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The Family Of Four, Five And Sixmembers Block Hybrid Simpson's Methods For Solution Of Stiff Ordinary Differential Equations

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ABSTRACT:
In this research work,the construction of two-step hybrid block Simpson's methods with two, threeand four off-grid points for the solutions of first order stiff systems of ordinary differential equations (ODEs) in studied.In the derivation of the method, power series is adopted as basis function to obtain the main scheme through collocation and interpolations approach. Taylor series was adopted alongside, the method to generate non-overlapping numerical results.This is achieved by transforming a k-step multi-step method into continuous form and evaluating at various grid points to obtain the discrete schemes. The performance of the methods is demonstrated on some numerical experiments. The results revealed that the hybrid block Simpson's method is efficient, accurate and convergent on mildly stiff problems.
Key words: power series, collocation, interpolation, hybrid, blocks method, multi-step method

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I. INTRODUCTION

Numerous problems such as chemical kinetics, orbital dynamics, circuit and control theory and Newton's second law applications involve second-order ODEs [1]. Ordinary differential equations (ODEs) are commonly used for mathematical modeling in many diverse fields such as engineering, operation research, industrial mathematics, behavioral sciences, artificial intelligence, management and sociology. This mathematical modeling is the art of translating problem from an application area into tractable mathematical formulations whose theoretical and numerical analysis provides insight, answers and guidance useful for the originating application [2].

We consider the conventional k-step linear multi-step methods for the solution of ordinary differential equations (ODE's) of the form

$$y' = f(x, y), \quad y(a) = y_0, \quad x \in [a, b] \qquad (1)$$

where y satisfies a given set of initial condition and we assume that the function f also satisfies the Lipschitz condition which guarantees existence, uniqueness and continuous differentiable solution, [3]. For the discrete solution of (1) linear multi-step methods has being studied by [4],and continuous solutions of (1), [5]. One important advantage of the continuous over the discrete approach is the ability to provide discrete schemes for simultaneous integration. These discrete schemes can as well be reformulated as general linear methods (GLM) [6]. The block methods are self-starting and can directly be applied to both initial and boundary value problems [7]. Block methods for solving ordinary differential equations have initially been proposed by [8] who advanced their use only as a means of obtaining starting values for predictor-corrector algorithms.In this paper we present a two-step hybrid block Simpson's method with two, three and four off-grid points for solving first order stiff ODEs of the form (1).

This paper is organized as follows: in the coming section we carried out the derivation of the method, where we considered two-step with two, three and four off-grid points through the approach of interpolation and collocation. The details of the analysis of the method were discussed in Section three. In the fourth section, some numerical problems were solved and finally, the conclusion was drawn in section five.

II. DERIVATION OF THE METHOD

In this section, a Two-step hybrid blockSimpson's method with two, three and four off-step points,
for solving problem (1) is derived [2] and [9]. Let the power series of the form

$$ {}^{j}y(x) = \sum_{i=0}^{v+m-1} a_i \left(\frac{x - x_n}{h} \right)^i , \quad j = 1, \ldots, m . \tag{2}$$

be the approximate solution to equation (1) for $x \in [x_n, x_2]$ where $n = 0, 1, 2, \cdots N-1$, $a's$ are the real coefficients to be determined, v is the number of collocation points, m is the number of interpolation points and $h = x_n - x_{n-1}$ is a constant step size of the partition of interval $[a, b]$, which is given by $a = x_0 < x_1 < \cdots < x_N = b$. Differentiating Equation (2) once yields:

$$ {}^{j}y'(x) = {}^{j}f\left(x^{j}, y^{j}, {}^{j}y\right) = \sum_{i=1}^{v+m-1} \frac{ia_i}{h} \left(\frac{x - x_n}{h} \right)^{i-1} , \quad j = 1, \cdots, m . \tag{3}$$

