RESEARCH ARTICLE OPEN ACCESS

Skolem Mean Labeling Of Four Star Graphs $K_{1,a_1} \cup K_{1,a_2} \cup K_{1,a_3} \cup K_{1,b}$ where $a_1 + a_2 + a_3 - 3 \le b \le a_1 + a_2 + a_3 - 2$

*D. S. T. Ramesh, S.O.Sopna, I.Gnanaselvi
Department of Mathematics Margoschis College, Nazareth- 628617 Tamil Nadu, India
Department of Mathematics Sree Devi kumara Women's College kuzhithurai. K.K. District -629163
Department of Mathematics Sarah Tucker College, Tirunelveli-627007 Tamil Nadu,India
Corresponding Author: D. S. T. Ramesh

ABSTRACT
A graph G = (V, E) with p vertices and q edges is said to be a skolem mean graph if there exists a function f from the vertex set of G to {1, 2, . . . , p} such that the induced map f* from the edge set of G to {2, 3, . . . , p} defined by $f^*(e = uv) = \frac{f(u)+f(v)}{2}$ if f(u) + f(v) is even and $\frac{f(u)+f(v)+1}{2}$ if f(u) + f(v) is odd, then the resulting edges get distinct labels from the set {2, 3, . . . , p}. In this paper, we prove that four star graph $G = K_{1,a_1} \cup K_{1,a_2} \cup K_{1,a_3} \cup K_{1,b}$ where $a_1 \le a_2 \le a_3$ is a skolem mean graph if $a_1 + a_2 + a_3 - 3 \le b \le a_1 + a_2 + a_3 - 2$.

***Keywords*: S**kolem mean graph, **s**kolem mean labeling, star graphs

---

Date of Submission: 13-09-2017 Date of acceptance: 28-09-2017

I. INTRODUCTION

All graphs in this paper are finite, simple and undirected. Terms not defined here are used in the sense of Harary [3]. The symbols V(G) and E(G) will denote the vertex set and edge set of the graph G. A graph with p vertices and q edges is called a (p, q) graph. In this paper, we prove that four star graph $G = K_{1,a_1} \cup K_{1,a_2} \cup K_{1,a_3} \cup K_{1,b}$ where $a_1 \le a_2 \le a_3$ is a skolem mean graph if $a_1 + a_2 + a_3 + 2 \le b \le a_1 + a_2 + a_3 + 3$.

Skolem mean labeling

Definition 1.1: A graph G is a non empty set of objects called vertices together with a set of unordered pairs of distinct vertices of G called edges. The vertex set and the edge set of G are denoted by V(G) and E(G) respectively. $|V(G)| = q$ is called the size of G, we say that u and v are adjacent and that u and v are incident with e.

Definition 1.2: A vertex labelling of a graph G is an assignment of labels to the vertices of G that induces for each edge xy a label depending on the vertex labels f(x) and f(y). Similarly, an edge labelling of a graph G is an assignment of labels to the edges of G that induces for each vertex v a label depending on the edge labels incident on it. Total labelling involves a function from the vertices and edges to some set of labels.

Definition 1.3: A graph G with p vertices and q edges is called a mean graph if it is possible to label the vertices $x \in V$ with distinct elements f(x) from 0, 1, 2, …q in such a way that when each edge e = uv is labeled with $\frac{f(u)+f(v)}{2}$ if f(u) + f(v) is even and $\frac{f(u)+f(v)+1}{2}$ if f(u) + f(v) is odd, then the resulting edge labels are distinct. The labeling f is called a mean labeling of G.

Definition 1.4: A graph G = (V,E) with p vertices and q edges is said to be skolem mean if it is possible to label the vertices $x \in V$ with distinct elements f(x) from 1, 2, …q in such a way that when each edge e = uv is labeled with $\frac{f(u)+f(v)}{2}$ if f(u) + f(v) is even and $\frac{f(u)+f(v)+1}{2}$ if f(u) + f(v) is odd, then the resulting edges get distinct labels from 2, 3, . . . , p. f is called a skolem mean labeling of G. A graph G = (V, E) with p vertices and q edges is said to be a **skolem mean graph** if there exists a function f from the vertex set of G to {1, 2, . . . , p} such that the induced map f* from the edge set of G to {2, 3, . . . , p} defined by

$$f^*(e=uv)=\begin{cases}\dfrac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even}\\[2mm] \dfrac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd}\end{cases}$$

the resulting edges get distinct labels from the set {2, 3, . . . , p}.

