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RESEARCH ARTICLE OPEN

Non-Linear Water Wave Transformation Model

Syawaluddin Hutahaean
Ocean Engineering Program, Faculty Of Civil And Environmental Engineering – Bandung Institute Of Technology (Itb) - Indonesia

ABSTRACT
This Research Develops One Dimension Shoaling And Breaking Model And Refraction-Diffraction Water Wave Model Using Non-Linear Potential Flow Water Wave That Are Formulated In Non-Linear Condition, I.E. The Formulations Were Done Without Assumptions Small Wave Amplitude And Deep Water Conditions. Also The Formulations Were Done For Sloping Bottom So That The Potential Flow Contains Shoaling Phenomenon. Breaking Phenomenon Also Exists At The Potential Flow That Can Be Stated With An Equation Similar In The Form To Miche's Breaking Criteria. The Effect Of Non-Linearity In Wave Amplitude Is Showed In The Wave Length, Where Wave Length Is Not Only Affected By Wave Period And Water Depth But Also By It's Amplitude. The Result Of Shoaling And Breaking Model Shows Conformity With The Result Of A Number Of Researchers Connecting Initial Waves Height And Wave Length In Deep Water With Breaking Wave Height.Based On The One Dimension Shoaling And Breaking Model, The Refraction-Diffraction Model Is Developed. The Execution Of Refraction-Diffraction Model In A Bay Shaped Bathymetry Shows A Phenomenon Of Wave Energy Spread, Whereas Execution Over Submerged Island Bathymetry Shows A Concentration Of Wave Energy At The Island Center. Those Two Phenomena Show That Refraction-Diffraction, Shoaling And Breaking Phenomena Can Be Simulated Well By The Model.
Index Terms: Nonlinear Water Wave Potential Flow, Shoaling And Breaking Water Wave Model , Reffraction, Diffraction And Breaking Of Water Wave Model.

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I. INTRODUCTION

Information On Waves Is Generally About The Wave Condition In The Deep Water. The Construction Of A Coastal Building Requires Information On Wave Height At The Shallow Coastal Water Where From The Deep Water Toward Shallow Coastal Water, Wave Will Experience Some Transformations Or Changes Which Include Among Others Are Refraction-Diffraction, Shoaling And Breaking. As A Consequence, Wave Transformation Model Needs To Be Continuously Developed.

In This Research, A Wave Transformation Model Is Developed Based On Potential Flow Equation Containing Breaking Phenomenon (Hutahaean 2007a-B, 2008a-B, 2015) With The Aim Of Obtaining A Model Capable Of Simulating Various Wave Transformation Phenomenon Well.

The Model Is Developed Using An Equation Of A Wave Moving In The Direction Of ξ Axis That Forms An Angle Of θ Against x Axis, And The Calculation Of Wave Parameter Change In The Direction Of x Axis Was Conducted. In Other Words, In This Research A Method Is Developed To Calculate Amplitude Change And Wave Direction Toward x, For A Wave Moving In The Direction Of ξ Axis That Forms An Angle Of θ Against x Axis, Where The Differential Equation That Is Formed Is Parabolic Differential Equation.

II. NONLINEAR WATER WAVE POTENTIAL FLOW.

Potential Flow Equation Is The Result Of Laplace Equation Solution, For Wave Moving In The Direction Of ξ Axis (Hutahaean 2007a-B, 2008a-B, 2015), With Axis System As Presented In **Figure (1).**

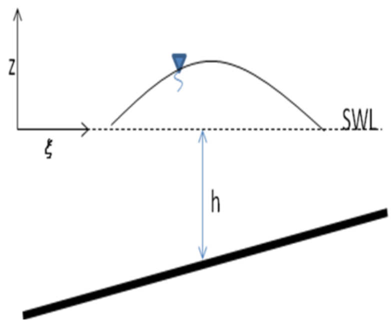


Figure (1). Water Wave Sketch And Axis System (ξ, z)

$$\phi = Ge^{kh}\beta(z)\cos k\xi \,\sin\sigma t$$
...............................(1)

G = Wave Constant,

$$G = \frac{\sigma A}{F}$$
......................................(2)

$$F = e^{kh}\left(k\beta_1\left(\frac{A}{2}\right) - \beta\left(\frac{A}{2}\right)\left(k - \frac{1}{2h}\frac{\partial h}{\partial \xi}\right)\frac{kA}{2}\right)$$
....(3)

$$\frac{\partial G}{\partial \xi} = -\frac{G}{2k}\frac{\partial k}{\partial \xi} = \frac{G}{2h}\frac{\partial h}{\partial \xi}$$
....................................(4)

k = Wave Number = $\frac{2\pi}{L}$, L = Wave Length,

Where,

$$\frac{\partial kh}{\partial \xi} = 0 \qquad \text{Or} \qquad \frac{\partial k\left(h + \frac{A}{2}\right)}{\partial \xi} = 0$$
......................................(5)

h = Water Depth, σ = Angular Frequency = $\frac{2\pi}{T}$ And

T = Wave Period

$$\beta(z) = \alpha e^{k(h+z)} + e^{-k(h+z)},$$
$$\beta_1(z) = \alpha e^{k(h+z)} - e^{-k(h+z)}$$
.....................(6)

$$\alpha = \frac{1}{2}\left(\frac{1 + \frac{\partial h}{\partial \xi}}{1 - \frac{\partial h}{\partial \xi}} + \frac{1 - \frac{\partial h}{\partial \xi}}{1 + \frac{\partial h}{\partial \xi}}\right)$$
......................(7)