Interpolating Equation (2) at the selected intervals, i.e., x_n and collocating Equation (3) at all points in the selected interval, i.e., $x_n, x_{n+\frac{1}{2}}, x_{n+1}, x_{n+\frac{3}{2}}, x_{n+2}$, gives thetwo step block hybrid Simpson's methods with two off- grid points, can be written in matrix form:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{h} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{h} & \frac{1}{h} & \frac{3}{4h} & \frac{1}{2h} & \frac{5}{16h} \\ 0 & \frac{1}{h} & \frac{2}{h} & \frac{3}{h} & \frac{4}{h} & \frac{5}{h} \\ 0 & \frac{1}{h} & \frac{3}{h} & \frac{27}{4h} & \frac{27}{4h} & \frac{405}{16h} \\ 0 & \frac{1}{h} & \frac{4}{h} & \frac{12}{h} & \frac{32}{h} & \frac{80}{h} \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{pmatrix} = \begin{pmatrix} {}^{j}y_n \\ {}^{j}y_{n+\frac{1}{2}} \\ {}^{j}y_{n+1} \\ {}^{j}y_{n+\frac{3}{2}} \\ {}^{j}y_{n+2} \end{pmatrix} \tag{4}$$

Applying the Gaussian elimination method on Equation (4) gives the coefficient $a_i's$, *for* $i = 0\,(1)10$.

These values are then substituted into Equation (2) to give the implicit continuous hybrid method of the form:

$$ {}^{j}y(x) = \sum_{i=\frac{1}{2},\,\frac{3}{2}} {}^{j}\beta_i(x)\,{}^{j}f_{n+i} + \sum_{i=0}^{2} {}^{j}\beta_i(x)\,{}^{j}f_{n+i}, \quad j = 1, \cdots, m . \tag{5}$$

We get four discrete schemes. Hence, the hybrid block methods are as follows

$$\begin{aligned} y_{n+\frac{1}{2}} &= y_n + \tfrac{h}{1440}[251f_n + 646f_{n+\frac{1}{2}} - 264f_{n+1} + 106f_{n+\frac{3}{2}} - 19f_{n+2}] \\ y_{n+1} &= y_n + \tfrac{h}{180}[29f_n + 124f_{n+\frac{1}{2}} + 24f_{n+1} + 4f_{n+\frac{3}{2}} - f_{n+2}] \\ y_{n+\frac{3}{2}} &= y_n + \tfrac{h}{160}[27f_n + 102f_{n+\frac{1}{2}} + 72f_{n+1} + 42f_{n+\frac{3}{2}} - 3f_{n+2}] \\ y_{n+2} &= y_n + \tfrac{h}{45}[7f_n + 32f_{n+\frac{1}{2}} + 12f_{n+1} + 32f_{n+\frac{3}{2}} + 7f_{n+2}] \end{aligned} \tag{6}$$

Interpolating Equation (2) at the selected intervals, i.e., x_n and collocating Equation (3) at all points in the selected interval, i.e., $x_n, x_{n+\frac{1}{2}}, x_{n+1}, x_{n+\frac{5}{4}}, x_{n+\frac{3}{2}}, x_{n+2}$, gives thetwo step block hybrid Simpson's methods with three off- grid points, can be written in matrix form:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{h} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{h} & \frac{1}{h} & \frac{3}{4h} & \frac{1}{2h} & \frac{5}{16h} & \frac{3}{16h} \\ 0 & \frac{1}{h} & \frac{2}{h} & \frac{3}{h} & \frac{4}{h} & \frac{5}{h} & \frac{6}{h} \\ 0 & \frac{1}{h} & \frac{4}{h} & \frac{12}{h} & \frac{32}{h} & \frac{80}{h} & \frac{9375}{512h} \\ 0 & \frac{1}{h} & \frac{3}{h} & \frac{27}{4h} & \frac{27}{4h} & \frac{405}{16h} & \frac{729}{16h} \\ 0 & \frac{1}{h} & \frac{4}{h} & \frac{12}{h} & \frac{32}{h} & \frac{160}{h} & \frac{192}{h} \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{pmatrix} = \begin{pmatrix} {}^{j}y_n \\ {}^{j}y_{n+\frac{1}{2}} \\ {}^{j}y_{n+1} \\ {}^{j}y_{n+\frac{5}{4}} \\ {}^{j}y_{n+\frac{3}{2}} \\ {}^{j}y_{n+2} \end{pmatrix} \tag{7}$$

Applying the Gaussian elimination method on Equation (7) gives the coefficient $a_i's$, *for* $i = 0\,(1)10$.

