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# Implementation of Numerical Methods of Euler and Runge-Kutta through MATLAB Software for the Solution of Ordinary Differential Equations Dedicated to Teaching

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**ABSTRACT**
Given the complexity of problems in the engineering field, new tools have become essential for solving them in academic society, so computer modeling and simulation through software has been one of the main alternatives found by researchers in the area to attend to these demands. Therefore, the objective of this work is to implement two numerical methods, namely Euler and Runge-Kutta, in which the purpose is the solution of Ordinary Differential Equations, having as a differential the computational modeling, which makes possible the solution of the problems that would be impossible or would require much time and energy of the involved researches, thus making the process simpler, besides reducing the probability of operational error. The scientific methodology of this study is based on applied research using MATLAB software, which seeks to obtain qualitative results, being that it uses the Significant Learning Theory, which argues that the new knowledge must be anchored to the knowledge acquired in the students, thus demonstrating a way or a direction to learn the methods to their practical applications, correlating with the problems faced in everyday life. In this way, the following study culminates in important results for the scientific society, being these the correlation existing between the methods, where it illustrates its particularities and precisions, besides connecting this knowledge with the teaching practice of the same ones.

**Keywords:** Euler, MATLAB Software, Numerical Methods, Ordinary Differential Equation, Runge-Kutta.

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## I. INTRODUCTION

The object of study of the following work are the Ordinary Differential Equations, and by definition, a differential equation is which the unknown is a function, and in addition, it has a relation with the derivatives of this function, and these are applicable in problems of mathematical modeling. In addition, the Ordinary Differential Equations – ODEs are classified when the unknown function involves those derived from only one independent variable [1]. In this way, ODEs are of great interest in the field of Exact Sciences, due to the fact that it is much used to solve technical-scientific problems equivalent to physical and mathematical quantity changes that describe them, as a consequence of their relations and laws that can be formulated to represent their conceptions and characteristics.

Having as problematic, the fact that many problems involving Ordinary Differential Equations are extremely complex, and in many cases, they are not feasible to be solved through the analytical method, due to the fact that they have needs of high energy expenditures and development times of all the process in addition to that may have a high rate of operational errors due to this type of obstacle. Therefore, the justification of the proposed study is to use computational modeling through MATLAB software, as a resource to soften or extinguish the problems arising from the ODEs solution process without the aid of simulation.

Thus, the objective of the work is to implement and compare the results obtained by the numerical methods of Euler and Runge-Kutta, based on the use of the same ODE in order to verify the particularity of each method, exposing their approximation. These results will be obtained through a computational modeling, followed by a mathematical sequence for the implementation of logical and algebraic operations, through MATLAB software. In addition, as a basis, the Theory of Significant Learning will be used which will demonstrate a path to learning the methods to their practical applications, correlating with the problems faced in everyday life.

## II. ORDINARY DIFFERENTIAL EQUATION - ODE

Following an equation $F(x) = 0$, in which the unknown $x$ is a function containing only one variable, is determined as the Ordinary Differential Equation. In this way, these are widely used to demonstrate general laws of disciplines such as Biology, Physics and Economics, which find their natural principles in these mathematical equations. From another perspective, a large number of questions involving calculus, such as differential geometry, topology and generative calculus of variations, are either elaborated by ODEs or are reduced to them [2].

According to [1], the Ordinary Differential Equations can be classified according to their order, in other words, according to the highest degree derived from the unknown function that occurs in the equation, as shown in equation (1) and equation (2).

$$\frac{dx}{dy} = F(x, y) \tag{1}$$

$$\frac{d^2x}{dt^2} = F\left(t, x, \frac{dx}{dt}\right) \tag{2}$$

### 2.1 Numerical Methods

Numerical methods seek to solve problems involving calculus, with processes that compose algorithms, using a finite sequence of operations of the simple arithmetic type, so that certain mathematical problems, previously unfeasible to be solved analytically, due to their high degree of complexity, become feasible [3]. Thus, in view of the fast development of the computational capacity present in today's world, numerical methods have reached a large space in the area of problem solving, where it has a fundamental role in the formation of courses, such as Chemistry, Economics, Physics, Engineering, Medicine, and others [4].