$\frac{\partial h}{\partial \xi}$ Is Seabed Slope. The Horizontal Velocity Of Particle In The Direction Of ξ Axis Is

$$u = Ge^{kh}\beta(z)\left(k\sin k\xi - \frac{1}{2h}\frac{\partial h}{\partial \xi}\cos k\xi\right)\sin\sigma t$$
............................(8)

Vertical Velocity Of Particle At The Direction Of z Axis

$$w = -Ge^{kh}k\beta_1(z)\cos k\xi \sin\sigma t$$
............................(9)

Water Surface Equation, $\eta = A\cos k\xi \cos\sigma t$
..........................(10)

2.1. Wave Length

At Potential Flow Equation, Equation (1), There Are 2 Unknown Constants, I.E. G And k. The Equations For Calculating Those Two Constants Are Formulated In This Section.

Hutahaean (2007a-B Dan 2008a-B, 2010) Formulated The Method Of Calculation Of The Two Constants Using Kinematic Free Surface Boundary Condition And Momentum Equation , Where The Free Surface Boundary Condition Is (Dean, 1984):

$$w_\eta = \frac{\partial \eta}{\partial t} + u_\eta \frac{\partial \eta}{\partial x}$$

u_η Is Particle Velocity Toward x Horizontal Direction And w_η Is Particle Velocity Of Vertical Direction, Both Are On The Surface. By Substituting u_η And w_η And $\frac{\partial \eta}{\partial x}$ From Equations (8), (9) And (10), And By Working On The Condition

$$\cos kx = \sin kx = \cos\sigma t = \sin\sigma t = \frac{\sqrt{2}}{2},$$

Equation

$$\sigma A = Ge^{kh}k\beta_1(\eta) - Gke^{kh}\beta(\eta)\frac{kA}{2} + \frac{G}{2h}\frac{\partial h}{\partial x}e^{kh}\beta(\eta)\frac{kA}{2}$$
..........................(11)

Can Be Obtained. The Kinematic Free Surface Boundary Condition Is An Equation For G And k That Can Be Written As

$$f_1(G,k) = -\sigma A + Ge^{kh}k\beta_1(\eta) - Gke^{kh}\beta(\eta)\frac{kA}{2} + \frac{G}{2h}\frac{\partial h}{\partial x}e^{kh}\beta(\eta)\frac{kA}{2} = 0$$
.............(12)

As The Next Equation Is Surface Momentum Equation, Hutahaean And Achyari (2017),

$$\frac{\partial u_\eta}{\partial t} = \frac{\beta_1(\eta)}{\beta(\eta)}ku_\eta\frac{\partial \eta}{\partial t} - \frac{1}{2}\frac{\partial}{\partial x}\left(u_\eta u_\eta + w_\eta w_\eta\right) - g\frac{\partial \eta}{\partial x}$$
.......................(13)

Or,

$$f_2(G,k) = \frac{\partial u_\eta}{\partial t} - \frac{\beta_1(\eta)}{\beta(\eta)}ku_\eta\frac{\partial \eta}{\partial t} + \frac{1}{2}\frac{\partial}{\partial x}\left(u_\eta^2 + w_\eta^2\right) + g\frac{\partial \eta}{\partial x} = 0$$
.............(14)

η At $\beta(\eta)$ And $\beta_1(\eta)$ At Kinematic Free Surface Boundary Condition Equations And

Surface Momentum Equation Is $\frac{A}{2}$, This Is Considering That Equation (14) Is Executed At The Conditions

$$\cos kx = \sin kx = \cos \sigma t = \sin \sigma t = \frac{\sqrt{2}}{2}.$$

At Equation (14), Velocity And Surface Water Equations Are Substituted And Executed At The Condition $\cos kx = \sin kx = \cos \sigma t = \sin \sigma t$. By Using Equations (12) And (14), G And k Can Be Calculated With The Input Wave Amplitude A, Wave Period T, Where $\sigma = \frac{2\pi}{T}$ And Water Depth, With Iteration Method From Newton-Rhapson, I.E.

$$\begin{bmatrix} \frac{\partial f_1}{\partial G} & \frac{\partial f_1}{\partial k} \\ \frac{\partial f_2}{\partial G} & \frac{\partial f_2}{\partial k} \end{bmatrix} \begin{Bmatrix} \delta G \\ \delta k \end{Bmatrix} = -\begin{Bmatrix} f_1(G,k) \\ f_2(G,k) \end{Bmatrix}$$

......................(15)

Table (1) Shows The Result Of The Calculation Of Wave Length For Wave Period Of 8 Seconds, With Varies Wave Amplitude, I.E. 0.40 M, 0.80 M, And 1.20 M. The Table Shows That Wave Length Is Affected By Wave Amplitude Where The Higher The Wave Amplitude, The Shorter The Wave Length. Whereas, Comparing With Wave Length Of Linear Wave Theory, It Is Show That Wave Length Of The Model Is Shorter Especially In Shallow Water.