These values are then substituted into equation (2) to give the implicit continuous hybrid method of the form:

$$ {}^{j}y(x) = \sum_{i=\frac{1}{2},\,\frac{5}{4},\,\frac{3}{2}} {}^{j}\beta_i(x)\,{}^{j}f_{n+i} + \sum_{i=0}^{2} {}^{j}\beta_i(x)\,{}^{j}f_{n+i}, \quad j = 1, \cdots, m \tag{8}$$

We get five discrete schemes. Hence, the hybrid block methods are as follows

$$
\begin{aligned}
y_{n+\frac{1}{2}} &= y_n + \tfrac{h}{720}[112f_n + 413f_{n+\frac{1}{2}} - 537f_{n+1} + 576f_{n+\frac{5}{4}} - 217f_{n+\frac{3}{2}} + 13f_{n+2}] \\
y_{n+1} &= y_n + \tfrac{h}{540}[81f_n + 412f_{n+\frac{1}{2}} - 108f_{n+1} + 256f_{n+\frac{5}{4}} - 108f_{n+\frac{3}{2}} + 7f_{n+2}] \\
y_{n+\frac{5}{4}} &= y_n + \tfrac{5h}{9216}[277f_n + 1400f_{n+\frac{1}{2}} - 150f_{n+1} + 1152f_{n+\frac{5}{4}} - 400f_{n+\frac{3}{2}} + 25f_{n+2}] \\
y_{n+\frac{3}{2}} &= y_n + \tfrac{h}{80}[12f_n + 61f_{n+\frac{1}{2}} - 9f_{n+1} + 64f_{n+\frac{5}{4}} - 9f_{n+\frac{3}{2}} + f_{n+2}] \\
y_{n+2} &= y_n + \tfrac{h}{45}[7f_n + 32f_{n+\frac{1}{2}} + 12f_{n+1} + 32f_{n+\frac{3}{2}} + 7f_{n+2}]
\end{aligned}
$$

(9)

Interpolating Equation (2) at the selected intervals, i.e., x_n and collocating Equation (3) at all points in the selected interval, i.e., x_n, $x_{n+\frac{1}{2}}$, x_{n+1}, $x_{n+\frac{5}{4}}$ $x_{n+\frac{3}{2}}$, $x_{n+\frac{7}{4}}$, x_{n+2} , gives thetwo step block hybrid Simpson's methods with four off- grid points, can be written in matrix form:

$$
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{h} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{h} & \frac{1}{h} & \frac{3}{4h} & \frac{1}{2h} & \frac{5}{16h} & \frac{3}{16h} & \frac{7}{128h} \\
0 & \frac{1}{h} & \frac{2}{h} & \frac{3}{h} & \frac{4}{h} & \frac{5}{h} & \frac{6}{h} & \frac{7}{h} \\
0 & \frac{1}{h} & \frac{4}{h} & \frac{12}{h} & \frac{32}{h} & \frac{80}{h} & \frac{9375}{512h} & \frac{109375}{4096h} \\
0 & \frac{1}{h} & \frac{3}{h} & \frac{27}{4h} & \frac{27}{4h} & \frac{405}{16h} & \frac{729}{16h} & \frac{5103}{64h} \\
0 & \frac{1}{h} & \frac{7}{h2} & \frac{147}{16h} & \frac{343}{16h} & \frac{12005}{256h} & \frac{50421}{512h} & \frac{823543}{4096h} \\
0 & \frac{1}{h} & \frac{4}{h} & \frac{12}{h} & \frac{32}{h} & \frac{160}{h} & \frac{192}{h} & \frac{448}{h}
\end{pmatrix}
\begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \end{pmatrix}
=
\begin{pmatrix} {}^{i}y_n \\ {}^{i}y_{n+\frac{1}{2}} \\ {}^{i}y_{n+1} \\ {}^{i}y_{n+\frac{5}{4}} \\ {}^{i}y_{n+\frac{3}{2}} \\ {}^{i}y_{n+\frac{7}{4}} \\ {}^{i}y_{n+2} \end{pmatrix}
$$

(10)

