13 : A function $f: X \rightarrow Y$ is called contra-semi- α gs-continuous if the inverse image of each α gs-open of Y is semi-closed in X .

DEFINITION 5.14 : A function $f: X \rightarrow Y$ is called contra-strongly- α gs-continuous if the inverse image of each α gs-open of Y is closed in X .

REMARK 5.15 : Since every open set is α gs-open and every closed set is semi-closed set and hence we have the following

- (i) Every contra-strongly- α gs-continuous function is contra-semi- α gs-continuous function.
- (ii) Every contra-semi- α gs-continuous function is contra-semi-continuous function.

THEOREM 5.16: The following are equivalent for a function $f: X \rightarrow Y$ is

- (i) f is contra-semi- α gs-continuous..
- (ii) For each α gs-closed subset F of Y , $f^{-1}(F) \in \text{SO}(X)$
- (iii) For each $x \in X$ and each α gs-closed subset F of Y containing $f(x)$, there exists semi-open set U in X containing point x such that $f(U) \subset F$.

Easy proof of the theorem is omitted.

THEOREM 5.17 : If a function $f: X \rightarrow Y$ is contra-semi- α gs-continuous and Y is α gs-regular, then f is semi- α gs-continuous.

PROOF: Obvious.

DEFINITION 5.18: A space X is called locally α gs-indiscrete if every semi-open set is α gs-closed in X .

Easy proofs of the following are omitted.

THEOREM 5.19 : If a function $f: X \rightarrow Y$ is contra-semi- α gs-continuous and X is locally α gs-indiscrete space, then f is α gs-irresolute.

THEOREM 5.21: Let $f: X \rightarrow Y$ be a function and let $g: X \rightarrow X \times Y$ be the graph function of f defined by $g(x) = (x, f(x))$ for every $x \in X$. If g is contra-semi- α gs-continuous, then f is contra-semi- α gs-continuous.

THEOREM 5.22: If $f: X \rightarrow Y$ is a contra-semi- α gs-continuous from a semi-connected space X onto any space Y , then Y is not a discrete space.

THEOREM 5.23: If $f: X \rightarrow Y$ is a contra-semi- α gs-continuous surjection and X is semi-connected space, then Y is α gs-connected.

We, define the following.

DEFINITION 5.24: A function $f: X \rightarrow Y$ is said to be perfectly semi- α gs-continuous if the inverse image of each α gs-open set of Y is both semi-clopen in X .

We, state the followings.

THEOREM 5.25: A function $f: X \rightarrow Y$ is perfectly semi- α gs-continuous if and only if the inverse image of every α gs-closed set in Y is both semi-open and semi-closed in X .

THEOREM 5.26: Every perfectly semi- α gs-continuous function is irresolute function.

REMARK 5.27: Let $f: X \rightarrow Y$ be a function, then

- (i) If f is perfectly semi- α gs-continuous function is semi- α gs-continuous function.
- (ii) If f is perfectly semi- α gs-continuous function is contra semi- α gs-continuous function.

THEOREM 5.28: The following are equivalent for a function $f: X \rightarrow Y$ is

- (i) f is perfectly semi- α gs-continuous.
- (ii) The inverse image of every α gs-open set in Y is both semi-open and semi-closed in X .
- (iii) The inverse image of every α gs-closed set in Y is both semi-open and semi-closed in X .

PROOF: Obvious .

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