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Strong and Weak Vertex-Edge Mixed Domination on S - Valued Graphs

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ABSTRACT
In [6] we have introduced the notion of vertex edge mixed domination in S-valued graphs and proved several results. In this paper we introduce the notion of strong and weak vertex - edge mixed domination on S-valued graphs.
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I. INTRODUCTION

Motivated by the definition of S-valued graph [2], Chandramouleeswaran and others [9] introduced the notion of semi ring valued graphs. In [5] and [8] the authors discussed regularity conditions on S-valued graphs. In [7] we have defined degree regularity on edges of S-valued graphs. The theory of domination was initiated by Berge [1] and was studied by many researchers. Motivated by the works on domination in crisp graph theory in [3] and [4] the authors introduced the notion of vertex domination and strong-weak vertex domination on S-valued graphs respectively. In [6] we have introduced the notion of vertex-edge mixed domination in S-valued graphs and proved several results. In this paper we introduce the notion of strong and weak vertex - edge mixed domination on S-valued graphs.

II. PRELIMINARIES

In this section, we recall some basic definitions that are needed for our work.

Definition 2.1: [2] A semi ring (S, +, .) is an algebraic system with a non-empty set S together with two binary operations + and . such that

(1) (S, +, 0) is a monoid.
(2) (S, .) is a semigroup.
(3) For all a, b, c $\in$ S , a . (b + c) = a . b + a . c and (a + b) . c = a . c + b . c
(4) 0 . x = x . 0 = 0 , $\forall$ x $\in$ S.

Definition 2.2:[2] Let (S, +, .) be a semiring. A Canonical Pre-order $\preceq$ in S defined as follows: for a, b $\in$ S , a $\preceq$ b if and only if, there exists an element c $\in$ S such that a + c = b.

Definition 2.3: [1] A subset D $\subseteq$ V of vertices in a graph G = (V, E) is called a vertex dominating set in G if every vertex v $\in$ V is either an element in D or is adjacent to an element in V – D. A subset D $\subseteq$ V is a vertex dominating set of G, if $\forall$ v $\in$ V – D, $N(v)\bigcap D \neq \phi$

Definition 2.4: [1] A subset D $\subseteq$ V is an independent set of G, if u, v $\in$ D, $N(u)\bigcap \{v\} = \phi$

Definition 2.5: [1] A subset D $\subseteq$ V is an Independent dominating set of G if D is both an independent and a dominating set.

Definition 2.6: [9] Let G = (V, E $\subset$ V $\times$ V) be a given graph with V, E $\neq \phi$. For any semiring (S, +, .), a semi ring-valued graph (or a S-valued graph), G^S, is defined to be the graph $G^S = (V, E, \sigma, \psi)$ where $\sigma : V \to S$ and $\psi : E \to S$ is defined to be

$$\psi(x,y) = \begin{cases} \min\{\sigma(x), \sigma(y)\}, if \sigma(x) \preceq \sigma(y) or \sigma(y) \preceq \sigma(x) \\ 0, otherwise \end{cases}$$

For every unordered pair (x,y) of E $\subset$ V $\times$ V. We call σ, a S- vertex set and ψ, a S-edge set of G^S.

Definition 2.7: [9] If σ (v) = a; $\forall$ v $\in$ V and some a $\in$ S then the corresponding S-valued graph G^S is called a vertex regular S-valued graph.

Definition 2.8: [3] Consider the S-valued graph G^S =(V, E $\subset$ V $\times$ V). The open neighbourhood of v_i in G^S is defined as the set

$$N_S(v_i) = \{(v_j, \sigma(v_j) where (v_i, v_j) \in E, \psi(v_i, v_j) \in S)\}.$$

Definition 2.9: [3] The closed neighbourhood of v_i in $G^S = (V, E, \sigma, \psi)$ is defined to be the set $N_S[v_i] = N_S(v_i) \cup \{(v_i, \sigma(v_i))\}$

Definition 2.10: [8] The degree of the vertex v_i of the S- valued graph G^S is defined as $\deg_S(v_i) = \left(\sum_{((v_i, v_j)) \in E} \psi((v_i, v_j)), l \right)$ where l is the number of edges incident with v_i .

