RESEARCH ARTICLE OPEN ACCESS

Subclasses of Analytic…] Subclasses of Analytic Functions Associated With (*J*, *K*)-Symmetrical Functions H.N. Kanthalakshmi, F.S Alsarari and S. Latha]

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ABSTRACT

The objective of the present paper is to study results that are defined using differential operator $\mathcal{D}_{\lambda,\delta}^{\sigma,s,l}$, and the notions of (j,k)-symmetrical functions. The integral representation and some properties for these class are obtained.

Keywords: Subordination, Convex functions, k-fold symmetric domain, k-fold symmetric function, (j,k)-symmetric points. **2010 AMS Subject Classification:** 30C45.

Date of Submission: 13-09-2017 Date of acceptance: 28-09-2017

I. INTRODUCTION

Let $\mathcal{A}$ denote the class of functions of form

$$f(z)=z+\sum_{n=2}^{\infty}a_nz^n, \tag{1}$$

which are analytic in the open unit disk **Error!**, and $\mathcal{S}$ denote the subclass of $\mathcal{A}$ consisting of all function which are univalent in $\mathcal{U}$.

For f and g be analytic in $\mathcal{U}$, we say that the function f is subordinate to g in $\mathcal{U}$, if there exists an analytic function w in $\mathcal{U}$ such that $|w(z)|<1$ with $w(0)=0$, and $f(z)=g(w(z))$, and we denote this by $f(z)\prec g(z)$. If g is univalent in $\mathcal{U}$, then the subordination is equivalent to $f(0)=g(0)$ and $f(\mathcal{U})\subset g(\mathcal{U})$. The convolution or Hadamard product of two analytic functions $f,g\in\mathcal{A}$ where f is defined by (1) and $g(z)=z+\sum_{n=2}^{\infty}b_nz^n$, is

$$(f*g)(z)=z+\sum_{n=2}^{\infty}a_nb_nz^n.$$

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In order to define a new class of symmetrical functions defined in the open unit disk $\mathcal{U}$, we first recall the notion of k-fold symmetric functions defined in k-fold symmetric domain, where k is any positive integer. A domain $\mathcal{D}$ is said to be k-fold symmetric if a rotation of $\mathcal{D}$ about the origin through an angle $\frac{2\pi}{k}$ carries $\mathcal{D}$ onto itself. A function f is said to be k-fold symmetric in $\mathcal{D}$ if for every z in $\mathcal{D}$ we have

$$f\left(e^{\frac{2\pi i}{k}}z\right)=e^{\frac{2\pi i}{k}}f(z),\ z\in\mathcal{D}.$$

The family of all k-fold symmetric functions is denoted by $\mathcal{S}^{k}$, and for k=2 we get class of odd univalent functions. In 1995, Liczberski and Polubinski [] constructed the theory of (j,k)-symmetrical functions for $j=0,1,2,\ldots,k-1$) and ($k=2,3,\ldots$. If $\mathcal{D}$ is k-fold symmetric domain and j any integer, then a function $f:\mathcal{D}\to C$ is called (j,k)-symmetrical if for each $z\in\mathcal{D}$, $f(\varepsilon z)=\varepsilon^{j}f(z)$. We note that the (j,k)-symmetrical functions is a generalization of the notions of even, odd, and k-symmetrical functions

The theory of (j,k)-symmetrical functions has many interesting applications; for instance, in the investigation of the set of fixed points of mappings, for the estimation of the absolute value of some integrals, and for obtaining some results of the type of Cartan's uniqueness theorem for holomorphic mappings, see [].