#### 2.1.1 Euler's Method

The Euler's method, whose name has the relation with Leonhard Euler, used in computational science and mathematics is a first-order numerical procedure whose purpose is to solve problems involving Ordinary Differential Equations, being this one of the simplest explicit methods for numerical integration in Ordinary Differential Equations [5].

In which the approximation of its result is determined by a line, and its formulation is expressed by equation (3) [6].

$$y_{i+1} = y_i + hf\ (x_i, y_i) \tag{3}$$

#### 2.1.2 Runge-Kutta's Method

The Runge-Kuttas's method is very used in Engineering, especially in the numerical analysis due to its degree of approximation, in which it allows numerical calculus with more accuracy due to having second and fourth order formulation, and when compared with others that have lower order, the same is more efficient [7].

Being that all Runge-Kutta's methods have their formulation according to the general form expressed by equation (4) [8].

$$y_{i+1} = y_i + h\Phi\ (x_i, y_i, h) \tag{4}$$

Therefore, for the second order and fourth order Runge-Kutta's method, their formulations can be represented respectively by equation (5) and equation (6) [8].

$$y_{i+1} = y_i + h\ (\tfrac{1}{2}k_1 + \tfrac{1}{2}k_2) \tag{5}$$

$$y_{i+1} = y_i + \tfrac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{6}$$

Where:

$k_1 = f(x_i, y_i)$

$k_2 = f(x_i + \dfrac{h}{6}, y_i + \dfrac{hk_1}{2})$

$k_3 = f(x_i + \dfrac{h}{2}, y_i + \dfrac{hk_2}{2})$

$k_4 = f(x_i + h, y_i + hk_3)$

## III. SIGNIFICANT LEARNING THEORY

One of the most efficient theories in engineering teaching is that of Significant Learning, where Ausubel was one of the proponents of this theory, which is to relate new information to a relevant pre-existing concept in the individual's cognitive structure. The same argued that the human brain is highly organized when it comes to storing information, thus facilitating this anchoring between new and existing information [9].

This theory has already been used for the development of new studies among renowned authors of scientific society, as in [10] carried out in his research, the development of a proposal for the teaching of Resistance of Materials, where he developed a Potentially Significant Material for calculation of tension and deformation in axial loads, using as base the theory of significant learning, which proved to be very efficient for teaching in engineering, through the results obtained by the work.

Conceptual maps are diagrams that indicate relationships between concepts addressed by the study in question, and these should be illustrated in such a way that the primordial tend to have a larger dimension and the less crucial ones are illustrated in a less obvious way. What is important is that these maps clearly represent the correlations between related concepts, so as to facilitate the learning and organization of principles by readers [11]. In this way, the one described in figure (1) represents the study concepts of this work.

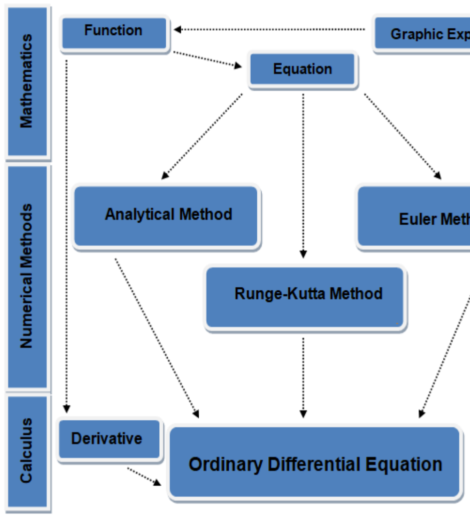


**Figure 01:** Conceptual Map.

## IV. METHODOLOGY

### 4.1 Materials

The study in question was developed by the authors in the Laboratory of Modeling and Simulation located at the Fluminense Federal University, to which it provides full access to MATLAB software for the development of scientific researches among students and researchers linked to the university.

Being that MATLAB is a software that uses an extremely effective and interactive language, which is widely used in several applications in engineering, especially aimed at numerical calculation. Integrating the technical computation for calculation with matrices, elaboration of graphs, numerical analysis, besides signal processing and algorithm development [12]. Consequently, such software was used due to the simplicity of its programming language, in addition to its characteristics and properties that facilitated the execution of the methods approached by the work.