Table (1) The Effect Of Amplitude On Wave Length For Wave Period Of 8 Seconds.

h	$A = 0.40m$	$A = 0.80m$	$A = 1.20m$	Linear
10	68.68	67.57	66.44	70.9
9	65.81	64.68	63.54	68.05
8	62.64	61.49	60.33	64.9
7	59.12	57.93	56.75	61.41
6	55.16	53.94	52.71	57.5
5	50.68	49.4	48.12	53.08
4	45.52	44.15	42.81	48.01
3	39.41	37.93	36.48	42.03
2	31.84	30.17	28.57	34.69

Table (2) The Effect Of Bottom Waters Slope On Wave Length, For Wave Period Of 8 Seconds, Amplitude $A = 0.80m$.

h	Bottom slope $\frac{dh}{d\xi}$			Linear Wave
	-0.005	-0.01	-0.05	
10	68.58	68.46	67.57	70.9
9	65.69	65.57	64.68	68.05
8	62.5	62.37	61.49	64.9
7	58.93	58.81	57.93	61.41
6	54.93	54.81	53.94	57.5
5	50.39	50.27	49.4	53.08
4	45.14	45.02	44.15	48.01
3	38.9	38.78	37.93	42.03
2	31.14	31.02	30.17	34.69

As Shown On **Table (2)**, Beach Slope Shortens Wave Length. According With Sleath, J.F.A (1984), Galvin (1972) Suggested That Breaker Type Depends On The Parameter $\frac{H_0}{L_0 \tan^2 \theta}$ Where θ Is The Beach Slope And $\frac{H_0}{L_0}$ Is The Deep Water Wave Steepness. This Parameter Indicates That Wave Length Is Affected By Beach Slope.

2.2. Breaking Characteristics

From Potential Flow Equation $\phi = Ge^{kh}\beta(z)\cos k\xi \sin \sigma t$,

Where $G = \frac{\sigma A}{F}$, Hence At $F \to 0$, ϕ Becomes Infinite, This Is A Breaking Condition.

From Equation (3), I.E.

$$F = e^{kh}\left(k\beta_1\left(\frac{A}{2}\right) - \beta\left(\frac{A}{2}\right)\left(k - \frac{1}{2h}\frac{\partial h}{\partial \xi}\right)\frac{kA}{2}\right)$$

, For $F = 0$ Applies

$$\left(k\beta_1\left(\frac{A}{2}\right) - \beta\left(\frac{A}{2}\right)\left(k - \frac{1}{2h}\frac{\partial h}{\partial \xi}\right)\frac{kA}{2}\right) = 0$$

$$\frac{kA}{2} = \frac{k\beta_1\left(\frac{A}{2}\right)}{\beta\left(\frac{A}{2}\right)\left(k - \frac{1}{2h}\frac{\partial h}{\partial \xi}\right)}$$

In The Case Of Very Small Slope,

$$\frac{kA}{2} = \frac{\beta_1\left(\frac{A}{2}\right)}{\beta\left(\frac{A}{2}\right)}$$

If The Effect Of Slope Is Totally Ignored

$$\frac{kA}{2}=\tanh k\left(h+\frac{A}{2}\right)$$

Substitute $k=\dfrac{2\pi}{L}$ And $A=\dfrac{H}{2}$

$$\frac{H}{L}=\frac{2}{\pi}\tanh k\left(h+\frac{A}{2}\right)$$

Miche's Criteria (Tomoya Shibayama (2009)) : $\dfrac{H}{L}\geq 0.143\tanh kh$. Both Equations Have Same Form But Differ In Their Constant Value. Comparison Breaking Conditon Of The Model With Miche's Criteria, Beyond Scope Of This Paper And Will Be Done In Next Paper.

2.3. Wave Profile

Hutahaean (2010) Developed Water Surface Equation By Integrating Kinematic Free Surface Boundary Condition Equation Against Time T With Inversion Integration Method. The Kinematic Free Surface Boundary Condition Is

$$w_\eta=\frac{\partial\eta}{\partial t}+u_\eta\frac{\partial\eta}{\partial x}$$

This Equation Is Written As Water Surface Equation,

$$\frac{\partial\eta}{\partial t}=w_\eta-u_\eta\frac{\partial\eta}{\partial x}$$

........................(16)

Substitute Equations (8),(9) And (10) To Equation (16),

$$\frac{\partial\eta}{\partial t}=-Ge^{kh}\beta_1(\eta)k\cos kx\sin\sigma t$$

$$-Ge^{kh}k\left(\sin kx-\frac{1}{2kh}\frac{\partial h}{\partial x}\cos kx\right)\beta(\eta)\sin\sigma t\frac{\partial\eta}{\partial x}$$

...............................(17)

The Equation Is Multiplied With dt And Integrated Against Time-t,

$$\eta(x,t)=-Gke^{kh}\sin kx\int_t\beta_1(\eta)\sin\sigma t\,dt$$

$$-Gke^{kh}\left(\sin kx-\frac{1}{2kh}\frac{\partial h}{\partial x}\cos kx\right)\int_t\beta(\eta)\sin\sigma t\frac{\partial\eta}{\partial x}dt$$

There Are Two Integrations Of Non-Linear Function, I.E. $\int_t\beta_1(\eta)\sin\sigma t\,dt$ And $\int_t\beta(\eta)\sin\sigma t\dfrac{\partial\eta}{\partial x}dt$. To Integrate Those Equations, Integral Inversion Method Is Conducted. Integral Inversion Method Can Be Seen In Hutahaean (2010) Where With Precision Level $O(\delta^2)$, Water Surface Equation Can Be Obtained, I.E.