Definition 2.11: [7] Let $G^S = (V, E, \sigma, \psi)$ be a S- valued graph. The degree of the edge e is defined as $\deg_S(e) = \left(\sum_{e_i \in N_S(e)} \psi((e_i), m \right)$ where m is the number of edges adjacent to e.

Definition 2.12: [5] A S- valued graph $G^S = (V, E, \sigma, \psi)$ is said to be a S-Star if its underlying graph G is a Star along with S-values.

Definition 2.13: [3] A vertex v $\in$ D of $G^S = (V, E, \sigma, \psi)$ is said to be an S-isolate vertex if $N_S(v) \subseteq V - D$.

Definition 2.14: [4] A subset D $\subseteq$ V is said to be a strong weight dominating vertex set if
(1) D is a weight dominating vertex set.
(2) For each vertex v $\in$ D; $\deg_S(u) \preceq \deg_S(v)$, $\forall$ u $\in$ $N_S[v]$.

Definition 2.15: [4] A subset D $\subseteq$ V is said to be a weak weight dominating vertex set if
(1) D is a weight dominating vertex set.
(2) For each vertex v $\in$ D; $\deg_S(v) \preceq \deg_S(u)$, $\forall$ u $\in$ $N_S[v]$.

Definition 2.16: [6] Consider the S-valued graph $G^S = (V, E, \sigma, \psi)$. Let D $\subseteq$ V. If every edge of G^S is weight m- dominated by any vertex in D, then D is said to be a ve- weight m- dominating set.

III. STRONG VERTEX - EDGE MIXED DOMINATION ON S-VALUED GRAPHS

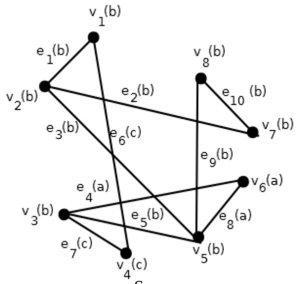
In [4] the authors have discussed strong and weak weight domination in S-valued graphs. They have studied the vertices in G^S, dominating the other vertices in its neighbourhood by comparing the weights and degrees.

Similarly in our earlier paper [6] we have discussed ve-weight m-dominating set by comparing the weight of the vertices which dominates the edges in the sub graph induced by the closed neighbourhood of the vertex.

Here we study the weight of the vertices which dominates the edges in the sub graph induced by the closed neighbourhood of the vertex under consideration (Example 3.1) along with their degrees (Example 3.2).

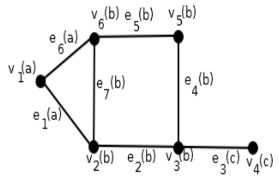
In this section, we introduce the notion of strong vertex - edge mixed domination in S-valued graph, analogous to the notion in crisp graph theory, and prove some simple results.

Example 3.1:



In this S-valued graph G^S , clearly $D_1 = \{v_1, v_3, v_5, v_7\}$, $D_2 = \{v_1, v_2, v_3, v_5, v_7\}$, $D_3 = \{v_1, v_2, v_3, v_5, v_8\}$, $D_4 = \{v_1, v_2, v_3, v_5, v_7, v_8\}$ are all ve- weight m- dominating sets.
Also in this S-valued graph G^S, there is no strong ve- weight m- dominating set.

Example 3.2:



In this S-valued graph G^S , clearly $D_1 = \{v_2, v_3, v_6\}$, $D_2 = \{v_2, v_3, v_5, v_6\}$ are strong weight dominating vertex sets.
Also in this S-valued graph G^S, there is no strong ve- weight m- dominating set.

Definition 3.3: Consider the S-valued graph $G^S = (V, E, \sigma, \psi)$. A subset D $\subseteq$ V is said to be a strong ve- weight m- dominating set, if (1) D is a ve- weight m- dominating (2) For each vertex v $\in$ D, $\deg_S(e_i) \preceq \deg_S(v)$ $\forall e_i \in \langle N_S[v] \rangle$.