We get six discrete schemes. Hence, the hybrid block methods are as follows

$$
\begin{aligned}
y_{n+\frac{1}{2}} &= y_n + \tfrac{1}{\text{[illegible]}}h[30585f_n + 143290f_{n+\frac{1}{2}} - 321888f_{n+1} + 519232f_{n+\frac{5}{4}} - 391818f_{n+\frac{3}{2}} \\
&\quad + 149952f_{n+\frac{7}{4}} - 23513f_{n+2}] \\
y_{n+1} &= y_n + \tfrac{1}{\text{[illegible]}}h[3735f_n + 22372f_{n+\frac{1}{2}} - 21672f_{n+1} + 47488f_{n+\frac{5}{4}} - 38052f_{n+\frac{3}{2}} + 14976f_{n+\frac{7}{4}} \\
&\quad - 2387f_{n+2}] \\
y_{n+\frac{5}{4}} &= y_n + \tfrac{1}{\text{[illegible]}}h[19137f_n + 114310f_{n+\frac{1}{2}} - 96600f_{n+1} + 267232f_{n+\frac{5}{4}} - 200550f_{n+\frac{3}{2}} \\
&\quad + 78240f_{n+\frac{7}{4}} - 12425f_{n+2}] \\
y_{n+\frac{3}{2}} &= y_n + \tfrac{1}{\text{[illegible]}}h[1107f_n + 6622f_{n+\frac{1}{2}} - 5712f_{n+1} + 16576f_{n+\frac{5}{4}} - 10542f_{n+\frac{3}{2}} + 4416f_{n+\frac{7}{4}} \\
&\quad - 707f_{n+2}] \\
y_{n+\frac{7}{4}} &= y_n + \tfrac{7}{\text{[illegible]}}h[1395f_n + 8330f_{n+\frac{1}{2}} - 7056f_{n+1} + 20384f_{n+\frac{5}{4}} - 11466f_{n+\frac{3}{2}} + 6624f_{n+\frac{7}{4}} \\
&\quad - 931f_{n+2}] \\
y_{n+2} &= y_n + \tfrac{1}{\text{[illegible]}}h[933f_n + 5600f_{n+\frac{1}{2}} - 4956f_{n+1} + 14336f_{n+\frac{5}{4}} - 8736f_{n+\frac{3}{2}} + 6144f_{n+\frac{7}{4}} - 91f_{n+2}]
\end{aligned}
$$

(11)

III. ANALYSIS OF THE METHOD

In this section,the analysis of the basic properties of the method derived shall be analyzed.

3.1 Order and error Constants of the Method

According to [4], the order of the new method in equation (6), (9) and (11) is obtained by using the Taylor series, and it is found that equation (6) is of mixed order five and six, with an error constant given by

$$C_{5,6} = \left[2.9297\times10^{-4} \;\; 1.7361\times10^{-4} \;\; 2.9297\times10^{-4} \;\; 6.6138\times10^{-5}\right]^T,$$

equation (9) is of uniform order six, with an error constant given by

$$C_6 = \left[-5.0443\times10^{-4} \;\; -4.0903\times10^{-5} \;\; -4.1124\times10^{-5} \;\; -4.0109\times10^{-5} \;\; -6.6138\times10^{-5}\right]^T,$$

and equation (11) is of uniform order eight, with an error constant given by

$$C_8 = \left[1.1130\times10^{-3} \;\; 9.6328\times10^{-5} \;\; 9.7066\times10^{-5} \;\; 9.6535\times10^{-4} \;\; 9.4482\times10^{-5} \;\; 9.7354\times10^{-5}\right]^T$$

3.2 Consistency

Definition 3.1:[5], The hybrid block method (6), (9) and (11) is said to be consistent if it has an order more than or equal to one, i.e., $P \geq 1$. Therefore, the method is consistent.

3.3 Zero Stability

Definition 3.2: [1],The hybrid block method (6), (9) and (11) said to be zero stable if the first characteristic polynomial $\pi(r)$ having roots such that $|r_z| \leq 1$ *and if* $|r_z| = 1$, then the multiplicity of r_z must not be greater than two. In order to find the zero-stability of hybrid block method (6), we only consider the first characteristic polynomial of the method according to Definition [3.2] as follows,

$$\prod(r) = \left| r\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \right| = r^3(r-1)$$

which implies $r = 0, 0, 0, 1$. Hence the method is zero-stable since $|r_z| \leq 1$.

Similarly, according to Definition [3.2], the hybrid block method (9) is as follows,

$$\prod(r) = \left| r\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \right| = r^4(r-1)$$

which implies $r = 0, 0, 0, 0, 1$. Hence the method is zero-stable since $|r_z| \le 1$.

and according to Definition [3.2], the hybrid block method (5) is as follows,

$$\Pi(r) =$$

$$\left| r\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \right| = r^5(r-1)$$

which implies $r = 0, 0, 0, 0, 0, 1$. Hence the method is zero-stable since $|r_z| \le 1$.