Theorem 2.1: The four star $G=K_{1,a_1}\cup K_{1,a_2}\cup K_{1,a_3}\cup K_{1,b}$ where $a_1\le a_2\le a_3$ is a skolem mean graph if $a_1+a_2+a_3-3\le b\le a_1+a_2+a_3-2$.

Proof: Let $A_i=\sum_{k=1}^{i} a_k$. That is,

$A_1=a_1$; $A_2=a_1+a_2$ and

$A_3=a_1+a_2+a_3$.

Consider the graph $G=K_{1,a_1}\cup K_{1,a_2}\cup K_{1,a_3}\cup K_{1,b}$. Let $V=\bigcup_{k=1}^{4}V_k$ be the vertex set of G where $V_k=\left\{v_{k,i}:0\le i\le a_k\right\}$ for $1\le k\le 3$ and $V_4=\left\{v_{4,i}:0\le i\le b\right\}$. Let $E=\bigcup_{k=1}^{4}E_k$ be the edge set of G where $E_k=\left\{v_{k,0}v_{k,i}:0\le i\le a_k\right\}$ for $1\le k\le 3$ and $E_4=\left\{v_{4,0}v_{4,i}:0\le i\le b\right\}$. The condition $a_1+a_2+a_3-3\le b\le a_1+a_2+a_3-2\Rightarrow A_3-3\le b\le A_3-2$

That is, there are two cases viz. $b=A_3-3$ and $b=A_3-2$. Let us prove in each of the two cases the graph G is a skolem mean graph.

Case 1: Let $b=A_3-3$

G has $A_3+b+4=2A_3+1$ vertices and $A_3+b=2A_3-3$ edges. The vertex labeling

$f:V\to\{1,2,3,\ldots,A_3+b+4=2A_3+1\}$

is defined as follows:

$f(v_{1,0})=2$; $f(v_{2,0})=4$; $f(v_{3,0})=6$;

$f(v_{4,0})=A_3+b+4=2A_3+1$

$f(v_{1,i})=2i-1 \qquad 1\le i\le a_1$

$f(v_{2,i})=2A_1+2i-1 \qquad 1\le i\le a_2$

$f(v_{3,i})=2A_2+2i-1 \qquad 1\le i\le a_3$

$f(v_{4,i})=2i+6 \qquad 1\le i\le b$

The corresponding edge labels are as follows:

The edge label of $v_{1,0}v_{1,i}$ is $1+i$ for $1\le i\le a_1$ (edge labels are $2,3,\ldots,a_1+1=A_1+1$), $v_{2,0}v_{2,i}$ is A_1+2+i for $1\le i\le a_2$ (edge labels are $A_1+3,A_1+4,\ldots,A_1+2+a_2=A_2+2$), $v_{3,0}v_{3,i}$ is A_2+3+i for $1\le i\le a_3$ (edge labels are $A_2+4,A_2+5,\ldots,A_2+3+a_3=A_3+3$), $v_{4,0}v_{4,i}$ is A_3+4+i for $1\le i\le b=A_3-3$ (edge labels are $A_3+5,A_3+6,\ldots,A_3+4+A_3-3=2A_3+1$).

These induced edge labels of graph G are distinct.

Hence G is a skolem mean graph.

