

## Effect of Slenderness Ratio on Euler Critical Load for Elastic Columns with ANSYS.

Prithviraj R Desai\*, Chintada Satish\*

\*(Department of Civil Engineering, National Institute of Technology Karnataka, Mangalore-575025)

Corresponding author : Prithviraj R Desai

### ABSTRACT

The slender columns are widely used in modern high rise buildings. The bearing capacity of such columns is, obviously, dependent on the effective length or, we can say, slenderness ratio. This paper presents the effect of slenderness ratio over the elastic structural steel columns based on the equivalent model. The critical load that a column can withstand for a defined slenderness ratio for the design is being determined by the buckling analysis of the column. The designed column being circular in cross-section is fixed at one of its ends and free at the other. A vertical load was applied to the slender column top, and the lateral displacement was to be calculated for the whole structure. The computational evaluations were performed with the general purpose finite element code ANSYS 19.0, which can effectively depict the behavior of the designed columns. Finite element model was developed to determine the overall structural response of several designed columns under the project. Then, Eigenvalue Buckling analysis was conducted and buckling critical load could be evaluated by the obtained load multiplier. Finally, the analytic and computational results are compared to assure the reasonable accuracy. It is followed by the graph depicting the effect of slenderness ratio over the buckling load.

**Keywords** - ANSYS, Buckling, Columns, Critical load, Slenderness ratio.

Date of Submission:03-05-2018

Date of acceptance: 19-05-2018

### I. INTRODUCTION

In the modern world, we are witnessing a huge number of tall buildings in various parts of the globe. The columns supporting the huge structures are subjected to enormous loading over it. Some columns not only show lateral displacements but also are fixed at the foot while a load would be applied on the free end. As an engineer, we must consider several possible modes of failure when designing a structure. When subjected to any axial loads, any column would deform and may buckle under the variety of loading conditions. When the member suddenly deflects laterally under axial compression, the bearing load is defined as the critical load in the compression member. At some value of the compressive axial load, the member no longer remains straight, but suddenly deflects laterally, bending like a beam. This lateral deflection caused by axial compression is called buckling. Buckling failures are often sudden and catastrophic, and engineers are ought to know how they can be prevented. A structural member which is subjected to axial compressive force is called as a strut. A strut may be horizontal, vertical or inclined. If it is a vertical member then it is called as columns. Whenever the columns are applied to a load, there are three modes of failure, either crushing or buckling or both. Generally, short columns fail due to crushing and long columns due to buckling. The

intermediates would usually fail due to both. Hence we classify the columns into short and long columns based on the parameter called as slenderness ratio. It is defined as the ratio of effective length to the minimum radius of gyration. The effective length is defined as the length of the columns between the lateral supports. The radius of gyration is the root of the ratio of Moment of Inertia to the cross-sectional area.

### II. EULER THEORY FOR BUCKLING OF COLUMNS

The following assumptions are made for the analysis.

- (1) The column is perfectly straight and the load is applied axially.
- (2) The cross-section of the column is uniform. The column is perfectly elastic, homogeneous and isotropic material.
- (3) The length of the column is very large compared to the cross-section.
- (4) The shortening of the column due to direct compression is neglected.
- (5) The column fails by buckling only.

The maximum load that a column could withstand without any lateral displacement is called buckling load. It is called as Euler buckling load or critical

load. The expression for the Euler buckling load is given by

$$P_{cr} = \frac{\pi^2 EI}{l_{eff}^2} \quad (1)$$

where  $P_{cr}$  is the critical load,  $E$  is Young's modulus,  $I$  is the moment of inertia and  $l_{eff}$  is the effective length of the column. The different cases of the columns that are designed conventionally are listed below followed by its expression for the critical load.

**Table 1:** Different cases of columns with their critical loads.

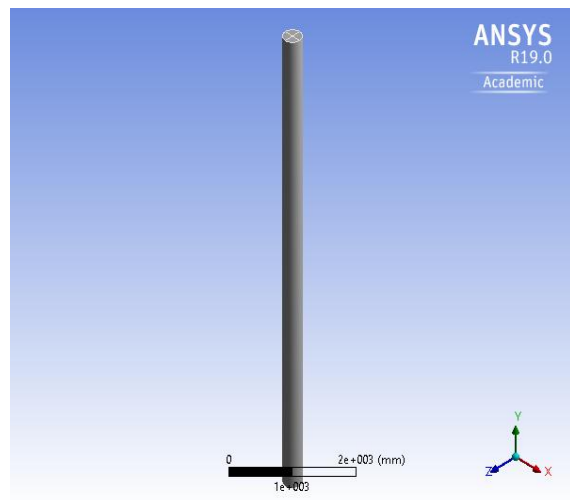
Different cases of columns	Effective length	Critical load
Column with pinned ends.	$l$	$P_{cr} = \frac{\pi^2 EI}{l^2}$
Column with one end fixed and other free.	$2l$	$P_{cr} = \frac{\pi^2 EI}{4l^2}$
Column with fixed ends.	$0.5l$	$P_{cr} = \frac{4\pi^2 EI}{l^2}$
Column with one end fixed and other pinned.	$0.7l$	$P_{cr} = \frac{2.05\pi^2 EI}{l^2}$

The current study is based on the condition where the column is fixed on one of its ends and other being free. We will investigate the changes occurring with the effect of the change in slenderness ratio of the columns. The computation results are obtained from the ANSYS and then the results are compared with the theoretical solutions.