### 4.2 Methods

The scientific methodology used in the study was based on applied research, in which the acquired knowledge is used for practical application in the software aiming at solving the proposed Ordinary Differential Equations. Regarding the way of approach, this study is classified as qualitative, because through the interpretation of the phenomenon resulting from the work, it was possible to attribute meanings to them. In addition, as for the technical procedures, bibliographical research is used to provide the theoretical basis necessary for the development of the whole process, besides the experimental research, where the entire computational programming process is carried out, applying the numerical methods in the software, besides of the analysis of the same, generating the results discussed in this work.

Therefore, was used the numerical methods of Euler and Runge-Kutta of the second and the fourth order, besides the analytical method for solving the same Ordinary Differential Equation, seeking to analyze the results obtained by the numerical methods by comparing the approximations found by them and the exact method of the analytical solution.

For this, the methods in question were adapted to a simple computer programming language, inserting their specific formulations of each method through the MATLAB software, so that it was able to perform the mathematical operations, besides the generation of graphs from the results obtained from these algebraic operations.

## V. RESULTS AND DISCUSSION

### 5.1 Analytical Method

The exact result of the proposed Ordinary Differential Equation was found by the analytical development of the calculations using the MATLAB software, and its result, shown in figure (2), served as a basis of comparison for the numerical methods in question, in this way it was possible to analyze the approximations and the supposed errors of each method.

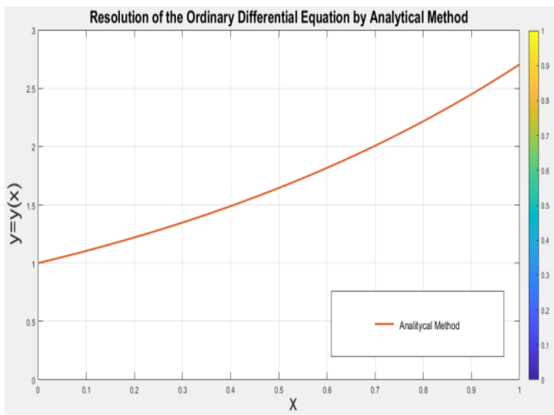


**Figure 02:** Resolution of the ODE by Analytical Method.

### 5.2 Euler's Method

The Euler method was implemented in MATLAB software by developing the calculation of its previously established formulation, according to figure (3).

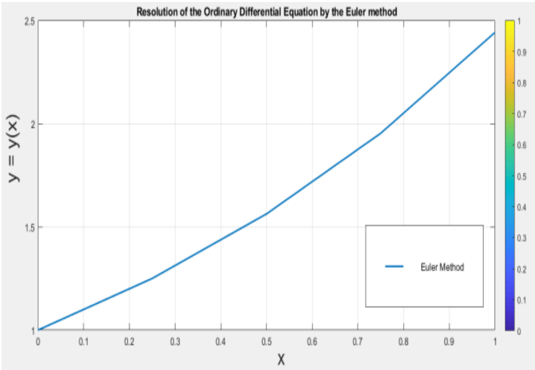


**Figure 03:** Resolution of the ODE by Euler's Method.

Afterwards, a comparison between the results of the Euler method and the Analytical method was performed. The error found was approximately 10% of the ODE, due to the step used, which was 5, according to figure (4).

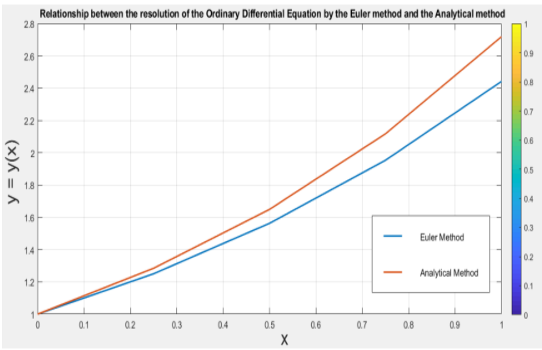


**Figure 04:** Relation between the resolutions of the ODE by Euler's Method and Analytical Method with step of 5.

Being that, the degree of approximation of the numerical result with the analytical result is defined according to the step size, so the larger the step, the better the accuracy of the Euler method. Thus, the same solution process was implemented in the software, but with the larger step, this being 100, finding a significantly lower error rate than previously, with approximately 0.5%, as shown in figure (5).