Example:

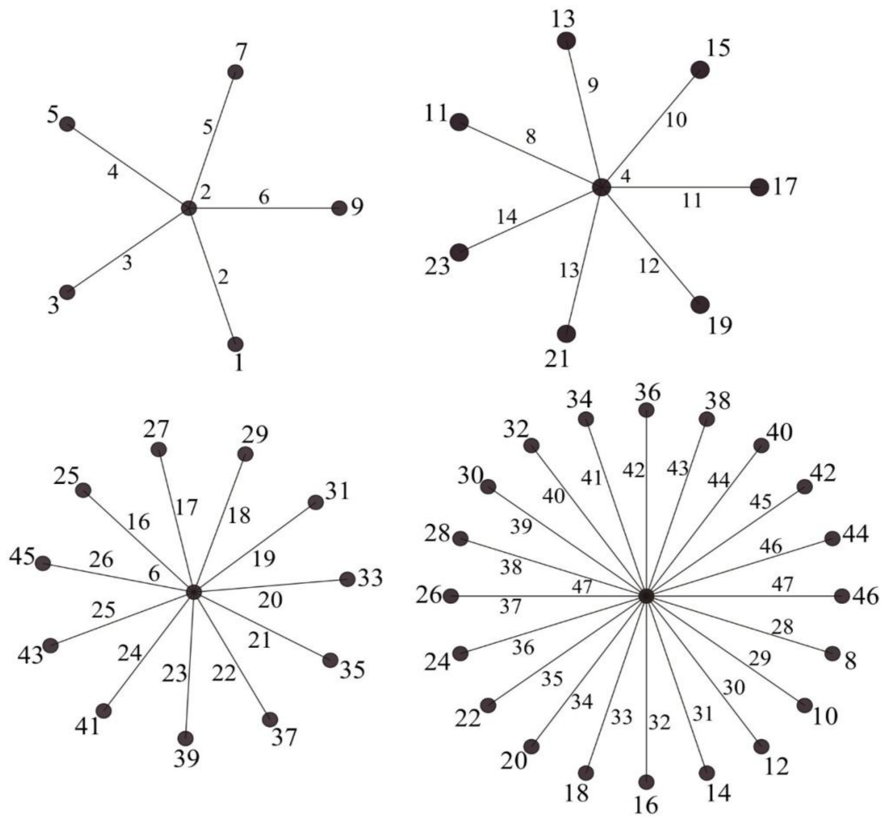

Figure $\left(K_{1,5} \cup K_{1,7} \cup K_{1,11} \cup K_{1,21}\right)$

Case 2: $b = A_3 - 2$

G has $A_3 + b + 4 = 2A_3 + 2$ vertices and $A_3 + b = 2A_3 - 2$ edges.The vertex labeling

$f : V \to \{1, 2, ..., A_3 + b + 4 = 2A_3 + 2\}$

is defined as follows:

$f(v_{1,0}) = 2;$ $f(v_{2,0}) = 4;$ $f(v_{3,0}) = 6;$

$f(v_{4,0}) = A_3 + b + 3 = 2A_3 + 1$

$f(v_{1,i}) = 2i - 1 \qquad 1 \le i \le a_1$

$f(v_{2,i}) = 2A_1 + 2i - 1 \qquad 1 \le i \le a_2$

$f(v_{3,i}) = 2A_2 + 2i - 1 \qquad 1 \le i \le a_3$

$f(v_{4,i}) = 2i + 6 \qquad 1 \le i \le b$

The corresponding edge labels are as follows:

The edge label of $v_{1,0}v_{1,i}$ is $1 + i$ for $1 \le i \le a_1$ (edge labels are $2, 3, ..., a_1 + 1 = A_1 + 1$), $v_{2,0}v_{2,i}$ is $A_1 + 2 + i$ for $1 \le i \le a_2$ (edge labels are $A_1 + 3, A_1 + 4, ..., A_1 + 2 + a_2 = A_2 + 2$), $v_{3,0}v_{3,i}$ is $A_2 + 3 + i$ for $1 \le i \le a_3$ (edge labels are $A_2 + 4, A_2 + 5, ..., A_2 + 3 + a_3 = A_3 + 3$), $v_{4,0}v_{4,i}$ is $A_3 + 4 + i$ for $1 \le i \le b = A_3 - 2$ (edge labels are $A_3 + 5, A_3 + 6, ..., A_3 + 4 + A_3 - 2 = 2A_3 + 2$).

These induced edge labels of graph G are distinct.

Hence G is a skolem mean graph.