$$\eta(x,t)=\frac{GF}{\sigma}\cos kx\cos\sigma t$$

...............................(18)

$$F=e^{kh}k\left(\beta_1(\eta)-\frac{k}{\sigma}\beta(\eta)\frac{\partial\eta}{\partial t}-\frac{k^2}{\sigma^2}\beta_1(\eta)\left(\frac{\partial\eta}{\partial t}\right)^2-\frac{k}{\sigma^2}\beta(\eta)\frac{\partial^2\eta}{\partial t^2}\right)$$

$$+e^{kh}k\left(1-\frac{1}{2kh}\frac{\partial h}{\partial x}\right)\left(\beta(\eta)\frac{\partial\eta}{\partial x}-\frac{k}{\sigma}\beta_1(\eta)\frac{\partial\eta}{\partial t}\frac{\partial\eta}{\partial x}-\frac{1}{\sigma}\beta(\eta)\frac{\partial^2\eta}{\partial t\partial x}\right)$$

..........(19)

The Value Of F Is Calculated With The Condition

$$\cos kx=\sin kx=\cos\sigma t=\sin\sigma t=\frac{\sqrt{2}}{2}$$

Where $\eta=A\cos kx\cos\sigma t=\dfrac{A}{2}$

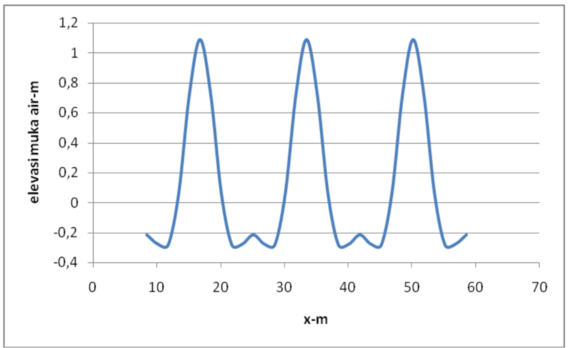


Figure (2). The Profile Of Cnoidal Wave For Waves With Period 8 Seconds, Amplitude $A=0.60m$, Water Depth 2.0 M.

Figure (2) Show Wave Profile Produced By Equations (18) And (19) Which Is Cnoidal Profile. Cnoidal Water Wave Profile Was First Discovered By Kortweg De Vries (Sharpkaya, 1981). With This Wave Profile, Where The Wave Trough And Its Trough Length Region Is Very Small, Then Wave Amplitude Can Be Considered As The Wave Height. In Addition, With That Cnoidal Profile, Waves Can Move In Very Swallow Water Where $\frac{H}{h} \geq 1$.

III. ONE DIMENSION SHOALING AND BREAKING MODEL

Shoaling Is The Enlargement Of Wave Height As A Result Of Water Shallowing, Where Waves Move From Deep Water To Shallower Water Always Experience This Shoaling Phenomenon. The Enlargement Of Wave Height Occurs Continuously And Breaking In The End

3.1. Wave Amplitude A

Equation (3) Can Be Written As An Equation For Amplitude A.

$$A = \frac{G}{\sigma} e^{kh}\left(k\beta_1(\eta) - k\beta(\eta)\frac{kA}{2} + \frac{1}{2h}\frac{\partial h}{\partial x}\beta(\eta)\frac{kA}{2}\right)$$

This Equation Is Differentiated Against ξ, By Taking Into Account The Equations Above,

$$\frac{\partial A}{\partial \xi} = \frac{1}{\sigma}\frac{\partial G}{\partial \xi} e^{kh}\left(k\beta_1(\eta) - k\beta(\eta)\frac{kA}{2} + \frac{1}{2h}\frac{\partial h}{\partial \xi}\beta(\eta)\frac{kA}{2}\right)$$

$$+\frac{G}{\sigma} e^{kh}\left(\frac{\partial k}{\partial \xi}\beta_1(\eta) - \frac{\partial k}{\partial \xi}\beta(\eta)\frac{kA}{2} - \frac{1}{2h^2}\frac{\partial h}{\partial \xi}\frac{\partial h}{\partial \xi}\beta(\eta)\frac{kA}{2}\right)$$

Substitute Equation (4), $\frac{\partial G}{\partial \xi} = \frac{G}{2h}\frac{\partial h}{\partial \xi}$, And Bearing In Mind Equation (5), $\frac{\partial kA}{\partial \xi} = 0$, Hence

$$\frac{\partial A}{\partial x} = \frac{A}{2h}\frac{\partial h}{\partial \xi} + \frac{G}{\sigma} e^{kh}\left(\frac{\partial k}{\partial \xi}\beta_1(\eta) - \frac{\partial k}{\partial \xi}\beta(\eta)\frac{kA}{2} - \frac{1}{2h^2}\frac{\partial h}{\partial \xi}\frac{\partial h}{\partial \xi}\beta(\eta)\frac{kA}{2}\right)$$

..........(20)

This Equation (20) Is Wave Amplitude Change Equation, Where From Equation (5), $\frac{\partial k}{\partial \xi} = -\frac{1}{h}\frac{\partial h}{\partial \xi}$ Whereas $\beta(\eta) = \beta\left(\frac{A_0}{2}\right)$ And $\beta_1(\eta) = \beta_1\left(\frac{A_0}{2}\right)$, Where A_0 Is Initial Wave Amplitude. This Amplitude Evolution Equation Is A Parabolic Differential Equation That Can Be Executed Using Finite Difference Method With Scheme Forward Difference.