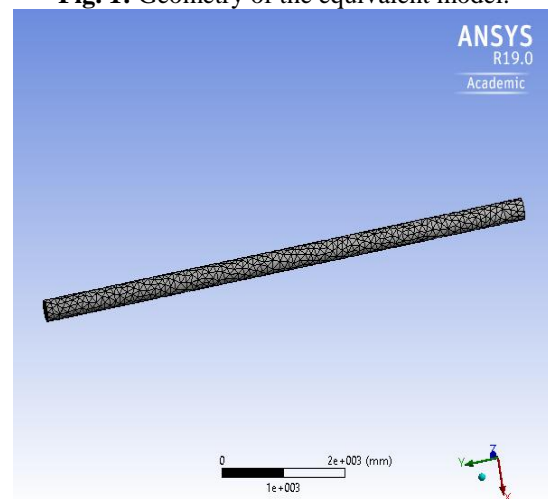
### III. ANSYS COMPUTATION

The cross-section is determined by the slenderness ratio for an 8000 mm column. Thus, the required models are designed in the ANSYS and eigenvalue buckling analysis is computed. The elements of the mesh were of size 100 mm each. A unit load was applied to the free end of the column while the other end was fixed. Such models with different slenderness ratio was developed and computed under the project.

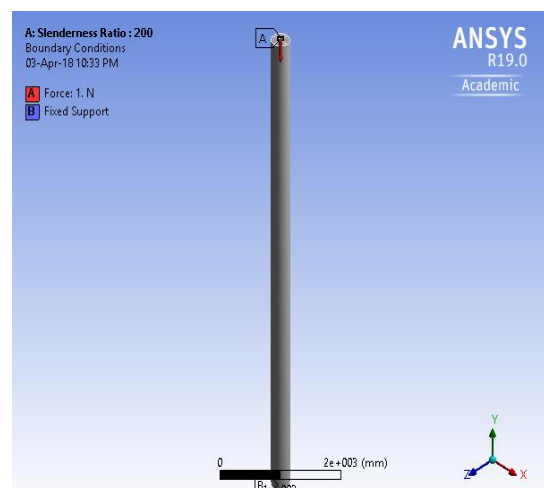
We shall perform and evaluate critical load for a column, for instance, having slenderness ratio 200. Similarly, the rest of the models were generated and results were obtained and tabulated.



**Fig. 1:** Geometry of the equivalent model.



**Fig. 2:** Meshing of the model with 100mm each.



**Fig. 3:** Boundary conditions and applied load.

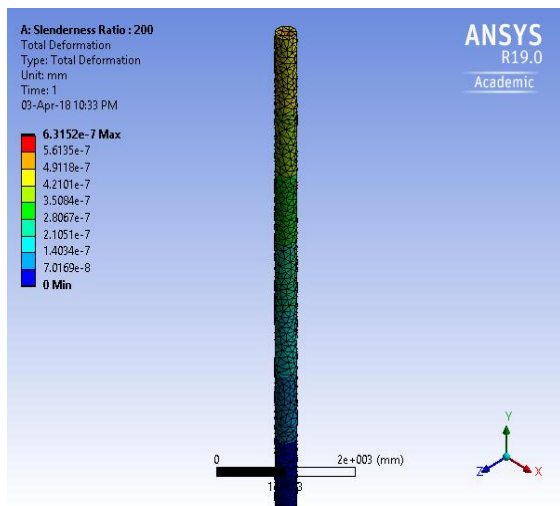


Fig. 4: Total deformation of the model.

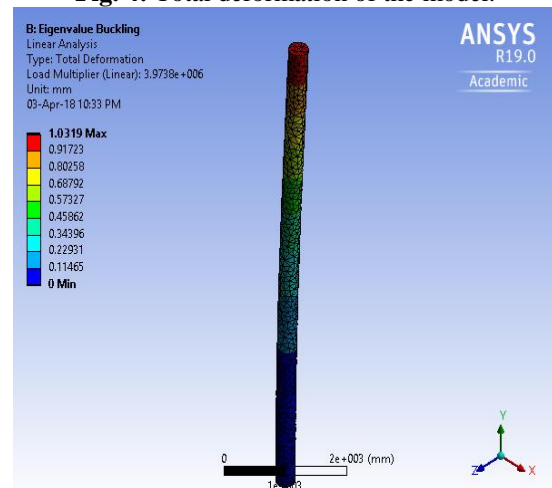


Fig. 5: Load multiplier with the application of unit load on the model.

#### IV. RESULTS

The Eigenvalue Buckling analysis has been carried out using ANSYS. We observe that the results for critical load are well matched with the theoretical results. Table 2 shows the comparisons of the study performed. Figures presented above show the 3D modeling of the columns and deformation caused by the application of unit axial load at the free end. The buckling behavior of several columns with gradual increase of their slenderness ratio was studied. There were 20 models investigated using finite element analysis. The results obtained show the critical load that a column could be subjected to, for the safe construction of buildings.