Denote the family of all (j,k)-symmetrical functions by $\mathcal{S}^{(j,k)}$. We observe that , $\mathcal{S}^{(0,2)}$, $\mathcal{S}^{(1,2)}$ and $\mathcal{S}^{(1,k)}$ are the classes of even, odd and k-symmetric functions respectively. We have the following decomposition theorem:

Theorem 1 [, Page 16] For every mapping $f:\mathcal{U}\mapsto C$, and a k-fold symmetric set $\mathcal{U}$, there exists exactly one sequence of (j,k)-symmetrical functions $f_{j,k}$ such that

$$f(z)=\sum_{j=0}^{k-1}f_{j,k}(z),$$

where

$$f_{j,k}(z)=\frac{1}{k}\sum_{v=0}^{k-1}\varepsilon^{-vj}f\left(\varepsilon^{v}z\right),\ z\in\mathcal{U}. \tag{2}$$

Now, we define the differential operator ${}_{\lambda,\delta}\mathcal{D}^{\sigma,s,l}$ as follows; The operator ${}_{\lambda,\delta}\mathcal{D}^{\sigma,s,l}$ can be written in terms of convolution as

$${}_{\lambda,\delta}\mathcal{D}^{\sigma,s,l}f(z)=\psi(z)*\psi(z)*\ldots\ldots\psi(z)_{\sigma-times}*\sum_{n=1}^{\infty}n^{s}z^{n}*f(z)$$

$$\psi(z)=\left[\frac{\lambda}{l+1}\frac{z}{(1-z)^{2}}-\frac{\lambda}{l+1}\frac{z}{1-z}+\frac{z}{(1-z)}\right]*\frac{z}{(1-z)^{\delta+1}},z\in\mathcal{U}.$$

$${}_{\lambda,\delta}\mathcal{D}^{\sigma,s,l}f(z)=z+\sum_{n=2}^{\infty}n^{s}\left(c(\delta,n)\left[\frac{\lambda(n-1)+l+1}{l+1}\right]^{\sigma}a_{n}\right)z^{n}$$

where

Error!

Note that ${}_{\lambda,\delta}\mathcal{D}^{0,1,0}f(z)={}_{1,0}\mathcal{D}^{1,0,0}f(z)=zf'(z)$ and ${}_{\lambda,\delta}\mathcal{D}^{0,0,0}f(z)=f(z)$. When $\sigma=0$ we get the *Sălăgean* differential operator [], when $s=l=0$, we obtain the Maslina Darus and Rabha W.Ibrahim differential operator [], when $s=0,\delta=0$, we get the multiplier transformation [] and when $\delta=s=l=0$, we obtain the Al-oboudi differential operator [].

We denote by $\mathcal{S}^{*}$ and $\mathcal{K}$ the familiar subclasses consisting of functions which, respectively starlkie and convex in $\mathcal{U}$. Al Sarari and Latha introduced and studied the classes which are related to (j,k)-symmetrical functions. For more details about the classes with (j,k)-symmetrical functions see [, , ,].

Definition 1 *Let* $\mathcal{K}_{j,k}^{\sigma,s,l}(\lambda,\delta,\alpha,\beta)$ *denote the class of functions* $f\in\mathcal{A}$ *satisfying the inequality*

$$\left|\frac{\left(z\left({}_{\lambda,\delta}\mathcal{D}^{\sigma,s,l}f(z)\right)'\right)'}{\left({}_{\lambda,\delta}\mathcal{D}^{\sigma,s,l}f_{j,k}(z)\right)'}-1\right|<\beta\left|\frac{\alpha\left(z\left({}_{\lambda,\delta}\mathcal{D}^{\sigma,s,l}f(z)\right)'\right)'}{\left({}_{\lambda,\delta}\mathcal{D}^{\sigma,s,l}f_{j,k}(z)\right)'}+1\right|, \tag{3}$$

where,

$$ {}_{\lambda,\delta}\mathcal{D}^{\sigma,s,l} f_{j,k}(z)=\frac{1}{k}\sum_{\nu=0}^{k-1}\varepsilon^{-\nu j}\,{}_{\lambda,\delta}\mathcal{D}^{\sigma,s,l} f(\varepsilon^{\nu}z), \varepsilon^{k}=1 \tag{4}$$

where
Error! *and j=0,1,2,...,k−1.*

II. MAIN RESULTS

Theorem 2.1 *A function* $f\in\mathcal{K}_{j,k}^{\sigma,s,l}(\lambda,\delta,\alpha,\beta)$ *if and only if*

$$\frac{(z({}_{\lambda,\delta}\mathcal{D}^{\sigma,s,l} f(z))')'}{{}_{\lambda,\delta}\mathcal{D}^{\sigma,s,l} f_{j,k}(z)')}\prec\frac{1+\beta z}{1-\alpha\beta z}$$