3.2 Wave Number k

From h_1 Water Depth To Shallower h_2 Water Depth, Wave Length Reduction Or Wave Number k Enlargement Will Occur**.** Equation (5) I.E. $\frac{\partial k\left(h + \frac{A}{2}\right)}{\partial \xi} = 0$, Can Be Written As,

$$k_2\left(h_2 + \frac{A_2}{2}\right) = k_1\left(h_1 + \frac{A_1}{2}\right),$$

Or

$$k_2 = k_1\frac{\left(h_1 + \frac{A_1}{2}\right)}{\left(h_2 + \frac{A_2}{2}\right)}$$

......................(21)

3.3. Constant G

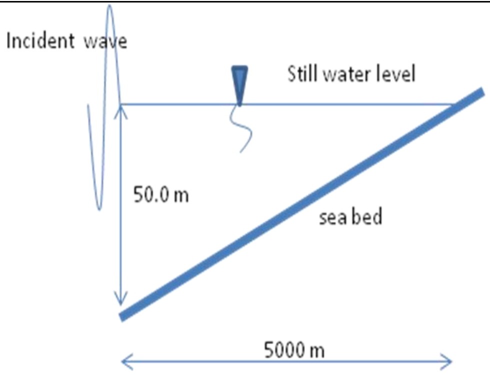
Value Of Constant G Decreased Also When Wave Moves To Shalower Water Depth, Equation (4) $\frac{\partial G}{\partial \xi} = -\frac{G}{2k}\frac{\partial k}{\partial \xi}$ Is Used. The Equation Can Be Written As, $\frac{\partial G}{G} = -\frac{1}{2}\frac{\partial k}{k}$. The Equation Is Integrated From Point 1 To Point 2,

$$G_2 = e^{\left(\ln G_1 - \frac{1}{2}(\ln k_2 - \ln k_1)\right)}$$

......................(22)

3.4. Example Of The Result Of Shoaling-Breaking Model

The Following Section Shows An Example Of The Result Of Shoaling And Breaking Model, For Waves With Periods Of 7 Seconds, 8 Seconds, And 9 Seconds With Wave Amplitude 0.80 M, And At The Bottom Water Slope 0.01. The Result Of The Model Is Presented In **Figure (3).**



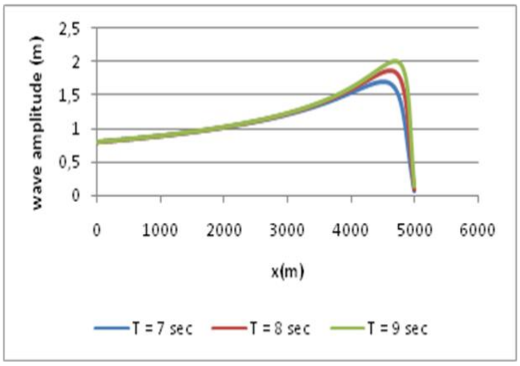


Figure (3) The Result Of Shoaling-Breaking Model

IV. CALIBRATION OF SHOALING-BREAKING MODEL AGAINST BREAKING INDEX EQUATION

In This Section, Result Of The Model Compare With Breaking Index Or Breaker Index Equation $\left(\frac{H_b}{H_0}\right)$ Result. The Selection Of This Equation Is Because Of Its Simplicity, I.E. Only One Unknown, E.G H_b. **Table (3)** Shows Some Breaking Index Equations With The Form $\left(\frac{H_b}{H_0}\right)$, As Cited From Tomoya Shibayama (2009).

Table (3) Breaking Index Equation In The Form Of $\left(\frac{H_b}{H_0}\right)$

No	Researcher	Code	Criteria
1	Komar and Gaughan (1972)	KG72	$\frac{H_b}{H_0} = 0.56\left(\frac{H_0}{L_0}\right)^{-\frac{1}{5}}$
2	Ogawa and Suto (1984)	OS84	$\frac{H_b}{H_0} = 0.68m^{0.09}\left(\frac{H_0}{L_0}\right)^{-0.25}$
3	Smith and Kraus (1990)	SK90b	$\frac{H_b}{H_0} = (0.34 + 2.74m)\left(\frac{H_0}{L_0}\right)^{-0.30+0.88m}$
4	Rattanpitikon and Shibayama (2000)	RS00b	$\frac{H_b}{H_0} = (10.02m^3 - 7.46m^2 + 1.32m + 0.55)\left(\frac{H_0}{L_0}\right)^{-\frac{1}{5}}$

For Another Comparison, Wave Height At Breaker Depth Of The Model Is Calculated Using Shoaling Equation Of Linear Wave Theory (Dean 1980). The Result Of The Comparison Is Shown On **Table (4)**

Table (4) Comparison Breaking Height Of The Model With Breaking Height Of Breaking Index Equation In The Form Of $\left(\frac{H_b}{H_0}\right)$

