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Dose rate in air for cylindrical shielded radioactive source

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ABSTRACT
In dosimetry, the cylindrical sealed radioactive source presents a special interest due to the radiotherapy units. In this article we develop the basic formulas for the calculation of gamma radiation source's dose rate in a given point, situated on cylinder axis at a certain distance from source base, in air. Our results indicate that there is dependence between dose rate and form factor which depends by the source's height.
Key words: *dose rate, gamma source, form factor*

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I. INTRODUCTION

In dosimetry, the cylindrical sealed radioactive source presents a special interest due to the radiotherapy units which is part of [1]. For gamma rays (indirectly ionizing radiation) kerma is used in order to define the ratio between the kinetic energy transferred by the gamma photons to the electrons from the unit mass of medium [2]. A particular case is the cylindrical source in disc form. The cylindrical source has two geometrical parameters: height cylinder, G and radius, R (figure 1). The calculation of source's dose rate in a given point P, situated on cylinder axis at a distance D from source base, in air, is very interesting and laborious involving many variables [3]. Inside of source we consider a circle centered on cylinder axis situated in a parallel plan with cylinder base, at a distance, g, from superior base of cylinder.

The two variables used in calculation, g and r are situated in the following ranges: $0 \le g \le G$ and $0 \le r \le R$.

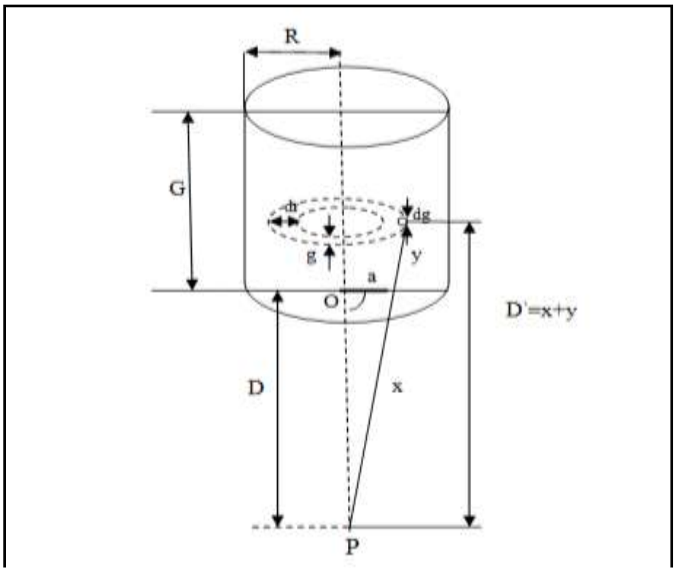

Figure 1. The calculation of source's dose rate in a given point P

A photon emitted by circle traverses the distance y in source's material and distance x in air and reach the point P. In order to express the x and y distances we can introduce the auxiliary distance, a (figure 1):

$$a = r\frac{D}{D+g} \quad (1)$$

so $x = \sqrt{D^2 + a^2} = \frac{D}{D+g}\sqrt{(D+g)^2 + r^2}$ (2)

$$y = \sqrt{g^2 + (r-a)^2} = \frac{g}{D+g}\sqrt{(D+g)^2 + r^2} \quad (3)$$

with $D'^2 = (x+y)^2 = (D+g)^2 + r^2$ (4)

In our case, $g \ll D, r \ll D$ and obtain:

$$y^2 = g^2 + (r-a)^2 = g^2(1 + \frac{r^2}{D^2+r^2}),$$
$$y \cong g \quad (5)$$

The considered circle has the elementary volume:
$dV = 2\pi r\, dr\, dg$ (6)
and elementary mass
$dm = \rho dV$ (7)
where ρ is the density of source material.