3.4 Convergence

Theorem (3.1):[1], the consistency and zero stability are sufficient condition for linear multistep method to be convergent. Since the method (6), (9) and (11) is consistent and zero stable, it implies the method is convergent for all point.

3.5 Region of Absolute Stability of the Block method

According to [4], the absolute stability region of the block method is obtained using (5) and (10) and is as shown below,

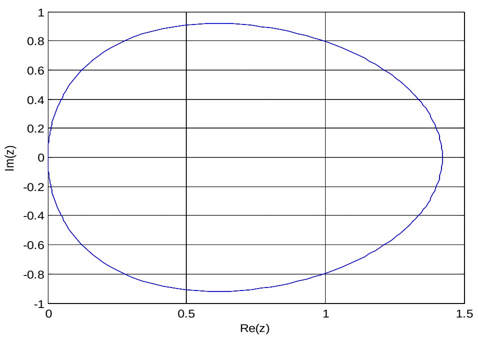

Figure 3.1: Region of absolute stability of the two-step block hybrid Simpson's method with two off-grid points.

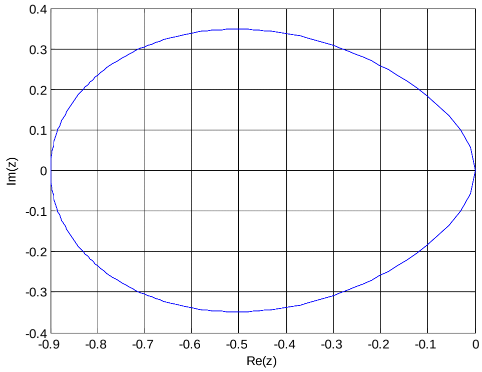

Figure 3.2: Region of absolute stability of the two-step block hybrid Simpson's method with three off-grid points.

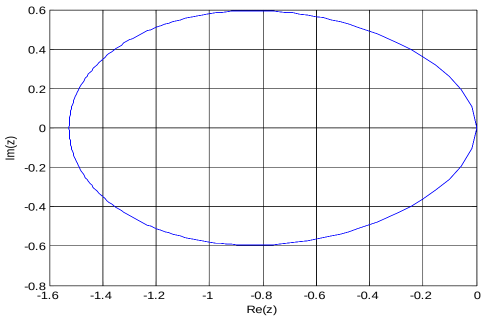

Figure 3.3: Region of absolute stability of the two-step block hybrid Simpson's method with four off-grid points.

IV. NUMERICAL IMPLEMENTATION

In this section, the efficiency and the performance of thegeneral two-step implicit hybrid block Simpson's method with two, three and four off-grid pointsis investigated on threestiffly problems. The performance of the method is examined using the following three systems of first-order initial value problems of ordinary differential equations.Tables 4.1, 4.2 and 4.3belowshow the comparison of the result obtained from the three problems below.

Problem 4.1

$$y_1' = -8y_1 + 7y_2$$
$$y_2' = 42y_1 - 43y_2$$

where

$$h = \tfrac{1}{10}, \qquad y_1(0) = 1, \qquad y_2(0) = 8$$

Exact Solution

$$y_1(x) = 2e^{-x} - e^{-50x}, \qquad y_2(x) = 2e^{-x} + 6e^{-50x}$$

with stiff ratio 5.0×10^1

Problem4.2

$$y_1' = 998y_1 + 1998y_2$$
$$y_2' = -999y_1 - 1999y_2$$
where
$$h = \tfrac{1}{10}, \qquad y_1(0) = 1, \qquad y_2(0) = 1$$
and exact solution
$$y_1(x) = 4e^{-x} - 3e^{-1000x}, \qquad y_2(x) = -2e^{-x} + 3e^{-1000x}$$
with stiff ratio 1.0×10^3