Varies Wave Period H_0=1.60 m, $h_0 = 50$m, $m = 0.01$								
Period (dt)	H_b (meter)					Shoaling -Linier	h_b (m)	$\left(\frac{H_b}{h_b}\right)$
	Model	Komar	Ogawa -Suto	Smith-Krauss	Rattanpitikon -Shibayama			
7	1,707	1,942	1,89	1,812	1,95	1,573	4,987	0,342
8	1,864	2,047	2,019	1,957	2,056	1,727	3,898	0,478
9	2,005	2,142	2,136	2,091	2,151	1,922	3,128	0,641
10	2,128	2,226	2,241	2,211	2,235	2,142	2,578	0,825
11	2,23	2,299	2,335	2,319	2,31	2,364	2,199	1,014
12	2,311	2,365	2,418	2,415	2,375	2,579	1,919	1,204
H_0 Varies $T = 10$dt, $h_0 = 50$m, $m = 0.01$								
H_0 (m)	H_b (meter)					Shoaling -Linier	h_b (m)	$\left(\frac{H_b}{h_b}\right)$
	Model	Komar	Ogawa -Suto	Smith-Krauss	Rattanpitikon -Shibayama			
1	1,555	1,528	1,576	1,585	1,535	1,444	1,849	0,841
1,5	2,038	2,114	2,136	2,112	2,123	2,028	2,469	0,826
2	2,471	2,661	2,65	2,59	2,672	2,582	3,038	0,813
2,5	2,868	3,181	3,133	3,034	3,195	3,115	3,578	0,802
3	3,242	3,68	3,592	3,452	3,696	3,63	4,108	0,789
Slope m Varies $T = 10$dt, $H_0 = 2.0$ m, $h_0 = 50$m								
m	H_b (meter)					Shoaling -Linier	h_b (m)	$\left(\frac{H_b}{h_b}\right)$
	Model	Komar	Ogawa -Suto	Smith-Krauss	Rattanpitikon -Shibayama			
0,005	2,473	2,661	2,49	2,541	2,644	2,583	3,028	0,817
0,0062	2,472	2,661	2,54	2,554	2,651	2,583	3,028	0,816
0,0083	2,472	2,661	2,607	2,574	2,663	2,583	3,028	0,816
0,0125	2,471	2,661	2,704	2,613	2,686	2,583	3,028	0,816
0,025	2,469	2,661	2,878	2,72	2,748	2,583	3,028	0,815

In The Review Of Varies Wave Period, Model's Result Is Close To Smith-Krauss's Criteria, But For High Wave Period Komar's Criteria Is Closest..

In The Review Of Varies Wave Amplitude, With Fixed Wave Period, The Breaking Index Equation That Gives Breaking Wave Height Closest To The Model Is Smith-Krauss Equation. From The Review Of Seabed Slope, Shoaling From Linear Wave Theory Give The Closest Result.

In General, The Breaking Height Produced By The Model Is Close To The Five Comparators. .

V. REFRACTION-DIFFRACTION MODEL

5.1. **Coordinate Rotation**

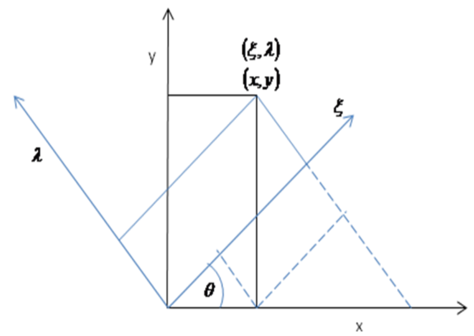


Figure 4. Rotation Of Coordinate Axis

The Relationship Between (ξ, λ) Axis And (x, y) Axis, That ξ Forms An Angle Of θ Against x Axis(**Figure(4)**) Is As Follows,

$$\xi = x\cos\theta + y\sin\theta$$

$$\lambda = -x\sin\theta + y\cos\theta$$

The Two Equations Produced:

$\frac{\partial\xi}{\partial x} = \cos\theta$ And $\frac{\partial\xi}{\partial y} = \sin\theta$

$\frac{\partial\lambda}{\partial x} = -\sin\theta$ And $\frac{\partial\lambda}{\partial y} = \cos\theta$

Chain Derivative Rule:

$\xi = \xi(x, y) \qquad \lambda = \lambda(x, y)$

$$f = f(\xi(x,y), \lambda(x,y))$$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial \xi}\frac{\partial \xi}{\partial x} + \frac{\partial f}{\partial \lambda}\frac{\partial \lambda}{\partial x}$$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial \xi}\cos\theta - \frac{\partial f}{\partial \lambda}\sin\theta$$

.....................(23)

5.2. Wave Parameters Evolution Due To Water Depth Change

a. Wave Amplitude

Equation (2) Can Be Written As Wave Amplitude Equation $A = \frac{GF}{\sigma}$ And Derived Against ξ,

$$\frac{\partial A}{\partial \xi} = \frac{F}{\sigma}\frac{\partial G}{\partial \xi} + \frac{G}{\sigma}\frac{\partial F}{\partial \xi}$$

Substitute Differential Nature Of G, Equation (4), I.E. $\frac{\partial G}{\partial \xi} = \frac{1}{2h}\frac{\partial h}{\partial \xi}$, Hence

$$\frac{\partial A}{\partial \xi} = \frac{GF}{2\sigma h}\frac{\partial h}{\partial \xi} + \frac{G}{\sigma}\frac{\partial F}{\partial \xi}$$