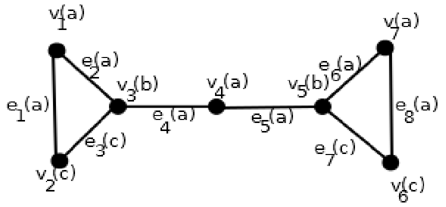
Example 3.4: Let (S = {0, a, b, c}, +, .) be a semiring with the following Cayley Tables:

+	0	a	b	c
0	0	a	b	c
a	a	a	b	c
b	b	b	b	b
c	c	c	b	c

.	0	a	b	c
0	0	0	0	0
a	0	0	a	0
b	0	a	b	c
c	0	0	c	c

Let $\preceq$ be a canonical pre-order in S, given by
$0 \preceq 0, 0 \preceq a, 0 \preceq b, 0 \preceq c, a \preceq a, a \preceq b, a \preceq c, b \preceq b, c \preceq b, c \preceq c$

Consider the S - graph $G^S = (V, E, \sigma, \psi)$,

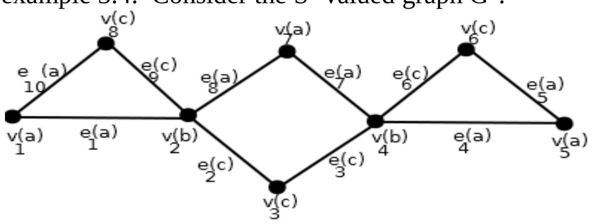


Define $\sigma : V \to S$ by
$\sigma(v_1) = \sigma(v_4) = \sigma(v_7) = a, \sigma(v_2) = \sigma(v_6) = c, \sigma(v_3) = \sigma(v_5) = b.$
and
$\psi : E \to S$ by
$\psi(e_1) = \psi(e_2) = \psi(e_4) = \psi(e_5) = \psi(e_6) = \psi(e_8) = a, \psi(e_3) = \psi(e_7) = c.$

Clearly D ={v_3, v_5} is a strong ve- weight m-dominating set.

Definition 3.5: Consider the S-valued graph $G^S = (V, E, \sigma, \psi)$. A subset D $\subseteq$ V is said to be a strong ve- weight m- dominating independent set, if
(1) D is a strong ve- weight m- dominating set.
(2) If u, v $\in$ D then $N_S(u) \cap (v, \sigma(v)) = \phi$.

Example 3.6: Let Let (S = {0, a, b, c}, +, .) be a semiring with the canonical preorder given in example 3.4. Consider the S- valued graph G^S:



Define $\sigma : V \to S$ by
$\sigma(v_1) = \sigma(v_5) = \sigma(v_7) = a, \sigma(v_2) = \sigma(v_4) = b, \sigma(v_3) = \sigma(v_6) = \sigma(v_8) = c.$
and $\psi : E \to S$ by
$\psi(e_1) = \psi(e_4) = \psi(e_5) = \psi(e_7) = \psi(e_8) = \psi(e_{10}) = a, \psi(e_2) = \psi(e_3) = \psi(e_6) = \psi(e_9) = c.$

Clearly D = {v_2, v_4} is a strong ve- weight m-dominating independent set.
The following theorem is obvious.

Theorem 3.7: A subset D $\subseteq$ V of a S- valued graph G^S is a strong ve- weight m- dominating independent set iff D is a strong ve- weight m-dominating set.

Theorem 3.8: In a S-Star, the strong ve- weight m-dominating set is unique.
Proof: Let G^S be a S-Star. Let the pole v_1 have the maximum weight.
Then $e_i \in \langle N_S[v_1] \rangle, \forall e_i \in G^S.$ Also $\deg_S(e_i) \preceq \deg_S(v_1) \forall e_i \in \langle N_S[v_1] \rangle.$
Hence {v_1} is the strong ve- weight m- dominating set.

Remark 3.9:(1) In a S- Wheel, there will be no strong ve- weight m- dominating set, since every spokes will have maximum degree than all vertices.
(2) In a Complete graph K_n^S , for n > 3 there will be no strong ve- weight m- dominating set, since every edges will have maximum degree than all vertices.
(3) In a Complete Bipartite graph $K_{m,n}^S$, for m, n > 3 there will be no strong ve- weight m- dominating set, since every edge will have maximum degree than all vertices.