If $\Lambda_m = \frac{\Delta\Lambda}{m}$, $\Lambda_m = 1\frac{Bq}{kg}$ (8)

is the mass activity, homogenous distributed in source volume, than elementary activity of circle is:
$d\Lambda = \Lambda_m d_m = 2\pi r \Lambda_m\, \rho\, dr\, dg$ (9)

Ions dose rate in a point P, on the circle axes will be:

$$d\dot{\Delta} = \frac{\Gamma d\Lambda}{D'^2} B e^{-\mu' y} e^{-\mu'' x} \quad (10)$$

where:
- B is the build-up factor for the material of source; the build-up factor for air is negligible,
- μ' and respective, μ'' are linear absorption coefficients of source's material and respective, for air

Ions dose rate (or kerma rate) in point P can be obtain through integration of the last equation:

$$\dot{\Delta} = 2\pi\Gamma\Lambda_m\varphi \int_0^G \int_0^R \frac{r B_s e^{-\mu' y} e^{-\mu'' x}}{(D+g)^2 + r^2} dr dg \quad (11)$$

Particular cases:

1. $B = 1$ and $D \gg g$ respective, $D \gg r$ si $\mu'' \to 0$ obtain, taking into account

$$\dot{\Delta} = 2\pi\Gamma\Lambda_m\rho \int_0^G e^{-\mu' g} dg \int_0^R r dr \quad (12)$$

$$\dot{\Delta} = 2\pi\Gamma\Lambda_m\rho \frac{R^2}{2}\frac{1}{\mu'}\left(-e^{-\mu' G} + 1\right)\frac{1}{D^2},$$
$$\Lambda = \Lambda_m \pi R^2 G\rho \quad (13)$$

$$\dot{\Delta} = \frac{\Gamma\Lambda}{D^2}\frac{1}{\mu' G}\left(1 - e^{-\mu' G}\right) \quad (14)$$

$\dot{\Delta} = \frac{\Gamma\Lambda}{D^2}$ – exponential of point source
$\frac{1}{\mu' G}\left(1 - e^{-\mu' G}\right)$ - form factor

In case of a ${}^{60}_{27}Co$ source (Z=27, A=60, $\rho = 8{,}9\frac{g}{cm^3}$) with gamma radiation energy, $\overline{E_\gamma} = 1{,}2MeV$, μ' can be calculated through following equation:

$$\mu' = \tau + \sigma + \chi \quad (15)$$

where,
μ' = linear absorption coefficient of source's material
τ = linear absorption coefficient due to photoelectric effect
σ = linear absorption coefficient due to Compton effect
χ = linear absorption coefficient due to pairs generation

At this energy level, the last term is negligible and we can use the semi-empirical formula:

$$\tau = 4{,}04 \cdot 10^{-7}\tau_{Pb}\frac{\rho Z^4}{A}$$
$$(\tau_{Pb} = 0{,}15cm^{-1}) \quad (16)$$

$$\sigma = 0{,}223 \cdot 10^{-7}\sigma_{Pb}\frac{\rho Z}{A}$$
$$(\sigma_{Pb} = 0{,}52cm^{-1}) \quad (17)$$

and obtain

$$\tau = 4{,}04 \cdot 10^{-7} \cdot 0{,}15 \cdot 8{,}9 \cdot \frac{27^4}{60} = 4.78 \cdot 10^{-3}cm^{-1} \quad (18)$$

$$\sigma = 0{,}223 \cdot 0{,}52 \cdot 8{,}9 \cdot \frac{27}{60} = 0{,}464cm^{-1} \quad (19)$$

and $\mu' = \tau + \sigma = 0{,}469cm^{-1}$ (20)

If we note with C form factor (G=3cm), we obtain:

$$C = \frac{1}{\mu' G}\left(1 - e^{-\mu' G}\right) = \frac{1}{1{,}407}(1 - 0{,}245) = 0{,}537 \quad (21)$$

Respect the point source case, dose rate is considerably reduced and confirms the experimental data.

2. Case $G \to 0$, source in disc form; for $\lim_{G\to 0} \frac{1-e^{-\mu G}}{\mu G} = 1$, the source can be assimilated with a point source.
3. If we take into account the scatter produced by the source's material, we take for scattering factor B, an empirical formula:

$$B = 1 + \mu' g \quad (22)$$

The integral becomes:

$$\dot{\Delta} = 2\pi \Gamma \Lambda_m \rho \int_0^G (1 + \mu' g) e^{-\mu' g} dg \int_0^R r dr \quad (23)$$

Forma factor is modified:

$C' = \frac{1}{\mu' G} \int_0^G x e^x dx$ with

$$x = -\mu' g \quad (24)$$

We have: $\int x e^x \, dx = (x-1) \cdot e^x$ (25)

and obtain: $C' = -\frac{1}{\mu' G}(1 + \mu' g) e^{-\mu' g} /_0^G = \frac{1}{\mu'} - \frac{1}{\mu'}(1 + \mu' G) e^{-\mu' G}$ (26)

$$C' = \frac{1}{\mu' G}\left[1 - (1 + \mu' G) e^{-\mu' G}\right] \quad (27)$$

the new form factor will be:

$$C_t = \frac{2}{\mu' G} - \frac{2}{\mu' G} e^{-\mu G} \quad (28)$$

4. The dependence of form factor function of source's thickness, G :

$$C(G) = \frac{1}{\mu' G}(1 - e^{-\mu' G}) \quad (29)$$

For ^{60}Co we have $\mu' = 0{,}469 cm^{-1}$ and the dependence we are presenting in the following table and graph:

Table 1. The form factor dependence function of source's height

$G\ (cm)$	$\mu' G$	$C(G)$
0	0	1,00
0,2	0,094	0,955
0,4	0,188	0,911
0,6	0,281	0,873
0,8	0,375	0,834
1,00	0,469	0.,789
1,2	0,563	0,765
1,4	0,657	0,733
1,6	0,750	0,703
1,8	0,844	0,675
2	0,938	0,649
2,2	1,032	0,624
2,4	1,126	0,600
2,6	1,219	0,578
2,8	1,313	0,557
3	1,407	0,537

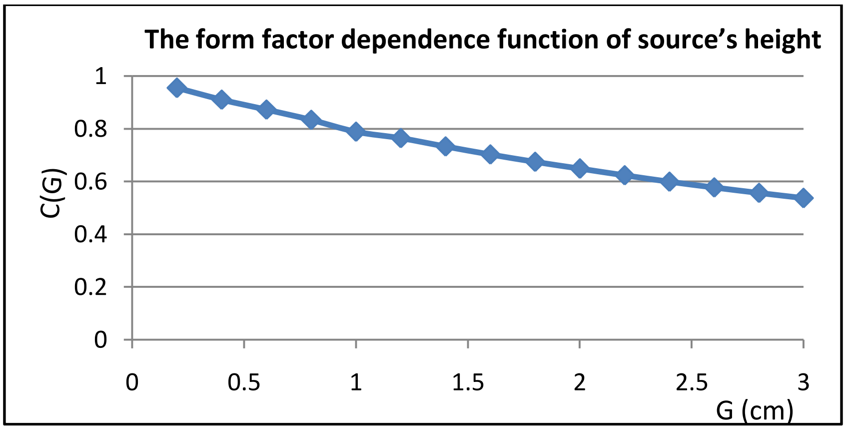

Figure 2. The form factor dependence function of source's height

Observation: It would be desirable that the thickness of source to be as small as possible, preferable a disc (G=0). The acceptable thickness would be G=2cm. The relation found for C (G) can be used only for maximum G=5cm and for this case we approximated G<<D. C (G) can be approximated through a linear development, taking the second grade terms also.

$$C(G) = 1 - \frac{\mu G}{2} \text{ (for small G)} \qquad (30)$$

5. If we take into account the scattering, we have Berger's formula[4]:

$$B = 1 + a\mu x e^{-b\mu x} \qquad (31)$$

where $\boldsymbol{a}$ and $\boldsymbol{b}$ are the parameters which depend by photon's energy and the atomic number (Z) of attenuation material.

In specialty literature are indicated more approximations of B, such as:

$$B = 1 + \mu x \quad \begin{cases} a = 1 \\ b = 0 \end{cases} \qquad (32)$$

For energies up to 3MeV and not heavy elements, with the same formula

$$B = 1 + \mu x \qquad 0{,}5 < a < 1 \qquad (33)$$

we find

$$\Delta C(G) = \frac{a}{\mu G}\left[1 - (1 + \mu G)e^{-\mu G}\right]$$

$$\lim_{G\to 0} \Delta C(G) = 0 \qquad (34)$$

Table 2. The form factor dependence function of source's height taking into account the Berger's formula