Substitute Equation (2)

$$\frac{\partial A}{\partial \xi} = \frac{A}{2h}\frac{\partial h}{\partial \xi} + \frac{G}{\sigma}\frac{\partial F}{\partial \xi}$$

................(24)

Differentiation Of Equation (3) Against ξ, For Small Changes h, Where $\frac{\partial^2 h}{\partial \xi^2} = 0$, Can Be Stated With Linear Equation,. Using $\frac{\partial k\left(h + \frac{A}{2}\right)}{\partial \xi} = 0$ And $\frac{\partial kh}{\partial \xi} = 0$, And $\frac{\partial \frac{kA}{2}}{\partial \xi} = 0$.

$$\frac{\partial F}{\partial \xi} = e^{kh}\left(\frac{\partial k}{\partial \xi}\beta_1\left(\frac{A}{2}\right) - \beta\left(\frac{A}{2}\right)\left(\frac{\partial k}{\partial \xi} + \frac{1}{2h^2}\frac{\partial h}{\partial \xi}\frac{\partial h}{\partial \xi}\right)\frac{kA}{2}\right)$$

..................(25)

Where $\frac{\partial h}{\partial \xi} = \frac{\partial h}{\partial x}\cos\theta + \frac{\partial h}{\partial y}\sin\theta$

Wave Amplitude A Evolution Equation In The Direction Of x Axis Can Be Obtained By Bearing In Mind $A = A(\xi(x,y), \lambda(x,y))$, Hence

$$\frac{\partial A}{\partial x} = \frac{\partial A}{\partial \xi}\frac{\partial \xi}{\partial x} + \frac{\partial A}{\partial \lambda}\frac{\partial \lambda}{\partial x}$$

Wave Moves In The Direction Of ξ Axis, Where The Value Of λ Is Constant (In This Case $\lambda = 0$), Or $\frac{\partial \lambda}{\partial x} = 0$, Hence Change In Wave Amplitude In The Direction Of x Axis Is, $\frac{\partial A}{\partial x} = \frac{\partial A}{\partial \xi}\frac{\partial \xi}{\partial x}$. Substitute $\frac{\partial \xi}{\partial x} = \cos\theta$,

$$\frac{\partial A}{\partial x} = \frac{\partial A}{\partial \xi}\cos\theta$$

................(26)

This Equation Is Wave Amplitude Evolution Equation In The Direction Of x Axis, In The Form Of Parabolic Differential Equation.

b. Wave Direction

Wave Direction Can Be Stated With The Following Equation,

$$\theta = a\tan\left(\frac{v_\eta}{u_\eta}\right)$$
................................(27)

Substitute $\xi = x\cos\theta + y\sin\theta$, To Potential Flow Equation (Equation 1),

$$\phi = Ge^{kh}\beta(z)\cos k(x\cos\theta + y\sin\theta)\sin\sigma t$$

The Surface Horizontal Velocity Of x Direction,

$$u_\eta = Ge^{kh}\beta(\eta)k\cos\theta\ \sin k(x\cos\theta + y\sin\theta)\sin\sigma t$$

$$-\frac{\partial G}{\partial x}e^{kh}\beta(\eta)\cos k(x\cos\theta + y\sin\theta)\sin\sigma t$$
..............(28)

The Surface Horizontal Velocity Of y Direction

$$v_\eta = Ge^{kh}\beta(\eta)k\sin\theta\sin k(x\cos\theta + y\sin\theta)\sin\sigma t$$

$$-\frac{\partial G}{\partial y}e^{kh}\beta(\eta)\cos k(x\cos\theta + y\sin\theta)\sin\sigma t$$
........(29)

c. Wave Number k

The Evolution Of Wave Number k_x For Wave Moving From Water Depth h_x With Wave Amplitude A_x To Shallower Water $h_{x+\delta x}$ Where Wave Amplitude $A_{x+\delta x}$ (Hutahaean 2015) Is

$$k_{x+\delta x} = e^{\left(\ln k_x - \ln\left(h_{x+\delta x} + \frac{A_{x+\delta x}}{2}\right) - \ln\left(h_x + \frac{A_x}{2}\right)\right)}$$
............(30)

d. G Constant

Whereas Changes In Wave Constant G (Hutahaean 2015) Is,

$$G_{x+\delta x} = e^{\left(\ln G_x - \frac{1}{2}(\ln k_{x+\delta x} - \ln k_x)\right)}$$
...................(31)

VI. THE RESULT OF REFRACTION-DIFFRACTION MODEL

In This Section The Refraction-Diffraction Model Is Executed At The Bathymetry Of Bay Shaped Coast And At Submerged Island. 8 Seconds And Initial Amplitude 0.80 M Wave, Is Used.