IV. WEAK VERTEX - EDGE MIXED DOMINATION ON S-VALUED GRAPHS

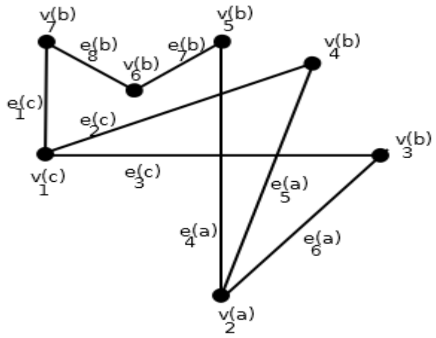
In this section, we introduce the notion of weak vertex - edge mixed domination in S-valued graph, analogous to the notion in crisp graph theory, and prove some simple results.

Definition 4.1: Consider the S-valued graph $G^S = (V, E, \sigma, \psi)$. A subset D $\subseteq$ V is said to be a weak ve- weight m- dominating set, if
(1) D is a ve- weight m- dominating set.
(2) For each vertex v $\in$ D, $\deg_S(v) \preceq \deg_S(e_i)$ $\forall e_i \in \langle N_S[v] \rangle$.

Example 4.2: Let (S = {0, a, b, c}, +, .) be a semiring with the following Cayley Tables:
Let $\preceq$ be a canonical pre-order in S, given by
0 $\preceq$ 0, 0 $\preceq$ a, 0 $\preceq$ b, 0 $\preceq$ c, a $\preceq$ a, a $\preceq$ b, a $\preceq$ c, b $\preceq$ b, c $\preceq$ b, c $\preceq$ c
Consider the S - graph $G^S = (V, E, \sigma, \psi)$,

+	0	a	b	c
0	0	a	b	c
a	a	a	b	c
b	b	b	b	b
c	c	c	b	c

.	0	a	b	c
0	0	0	0	0
a	0	0	a	0
b	0	a	b	c
c	0	0	c	c



Define $\sigma : V \to S$ by
$\sigma(v_1) = c, \sigma(v_2) = a, \sigma(v_3) = \sigma(v_4) = \sigma(v_5) = \sigma(v_6) = \sigma(v_7) = b.$

and $\psi : E \to S$ by
$\psi(e_1) = \psi(e_2) = \psi(e_3) = c, \psi(e_4) = \psi(e_5) = \psi(e_6) = a, \psi(e_7) = \psi(e_8) = b.$
Clearly $D_1 = \{v_3, v_4, v_5, v_7\}$ is a weak ve- weight m-dominating set.
Also $D_2 = \{v_3, v_4, v_5, v_6, v_7\}$ is a weak ve- weight m- dominating set.

Definition 4.3: Consider the S-valued graph $G^S = (V, E, \sigma, \psi)$. A subset D $\subseteq$ V is said to be a minimal weak ve- weight m- dominating set, if
(1) D is a weak ve- weight m- dominating set.
(2) No proper subset of D is a weak ve- weight m-dominating set.
In the example 4.2, $D_1 = \{v_3, v_4, v_5, v_7\}$ is a minimal weak ve- weight m- dominating set.

Definition 4.4: Consider the S-valued graph $G^S = (V, E, \sigma, \psi)$. A subset D $\subseteq$ V is said to be a maximal weak ve- weight m- dominating set, if
(1) D is a weak ve- weight m- dominating set.
(2) there is no weak ve- weight m- dominating set D' $\subset$ V such that D $\subset$ D' $\subset$ V .
In the example 4.2, $D_2 = \{v_3, v_4, v_5, v_6, v_7\}$ is a maximal weak ve- weight m- dominating set.

Definition 4.5: Consider the S-valued graph $G^S = (V, E, \sigma, \psi)$. A subset D $\subseteq$ V is said to be a weak ve- weight m- dominating independent set, if
(1) D is a weak ve- weight m- dominating set.
(2) if u,v $\in$ D then $N_S(u) \cap (v, \sigma(v)) = \phi$.
In the example 4.2, $D_1 = \{v_3, v_4, v_5, v_7\}$ is a weak ve- weight m- dominating independent set.

Theorem 4.6: In a tree G^S, a ve- weight m-dominating set D will be strong ve- weight m-dominating set if all the vertices of D are intermediate vertices. If the vertices of D are pendent vertices then D is a weak ve- weight m-dominating set.