[Sorry. Ignored \begin{proof} ... \end{proof}]

Remark 2 *From Theorem* ***Error! Reference source not found.*** *we have*

$$\Re\left\{\frac{(z({}_{\lambda,\delta}\mathcal{D}^{\sigma,s,l} f(z))')'}{{}_{\lambda,\delta}\mathcal{D}^{\sigma,s,l}(f_{j,k}(z))'}>0\right\} \tag{5}$$

since

$$\Re\left\{\frac{1+\beta z}{1-\alpha\beta z}\right\}>0$$

Theorem 2.3 *If* $f\in\mathcal{K}_{j,k}^{\sigma,s,l}(\lambda,\delta,\alpha,\beta)$, *then* ${}_{\lambda,\delta}\mathcal{D}^{\sigma,s,l} f_{j,k}\in\mathcal{K}$,

[Sorry. Ignored \begin{proof} ... \end{proof}]

Remark 4 *From Theorem and inequality (5), we know that if* $f\in\mathcal{K}_{j,k}^{\sigma,s,l}(\lambda,\delta,\alpha,\beta)$,

then ${}_{\lambda,\delta}\mathcal{D}^{\sigma,s,l} f$ *is a quasi- convex function.*

2.1 The Integral Representation

In this section, we give the integral representation of functions in the class $\mathcal{K}_{j,k}^{\sigma,s,l}(\lambda,\delta,\alpha,\beta)$.

Theorem 2.5 *Let* $f\in\mathcal{K}_{j,k}^{\sigma,s,l}(\lambda,\delta,\alpha,\beta)$. *Then*

where ${}_{\lambda,\delta}\mathcal{D}^{\sigma,s,l} f_{j,k}(z)$ *is defined by the equality (4),* ω *is analytic in* $\mathcal{U}$ *and* $\omega(0)=0$, $|\omega(z)|<1$.

[Sorry. Ignored \begin{proof} ... \end{proof}]

Theorem 2.6 *Let* $f\in\mathcal{K}_{j,k}^{\sigma,s,l}(\lambda,\delta,\alpha,\beta)$. *Then*

Error! (6)

$$ {}_{\lambda,\delta}\mathcal{D}^{\sigma,s,l} f(z)=\int_0^z\frac{1}{\xi}\int_0^{\xi}\exp\left\{\frac{1}{k}\sum_{\mu=0}^{k-1}\int_0^{\varepsilon^{\mu}z}\frac{(1+\alpha)\,\omega(t)}{t(1-\alpha\beta\omega(t))}dt\right\}\frac{1+\beta\omega(\eta)}{1-\alpha\beta\omega(\eta)}d\eta d\xi \tag{7}$$

where ω *is analytic in* $\mathcal{U}$ *and* $\omega(0)=0$, $|\omega(z)|<1$.

[Sorry. Ignored \begin{proof} ... \end{proof}]

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International Journal of Engineering Research and Applications (IJERA) is **UGC approved** Journal with Sl. No. 4525, Journal no. 47088. Indexed in Cross Ref, Index Copernicus (ICV 80.82), NASA, Ads, Researcher Id Thomson Reuters, DOAJ.

H.N. Kanthalakshmi. "Subclasses of Analytic…] Subclasses of Analytic Functions Associated With (J, K)-Symmetrical Functions H.N. Kanthalakshmi, F.S Alsarari and S. Latha] ." International Journal of Engineering Research and Applications (IJERA) , vol. 7, no. 9, 2017,