Example:

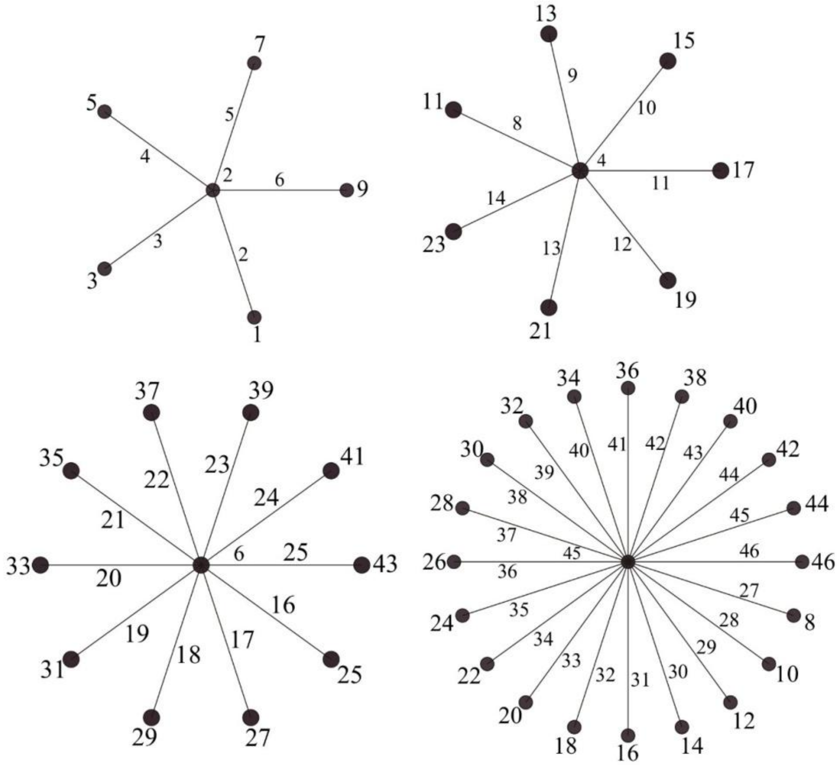

Figure $\left(K_{1,5} \cup K_{1,7} \cup K_{1,10} \cup K_{1,20}\right)$

REFERENCE

[1]. V. Balaji, D. S. T. Ramesh and A. Subramanian, Skolem Mean Labeling, Bulletin of Pure and Applied Sciences, Vol. 26E No. 2 (2007), 245-248.

[2]. V. Balaji, D. S. T. Ramesh and A. Subramanian, Some Results on Skolem Mean graphs, Bulletin of Pure and Applied Sciences, Vol. 27E No. 1 (2008), 67-74.

[3]. J. A Gallian, A Dynamic Survey of Graph Labeling, The Electronic Journal of Combinatorics, 14 (2016),#DS6.

[4]. F.Harary, Graph Theory, Addison- Wesley, Reading Mars,(1972).

[5]. D.S.T.Ramesh, S.O.Sopna, I.Gnanaselvi and M.P.Syed Ali Nisaya Skolem Mean Labeling Of Four Star Graphs $K_{1,a_1} \cup K_{1,a_2} \cup K_{1,a_3} \cup K_{1,b}$ where $a_1 + a_2 + a_3 + 2 \le b \le a_1 + a_2 + a_3 + 3$, International Journal of Scientific Research Vol.6, Issue 8 (Aug. 2017) PP.190-193.

[6]. D.S.T. Ramesh, S.O.Sopna, I.Gnanaselvi "Skolem Mean Labeling Of Four Star Graphs $K_{1,a_1} \cup K_{1,a_2} \cup K_{1,a_3} \cup K_{1,b}$ where $a_1 + a_2 + a_3 - 1 \le b \le a_1 + a_2 + a_3 + 1$, IOSR Journal of Engineering (IOSRJEN), Issue 07 (July 2017), ||V1|| PP 05-11.

International Journal of Engineering Research and Applications (IJERA) is **UGC approved** Journal with Sl. No. 4525, Journal no. 47088. Indexed in Cross Ref, Index Copernicus (ICV 80.82), NASA, Ads, Researcher Id Thomson Reuters, DOAJ.

D. S. T. Ramesh. "Skolem Mean Labeling Of Four Star Graphs where ." International Journal of Engineering Research and Applications (IJERA) , vol. 7, no. 9, 2017, pp. 29–32.