$\boldsymbol{G\ (cm)}$	$\boldsymbol{\mu' G}$	$\Delta \boldsymbol{C(G)}$ $\boldsymbol{a = 0,5}$	$\Delta \boldsymbol{C(G)}$ $\boldsymbol{a = 1}$
0,2	0,094	0,022	0,044
0,4	0,188	0,042	0,083
0,6	0,281	0,058	0,117
0,8	0,375	0,073	0,146
1,00	0,469	0,086	0,171
1,2	0,563	0,098	0,195
1,4	0,657	0,107	0,215
1,6	0,750	0,116	0,231
1,8	0,844	0,123	0,245
2	0,938	0,129	0,258
2,2	1,032	0,134	0,267
2,4	1,126	0,138	0,277
2,6	1,219	0,141	0,282
2,8	1,313	0,144	0,288
3	1,407	0,146	0,291

For a=0.5, the graph indicate us the linear assessment of variable

$$C(G) = 1 - \alpha G \begin{cases} G = 0,\ C(G) = 1{,}000 \\ G = 3cm, C(G) = 0{,}683 \end{cases} \qquad (35)$$

$$\alpha = \frac{1 - C(G)}{G} = \frac{1 - 0{,}683}{3} = 0{,}1057cm^{-1} = 1{,}057mm^{-1} \qquad (36)$$

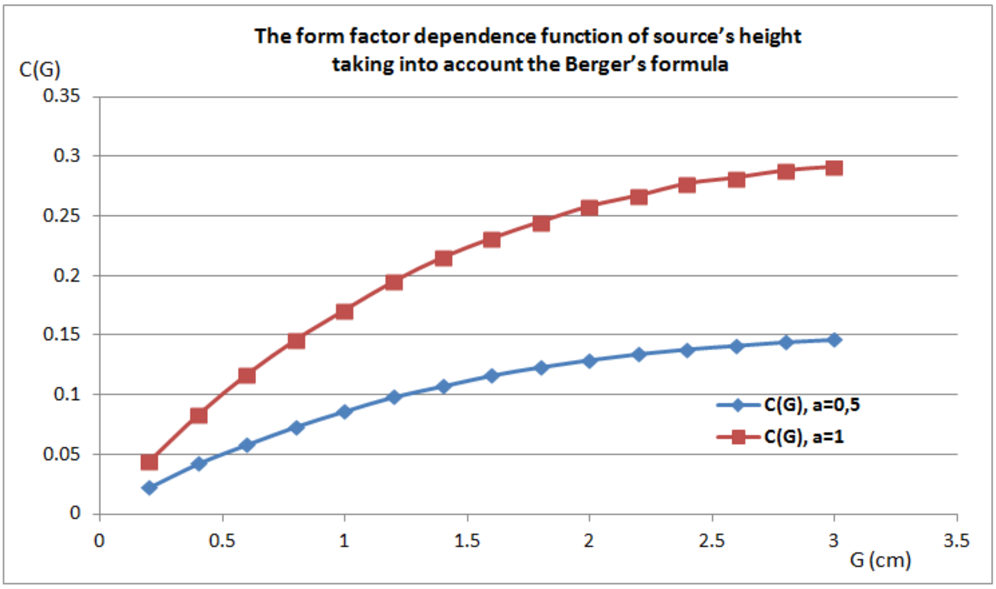

Figure 3. The form factor dependence function of source's height taking into account the Berger's formula

II. DISCUSSIONS AND CONCLUSIONS

In the specialty literature a plenty of methods for calculation of gamma dose and gamma dose rates have been developed [3] due to the special interest of cylindrical sealed radioactive source which is part of radiotherapy units [1]. For gamma rays (indirectly ionizing radiation) kerma is used in order to define the ratio between the kinetic energy transferred by the gamma photons to the electrons from the unit mass of medium [2].

Our article showed that dose rate, $\dot{\Delta}$ depends on form factor C, which depends on height source, G and on the linear absorption coefficient of source's material, μ' .

Our results indicated that it would be preferable that the thickness of source to be as small as possible, preferable a disc (G=0). The acceptable thickness would be G=2cm. The form factor can be approximated through a linear development and the relation found for it can be used only for maximum G=5cm and for this case we approximated G<<D.

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