Proof: Let G^S be a tree. Let D $\subseteq$ V be a ve - weight m- dominating set. If all the vertices of D are intermediate vertices, then for every vertex v $\in$ D, $deg_S(e_i) \preceq deg_S(v)$ $\forall$ $e_i \in \langle N_s[v] \rangle$. Hence D is a strong ve- weight m- dominating set.
If all the vertices of D are pendent vertices, then for every vertex v $\in$ D, $deg_S(v) \preceq deg_S(e_i)$ $\forall$ $e_i \in \langle N_s[v] \rangle$. Therefore D is a weak ve- weight m-dominating set.

Theorem 4.7: A weak ve- weight m- dominating set D of a graph G^S is a minimal weak ve- weight m- dominating set of G^S iff every vertex v $\in$ D satisfies atleast one of the following properties;
(1) there exist a vertex u $\in$ V - D such that $N_s(u) \cap \{D \times S\} = \{(v, \sigma(v))\}$
(2) v is adjacent to no vertex of D.

Proof : If each v $\in$ D satisfies at least one of the above two properties, then D – {v} is not a weak ve- weight m- dominating set.
$\therefore$ D is a minimal weak ve- weight m- dominating set.
Conversely, assume that D is a minimal weak ve-weight m- dominating set of G^S.
Then for each v $\in$ D, D-{v} is not a minimal weak ve- weight m- dominating set of G^S.
$\therefore$ there exist a vertex u $\in$ V - (D - {v}) that is adjacent to no vertex of (D-{v}).
If u = v, then v is adjacent to no vertex of D.
If u $\neq$ v, then D is a weak ve- weight m-dominating set and u $\notin$ D $\Rightarrow$ u is adjacent to atleast one vertex of D. However u is not adjacent to any vertex of D – {v} $\Rightarrow$ $N_s(u) \cap \{D \times S\} = \{(v, \sigma(v))\}$.

Theorem 4.8: If D $\subseteq$ V of G^S is a minimal weak ve- weight m- dominating set without S- isolate vertices then V -D is also a weak ve- weight m-dominating set of G^S, whenever G^S is vertex regular S- valued graph.

Proof: Assume that $G^S = (V, E, \sigma, \psi)$ be a vertex regular S- valued graph.
Let v $\in$ D then by theorem 4.7 ,
(1) there exist a vertex u $\in$ V - D such that $N_s(u) \cap \{D \times S\} = \{(v, \sigma(v))\}$
(2) v is adjacent to no vertex of D.
In the first case, v is adjacent to some vertex in V - D.
In the second case, v is an S- isolate vertex of the subgraph spanned by $\langle D \rangle$.
But v is not S- isolated in G^S.
Hence v is adjacent to some vertex of V - D:
Thus V - D is a weak ve- weight m- dominating set of G^S; whenever G^S is vertex regular S- valued graph.

Remark 4.9: In the above theorem, the vertex regularity of G^S is essential. That is, if the graph G^S is not vertex regular then the theorem fails as given by the following example.
In the example 4.2, $D_1 = \{v_3, v_4, v_5, v_7\}$ is a minimal weak ve- weight m- dominating set.
And V - $D_1 = \{v_1, v_2, v_6\}$.
Since the vertices of V - D_1 has minimum weight, V - D_1 is not a weak ve- weight m- dominating set.

Theorem 4.10: A subset D $\subseteq$ V of a S- valued graph G^S is weak ve- weight m- dominating independent set iff D is a maximal independent vertex set in G^S .

Proof: Clearly every maximal independent vertex set D in G^S is a weak ve- weight m- dominating independent set.

Conversely assume that D is weak ve- weight m-dominating independent set. Then D is independent and every vertex not in D is adjacent to a vertex of D and therefore D is a maximal independent vertex set in G^S.

Theorem 4.11: Every maximal independent vertex set D in G^S is a minimal weak ve- weight m-dominating set.

Proof : Let D be a maximal independent vertex set in G^S . Then by theorem 4.10, D is a weak ve-weight m- dominating independent set. Since D is independent, certainly every vertex of D is adjacent to no vertex of D. Thus, every vertex of D satisfies the second condition of theorem 4.7 Hence D is a minimal weak ve- weight m-dominating set in G^S.

Theorem 4.12: Every weak ve- weight m-dominating independent set D in G^S is a minimal weak ve- weight m- dominating set.

Proof : The proof follows from the theorem 4.10 and the theorem 4.11.

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