Problem 4.3
$$y_1' = -y_1 + 95y_2$$
$$y_2' = -y_1 - 97y_2$$
where
$$h = \tfrac{1}{10}, \qquad y_1(0) = 1, \qquad y_2(0) = 1$$
and exact solution
$$y_1(x) = \tfrac{95}{47}e^{-2x} - \tfrac{48}{47}e^{-96x}, \qquad y_2(x) = -\tfrac{48}{47}e^{-96x} - \tfrac{1}{47}e^{-2x}$$
with stiff ratio 4.8×10^1

Table 4.1: Comparing the Absolute Stability Errors for problem4.1

	BHSM with two off-grid points		BHSM with three off-grid points		BHSM with four off-grid points	
x	$y_1(x)$	$y_2(x)$	$y_1(x)$	$y_2(x)$	$y_1(x)$	$y_2(x)$
0.1	$1.284E^{-2}$	$7.706E^{-4}$	$1.358E^{0}$	$3.201E^{0}$	$2.011E^{-1}$	$1.815E^{0}$
0.2	$4.574E^{-2}$	$2.744E^{-1}$	$9.020E^{-1}$	$7.357E^{-1}$	$4.699E^{-1}$	$1.504E^{0}$
0.3	$8.961E^{-4}$	$5.377E^{-3}$	$1.093E^{0}$	$2.575E^{0}$	$3.821E^{-1}$	$1.484E^{0}$
0.4	$2.096E^{-4}$	$1.258E^{-2}$	$8.091E^{-1}$	$5.315E^{-1}$	$4.668E^{-1}$	$1.316E^{0}$
0.5	$4.104E^{-5}$	$2.462E^{-4}$	$8.844E^{-1}$	$2.097E^{0}$	$4.715E^{-1}$	$1.219E^{0}$
0.6	$9.595E^{-5}$	$5.757E^{-4}$	$7.222E^{-1}$	$3.754E^{0}$	$4.951E^{-1}$	$1.098E^{0}$
0.7	$1.877E^{-8}$	$1.127E^{-5}$	$7.150E^{-1}$	$1.708E^{0}$	$4.951E^{-1}$	$9.981E^{-1}$
0.8	$4.393E^{-8}$	$2.635E^{-5}$	$6.418E^{-1}$	$2.568E^{0}$	$4.917E^{0}$	$9.018E^{-1}$
0.9	$8.738E^{-8}$	$5.146E^{-5}$	$5.776E^{-1}$	$1.391E^{0}$	$4.791E^{-1}$	$8.166E^{-1}$
1.0	$2.792E^{-7}$	$1.205E^{-6}$	$5.682E^{-1}$	$1.674E^{-1}$	$4.624E^{-1}$	$7.384E^{-1}$
1.1	$4.596E^{-10}$	$2.250E^{-6}$	$4.664E^{-1}$	$1.132E^{0}$	$4.420E^{-1}$	$6.681E^{-1}$
1.2	$9.100E^{-8}$	$5.460E^{-8}$	$5.015E^{-1}$	$1.009E^{-1}$	$4.191E^{-1}$	$6.043E^{-1}$

Table 4.2: Comparing the Absolute Stability Errors for problem4.2

	BHSM with two off-grid points		BHSM with three off-grid points		BHSM with four off-grid points	
x	$y_1(x)$	$y_2(x)$	$y_1(x)$	$y_2(x)$	$y_1(x)$	$y_2(x)$
0.1	$1.819E^{0}$	$3.673E^{0}$	$9.742E^{-2}$	$3.522E^{0}$	$1.978E^{0}$	$1.813E^{0}$
0.2	$1.623E^{0}$	$3.190E^{0}$	$1.460E^{0}$	$4.735E^{0}$	$8.443E^{-1}$	$1.728E^{0}$
0.3	$1.482E^{0}$	$2.963E^{0}$	$4.743E^{-2}$	$3.011E^{0}$	$1.853E^{0}$	$1.494E^{0}$
0.4	$1.341E^{0}$	$2.682E^{0}$	$7.110E^{-2}$	$1.970E^{0}$	$1.779E^{0}$	$1.345E^{0}$
0.5	$1.213E^{0}$	$2.426E^{0}$	$2.309E^{-2}$	$2.403E^{0}$	$1.683E^{0}$	$1.220E^{0}$
0.6	$1.098E^{0}$	$2.195E^{0}$	$3.461E^{-2}$	$2.541E^{0}$	$1.586E^{0}$	$1.104E^{0}$
0.7	$9.932E^{-1}$	$1.986E^{0}$	$1.124E^{-2}$	$1.998E^{0}$	$1.492E^{0}$	$9.984E^{-1}$
0.8	$8.987E^{-1}$	$1.797E^{0}$	$1.685E^{-2}$	$1.629E^{0}$	$1.389E^{0}$	$9.029E^{-1}$
0.9	$8.131E^{-1}$	$1.626E^{0}$	$5.411E^{-3}$	$1.621E^{0}$	$1.292E^{0}$	$8.096E^{-1}$
1.0	$7.358E^{-1}$	$1.472E^{0}$	$8.202E^{-2}$	$1.554E^{0}$	$1.198E^{0}$	$7.329E^{-1}$
1.1	$6.657E^{-1}$	$1.331E^{0}$	$2.663E^{-3}$	$1.334E^{0}$	$1.108E^{0}$	$6.681E^{-1}$
1.2	$6.024E^{-1}$	$1.205E^{0}$	$3.993E^{-2}$	$1.165E^{0}$	$1.021E^{0}$	$6.043E^{-1}$