Table 2: Comparison between the Theoretical and Computation values.

Slenderness ratio	Diameter (mm)	Theoretical value (N* x 10 <sup>6</sup> )	Computational value (N* x 10 <sup>6</sup> )	Accuracy (%)
40	1600	2483.498	2302.000	7.30

			0	
60	1066.67	490.5676	473.1500	3.55
80	800	155.2186	154.4600	0.48
100	640	63.5775	63.4560	0.19
120	533.34	30.6604	30.6510	0.03
140	457.14	16.5497	16.5470	0.01
150	426.67	12.5585	12.5620	0.02
160	400	9.7011	9.7030	0.02
170	376.47	7.6121	7.6138	0.02
180	355.56	6.0563	6.0581	0.03
190	336.84	4.8785	4.8786	0.01
200	320	3.9735	3.9738	0.01
210	304.76	3.2691	3.2694	0.01
220	290.91	2.7140	2.7143	0.01
230	278.26	2.2718	2.2719	0.01
240	266.67	1.9162	1.9166	0.02
250	256	1.6275	1.6284	0.05
260	246.15	1.3912	1.3913	0.01
280	228.57	1.0343	1.0344	0.01
300	213.34	0.7850	0.7851	0.01

N\* - Newton

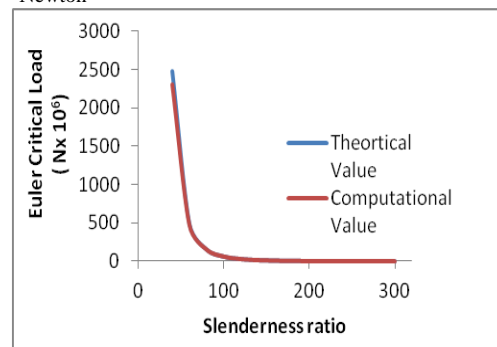


Fig. 6: Euler Critical Load and Slenderness ratio

We could observe, with the increase in the slenderness ratio the critical loads decreases. We shall plot the same graph beyond the slenderness ratio 120 to depict the curve precisely.

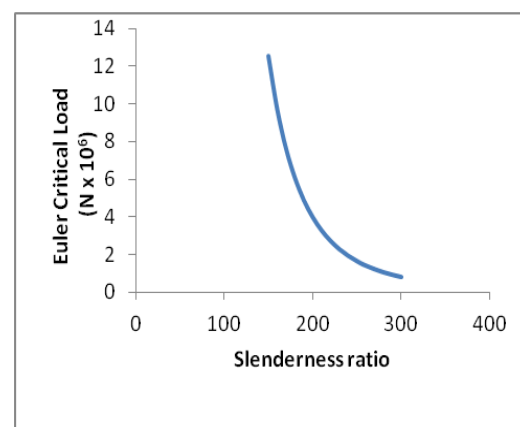


Fig. 7: Euler Critical Load and Slenderness ratio

#### V. CONCLUSIONS

The buckling analysis of the circular cross-sectional columns was successfully determined in analytic and computational methods. The critical loading was independently calculated by using Euler theory. It is always advisable to keep the subjected

load over the column lesser than the determined critical loads. It is also observed that it is easy to determine the slenderness ratio knowing the designed load that a column would bear in the structure. We also noticed that the critical loads were more accurate for the higher ratios. It justifies that the Euler theory is more precise for longer columns. The theory would fail, in case of slenderness ratio 40, with the fact that the short columns have the crushing mode of failure being a factor.

#### ACKNOWLEDGEMENTS

The work was carried out under the supervision of our beloved teachers. We deeply express our gratitude towards their immense support and efforts.

#### REFERENCES

- [1]. Hongwei Ma, Liang Ye, Lang Lin, A study on effective length of slender columns with elastic restraints (Procedia Engineering, 6<sup>th</sup> International Workshop on Performance, Protection & Strengthening of Structures under Extreme Loading, Vol. 210,2017 228-234)
- [2]. Stephen P. Timoshenko, James M. Gere, Mechanics of Materials (Saint-Petersburg, Moscow, 2002)
- [3]. Roy R.Craig, Jr., Mechanics of Materials (John and Wiley Sons, 3<sup>rd</sup> Edition)
- [4]. A. Webber, J.J. Orr, P.Shepherd, K. Crothers, The effective length columns in multi-storey frames (Engineering structures, Vol. 102, 2015 132-143)
- [5]. Shruti Deshpande, Buckling And Post Buckling Of Structural Components (The University Of Texas At Arlington, pp. 1 – 65, December 2010.)
- [6]. Ansys Manuals.
- [7]. Buckling and Euler's Critical Load (Wikipedia)
- [8]. NPTEL Online courses

Prithviraj R Desai "Effect of Slenderness Ratio on Euler Critical Load for Elastic Columns with ANSYS." *International Journal of Engineering Research and Applications (IJERA)* . vol. 8. no.5. 2018. pp. 40-43