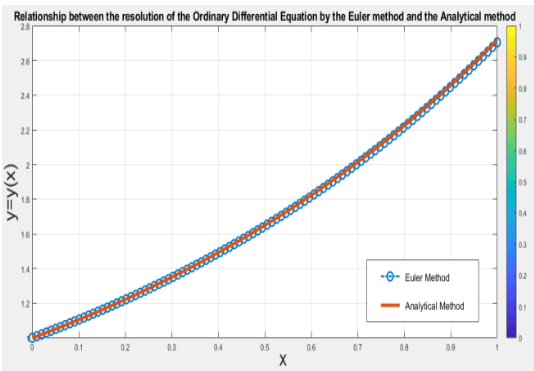


**Figure 05:** Relation between the resolutions of The ODE by Euler's Method and Analytical Method with step of 100.

### 5.3 Runge-Kutta's Method

At this stage of work, the second and fourth order Runge-Kutta method was implemented in the MATLAB software by developing the calculation of its previously established formulations, according to figure (6) and figure (7).

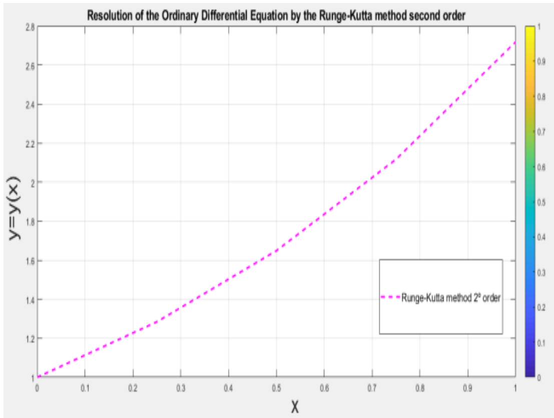


**Figure 06:** Resolution of the ODE by Runge-Kutta's Method of second order.

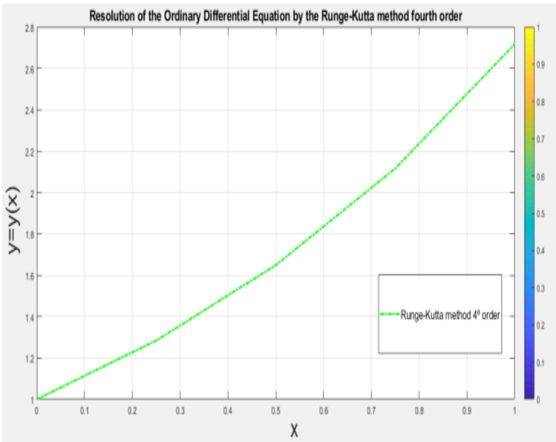


**Figure 07:** Resolution of the ODE by Runge-Kutta's Method of fourth order.

Then, a comparison was made between the second and fourth order Runge-Kutta methods, and the Analytical method. As can be observed in figure (8), the results of second and fourth order Runge-Kutta methods obtained a minimum error, which is almost imperceptible.

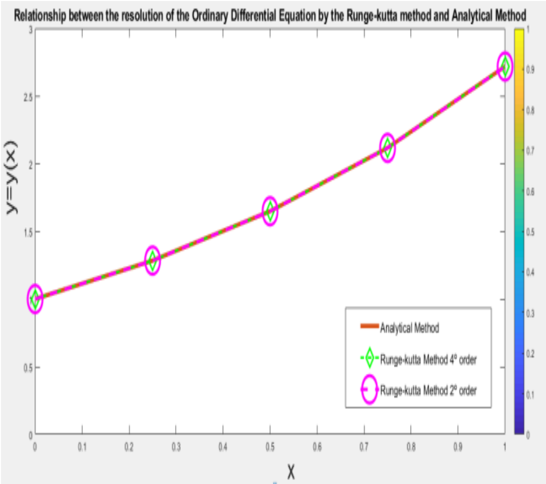


**Figure 08:** Relation between the resolutions of the ODE by Runge-Kutta's Method and Analytical Method.

### 5.4 Relation Between All Methods

As shown in figure (9), it is evident that when we analyze the comparison between the methods using only 5 steps in its development, the numerical method that demonstrates a better accuracy in terms of result is the second and fourth order Runge-Kutta, compared to the Euler method.