Table 4.3 : Comparing the Absolute Stability Errors for problem4.3

	BHSM with two off-grid points		BHSM with three off-grid points		BHSM with four off-grid points	
x	$y_1(x)$	$y_2(x)$	$y_1(x)$	$y_2(x)$	$y_1(x)$	$y_2(x)$
0.1	$1.475E^{-1}$	$1.871E^{0}$	$1.922E^{0}$	$1.750E^{0}$	$5.950E^{-3}$	$5.950E^{-3}$
0.2	$2.819E^{-1}$	$1.656E^{0}$	$1.729E^{0}$	$1.461E^{0}$	$4.902E^{-3}$	$4.902E^{-3}$
0.3	$3.723E^{-1}$	$1.493E^{0}$	$1.851E^{0}$	$1.467E^{0}$	$3.489E^{-4}$	$2.889E^{-6}$
0.4	$4.324E^{-1}$	$1.350E^{0}$	$1.761E^{0}$	$1.319E^{0}$	$2.353E^{-4}$	$2.353E^{-6}$
0.5	$4.695E^{-1}$	$1.220E^{0}$	$1.682E^{0}$	$1.206E^{0}$	$1.355E^{-7}$	$6.049E^{-4}$
0.6	$4.888E^{-1}$	$1.104E^{0}$	$1.586E^{0}$	$1.090E^{0}$	$1.098E^{-7}$	$7.294E^{-8}$
0.7	$4.947E^{-1}$	$9.984E^{-2}$	$1.488E^{0}$	$9.879E^{-1}$	$2.400E^{-8}$	$7.060E^{-10}$
0.8	$4.906E^{-1}$	$9.030E^{-2}$	$1.389E^{0}$	$8.943E^{-1}$	$2.100E^{-8}$	$5.460E^{-12}$
0.9	$4.790E^{-1}$	$8.167E^{-2}$	$1.292E^{0}$	$8.096E^{-1}$	$1.600E^{-8}$	$1.999E^{-7}$
1.0	$4.623E^{-1}$	$7.386E^{-2}$	$1.198E^{0}$	$7.329E^{-1}$	$1.340E^{-8}$	$2.999E^{-4}$
1.1	$4.418E^{-1}$	$6.681E^{-2}$	$1.108E^{0}$	$6.634E^{-1}$	$2.300E^{-8}$	$1.200E^{-10}$
1.2	$4.190E^{-1}$	$6.043E^{-2}$	$1.021E^{0}$	$6.045E^{-1}$	$2.000E^{-8}$	$2.900E^{-11}$

V. CONCLUDING REMARKS

In this paper the newly constructed hybrid block Simpson's methods were demonstrated onsome three stiff initial value problems (IVPs). It is evident, from the tables (1-3), the block hybrid Simpson's method with two off-grid points has been shown to be more efficient and converges very well on problem 1 and performs fairly on the problem 2 and 3. Also the block hybrid Simpson's methods with four off-grid points has been shown to be more efficient and converges very well on problem 3 and performs fairly on the problems 1 and 2, while the block hybrid Simpson's methods with three off-grid pointsperforms fairly convergent throughout the three problems.It is obvious that, the newly constructed block hybrid Simpson's methods are efficient, accurate and convergent on mildly stiff problems.

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