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On The Heptic Diophantine Equation With Three Unknowns $7(x^2+y^2)-13xy=28z^7$

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ABSTRACT
We obtain two different patterns of non-zero integral solutions of the Heptic Diophantine equation with three unknowns $7(x^2+y^2)-13xy=28z^7$ by employing suitable transformations.
KEYWORDS: Heptic equation with three unknowns, Integral solutions.

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I INTRODUCTION

Diophantine equations, homogeneous and non- homogeneous have aroused the interest of numerous mathematicians since antiquity as can be seen from [1-2]. The problem of finding all integral solutions of on Interminate equation with three or more variables in general presents a good deal of difficulties. Cubic equations in two variables fall into the theory of elliptic curves which is a very developed theory but still an important topic of current research [3-9]. For equations with degree at least three very little is known. In this communication a Heptic Polynomial equation with three variables represented by $7(x^2+y^2)-13xy=28z^7$ is considered and two different patterns of non-zero integral solutions have been presented.

II METHOD OF ANALYSIS:

The equation under consideration is

$$7(x^2+y^2)-13xy=28z^7 \quad \text{---} \quad (1)$$

Assigning the transformations

$$x=u+v,\ y=u-v \quad \text{---} \quad (2)$$

in (1) leads to

$$u^2+27v^2=28z^7 \quad \text{---} \quad (3)$$

The equation (3) is solved through different approaches and they, one obtains distinct sets of solutions is (1)

Pattern 1:

Assume that $z=a^2+27b^2$ --- (4)

Write $28=\left(1+i\sqrt{27}\right)\left(1-i\sqrt{27}\right)$ --- (5)

use (5) & (4) in (3) and applying the method of factorization, define

$$u+i\sqrt{27}v=\left(1+i\sqrt{27}\right)\left(a+i\sqrt{27}b\right)^7 \quad \text{---} \quad (6)$$

Equating the real and imaginary parts, we have

$$u=u(a,b)=a^7-189a^6b-567a^5b^2+25515a^4b^3+25515a^3b^4-413343a^2b^5-137781ab^6+531441b^7$$

$$v=v(a,b)=a^7+7a^6b-567a^5b^2-945a^4b^3+25515a^3b^4+15309a^2b^5-137781ab^6-19683b^7$$

Substituting the above values of u and v in equation (2), and hence the non-zero integral solutions of (1) are

$$x=2a^7-182a^6b-1134a^5b^2+24570a^4b^3+51030a^3b^4-398034a^2b^5-275562ab^6+511758b^7$$

$$y=-196a^6b+26460a^4b^3-428652a^2b^5+551124b^7 \quad (7)$$

$$z=a^2+27b^2$$

Pattern 2:

Equ (3) can be written as

$$u^2+27v^2=28z^7*1 \quad \text{---} \quad (8)$$

Write $28=\frac{(2+2i\sqrt{27})(2-2i\sqrt{27})}{4}$ --- (9)

and also 1 as

$1 = \frac{(3+i\sqrt{27})(3-3i\sqrt{27})}{36}$ --- (10)

use (4), (10), (9) in (8) and applying the method of factorization, define

$u + i\sqrt{27}v = \frac{1}{12}\{(2 + 2i\sqrt{27})(3 + i\sqrt{27})(a + i27b7$ --- (11)

Equating the real and imaginary part, we have

$u = u(a,b) = \frac{1}{12}\{-48a^7 - 1512a^6b + 27216a^5b^2 + 204120a^4b^3 - 1224720a^3b^4 - 3306744a^2b^5 + 6613488ab^6 + 4251528b^7\}$

$v = v(a,b) = \frac{1}{12}\{8a^7 - 336a^6b - 4536a^5b^2 + 45360a^4b^3 + 204120a^3b^4 - 734832a^5b^5 - 1102248ab^6 + 944784b^7\}$

Substituting the values of u and v in equ (2), then the values of x and y are given by

$x = \frac{1}{12}\{-40a^7 - 1848a^6b + 22680a^5b^2 + 249480a^4b^3 - 1020600a^3b^4 - 4041576a^2b^5 + 5511240ab^6 + 5196312b^7\}$ (12)

$y = \frac{1}{12}\{-56a^7 - 1176a^6b + 31752a^5b^2 + 158760a^4b^3 - 1428840a^3b^4 - 2571912a^2b^5 + 7715736ab^6 + 3306744b^7\}$

As our interest is on finding integer solutions, we choose a and b suitably so that the values of x and y are in integers.

Replace a by 12A and b by 12B in (4) and (12) we get

$x = 12^6\{-40A^7 - 1848A^6B + 22680A^5B^2 + 249480A^4B^3 - 1020600A^3B^4 - 4041576A^2B^5 + 5511240AB^6 + 5196312B^7\}$ (13)

$y = 12^6\{-56A^7 - 1176A^6B + 31752A^5B^2 + 158760A^4B^3 - 1428840A^3B^4 - 2571912A^2B^5 + 7715736AB^6 + 3306744B^7\}$

$z = A^2 + 27B^2$

III CONCLUSION:

In this paper we have presented two different patterns of non-zero integral solutions of the Heptic Diophantine equation with three unknown (1). One may search for other patterns of solutions and their corresponding properties.

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