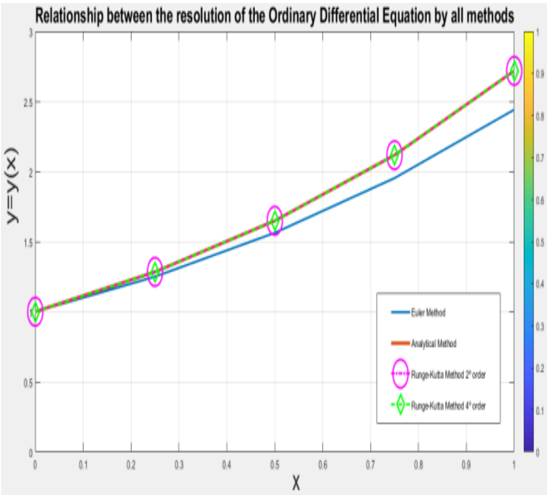


**Figure 09:** Relation between the resolutions of the ODE by All Methods with step of 5.

However, when using a better amount of steps, all methods proved to be effective for the solution of the proposed Ordinary Differential Equation, as shown in figure (10).


**Figure 10:** Relation between the resolutions of the ODE by All Methods with step of 100.

## VI. CONCLUSION

As a conclusion to the proposed study, it is well known that the use of computational tools, such as MATLAB software, is extremely necessary both to solve the problems proposed, especially in numerical calculation, and to illustrate in a didactic way the graphical representation of the results obtained by the methods.

Thus, it is of utmost importance for the learning to demonstrate clearly the concepts approached theoretically, this being possible through the aid of computational modeling, where it makes the connection between them, through graphical illustrations, of normally complex results and not so easy to be assimilated, consequently providing the student a better absorption of the content.

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## REFERENCES

[1] H. L. Guidorizzi, Um Curso De Cálculo (LTC Editora, Ed. 5, V. 4, 548 P., 2002).
[2] J. C. Butcher, Numerical Methods For Ordinary Differential Equations (John Wiley & Sons, Third Edition, 538 P., 2016).
[3] F. Stenger, Handbook Of Sinc Numerical Methods (CRC Press, 482 P., 2016).
[4] J. D. Faires, And R. L. Burden, Numerical Methods (Cengage Learning, Fourth Edition, 608 P., 2012).

[5] S. C. Chapra, And R. P. Canale, Métodos Numéricos Para Engenharia (AMGH Editora, Ed. 5, 825 P., 2011).
[6] C. Li, And F. Zeng, Numerical Methods For Fractional Calculus (CRC Press, 300 P., 2015).
[7] N. B. Franco, Cálculo Numérico (Pearson, Ed. 1, 520 P., 2006).
[8] T. Young, And M. J. Mohlenkamp, Introduction To Numerical Methods And MATLAB Programming For Engineers (Athens, 182 P., 2017).
[9] M. A. Moreira, Al Final, Qué Es Aprendizaje Significativo, Qurriculum: Revista De Teoría, Investigación Y Prática Educativa, La Laguna -Espanha, N. 25, P. 29-56, 2012.
[10] C. F. L. Junior, Et Al., Uma Proposta Para O Ensino De Resistência Dos Materiais Com O Software Geogebra: Desenvolvimento De Um Material Potencialmente Significativo Para Cálculo De Tensão E Deformação Em Carregamentos Axiais, EDU.TEC – Revista Científica Digital Da FAETEC, V. 1, N. 1, Pp. 1-16, 2017.
[11] R. Tavares, Construindo Mapas Conceituais, Ciência & Cognição, V. 12, P. 75-85, 2007.
[12] A. Gilat, Matlab: An Introduction With Applications (John Wiley & Sons, Fourth Edition, 416 P., 2011).

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