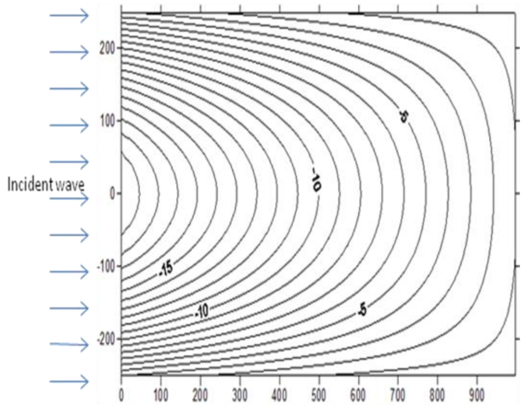


Figure (5.A) Bay-Shape Seabed Bathymetry

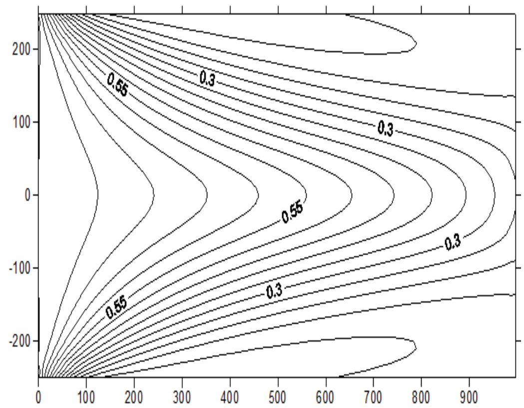


Figure (5.B) Wave Amplitude Contour Over Bay Shape Bathymetry

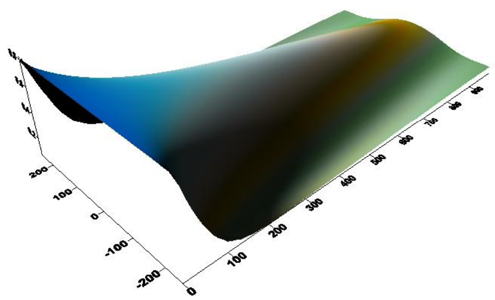


Figure (5.C) 3 D Wave Amplitude Contour Over Bay Shape-Bathymetry

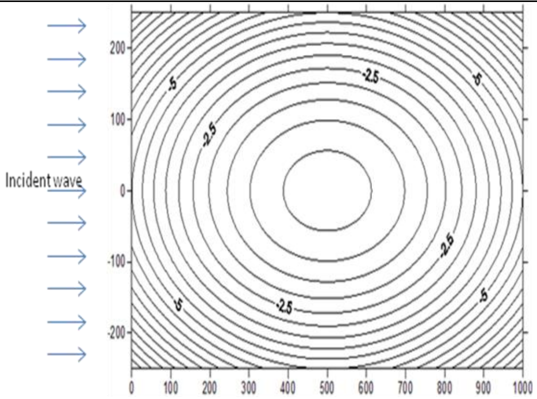


Figure (6.A) Submerged Island Bathymetry

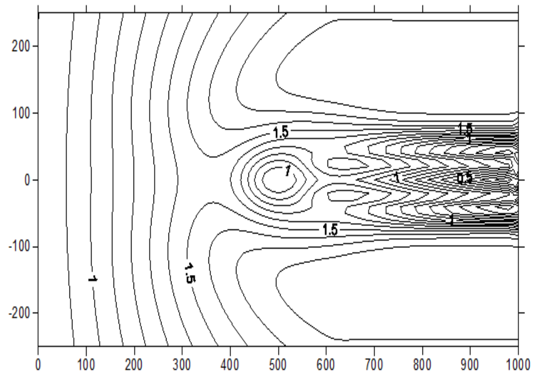


Figure (6.B) Wave Amplitude Contour Over Submerged Island

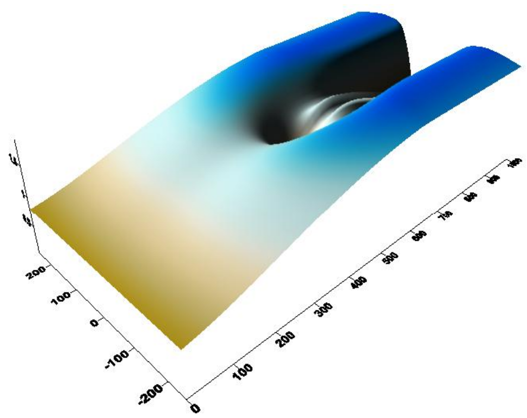


Figure (6.C) 3 D Wave Amplitude Contour Over Submerged Island

Execution Model At Bay-Shaped Bathymetry Shows The Occurrence Of Wave Energy Spread At The Bay Side **(Figure (5.B And C))**. Reduction Of Wave Amplitude Due To Wave Energy Spread More Dominant Than Shoaling, Seems Shoaling Does Not Occur.

Execution Models At Submerged Island Bathymetry **(Figure (6.B And C))** Show That The Wave Refracted To The Center Of The Island, Where Shoaling And Breaking Occurs. Breaking Wave Height That Occurs Is More Or Less Similar With The One Produced By Shoaling And Breaking Model, I.E. About 1.80 M

VII. CONCLUSION

Model Can Simulate Shoaling-Breaking Phenomenon Quite Well, Where Breaking Wave Height Produced By The Model Is Quite Close To Research Result Of Some Researchers, Specifically From Smith-Krauss Equation.

Wave Refraction-Diffraction Equation That Is Produced In This Research Is In The Form Of Parabolic Equation That Can Be Solved Easily Using Finite Difference Method. In General, Model Can Simulate Refraction-Diffraction Phenomenon, Shoaling And Breaking. Diffraction As A Result Of Energy Difference With Normal Direction Toward Wave Direction, As Occurs In The Breakwater, Has Not Been Accounted For. Therefore, Further Development That Needs To Be Done Is Involving Diffraction Phenomenon As A Result Of Energy Difference.

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