

## Calculation of Transmittance of Superconductor-Dielectric Periodic Structure

J P Pandey

M L K P G College, Balrampur (Up)-271201, India

### ABSTRACT

Transfer matrix method (TMM) is used to study the optical properties of photonic crystals (PCs) with superconducting constituent for a stratified medium. The filtering properties in superconductor-dielectric photonic band gap material can be studied by reflectance and transmittance at different incidence angles and temperatures. Here, I have calculated the dispersion and transmittance spectrum of superconductor-dielectric periodic structure based on the transcendental equations derived from TMM and Bloch theorem.

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### I. INTRODUCTION

Much work has been done on the computation of the band structures of electromagnetic wave propagation in one-, two- and three-dimensional dielectric periodic structure [1-4]. It has already been proposed that Photonic Crystals (PCs) constructed with ferroelectric or ferromagnetic materials could be tuned by means of an external electric or magnetic field [5]. Recently, it was proposed that photonic crystal of the inverse opal type doped with a liquid crystal [6] and photonic crystals made of semiconductor constituents are suitable to constitute tunable photonic crystal [7]. The idea of the former is based on liquid crystal dielectric response to the external electric field, while the latter is mainly due to the variations of the concentration of free carriers within the semiconductor by means of change in temperature or in the density of dopants.

To foresee novel applications and interesting possibilities of incorporating the photonic crystals concept into superconducting devices, it is interesting to investigate new possibilities of merging new areas of electronics and photonics. The study of photonic band gap consisting of a superconducting material and a dielectric has been reported [8-12]. The gap size is characterized by polarization, penetration depth, and highly dependent on temperature at the vicinity of superconducting transition temperature. The analysis shows that the property of thin photonic crystals may have application in optical region if extremely low relaxation time superconductor is used. This may be asset for superconducting electronics-photonics integration.

### Theoretical formulation

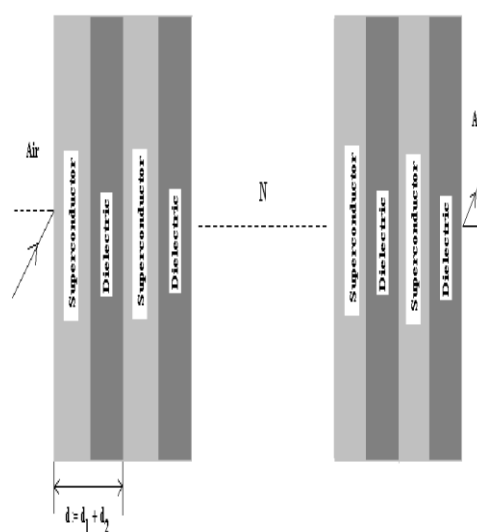


Figure 1: Schematic presentation of superconductor/dielectric periodic structure.

The refractive index profile of the structure is given by

$$n(x) = \begin{cases} n_1, & 0 < x < d_1 \\ n_2, & d_2 < x < d \end{cases} \quad (1)$$

with  $n(x) = n(x + d)$  where  $n_1, n_2$  are the refractive indices of the superconductor and dielectric materials. The electric field  $E(x)$  in the  $m^{\text{th}}$  unit cell can be written as

$$E(x) = \begin{cases} a_m e^{-ik_1(x-md)} + b_m e^{ik_2(x-md)}; & (md - d_1) < x < md \\ c_m e^{-ik_2(x-md+d_1)} + d_m e^{ik_2(x-md+d_1)}; & (m-1)d < x < (md - d_1) \end{cases} \quad (2) \quad \text{where}$$

$$k_1 = \left[ \left( \frac{\omega}{c} n_1 \right)^2 - \beta^2 \right]^{1/2} = \frac{n_1 \omega}{c} \cos \theta_1$$

$$k_2 = \left[ \left( \frac{\omega}{c} n_2 \right)^2 - \beta^2 \right]^{1/2} = \frac{n_2 \omega}{c} \cos \theta_2 \quad (3)$$

For the superconductor, the index of refraction can be described on the basis of the conventional two fluid models [13-14]. The two-fluid model is used to describe the electromagnetic behavior of superconductor at non-zero temperature. Normally, the conductivity of the superconductor is complex. With some approximations (lossless), the complex conductivity of a superconductor

approximates to 
$$\sigma(\omega) = \frac{-i e^2 n_s}{m \omega}$$

(4) only when the conduction  $n_s \tau < n_s / \omega$  is satisfied. Where  $\sigma(\omega)$  is the conductivity of the superconductor,  $e$  and  $m$  are the charge and mass of electron respectively,  $n_s$  is the density of electron and  $\omega$  is the frequency of external electromagnetic wave. The approximation condition can be found in Ref. [15]. The conductivity equation (4) can be expressed in terms of London-penetration depths  $\lambda_L$ , since

$$\lambda_L^2 = \frac{m}{\mu_0 n_s e^2} \quad (5a) \quad \text{So that}$$

$$\sigma(\omega) \approx -\frac{i}{\mu_0 \omega \lambda_L^2} \quad (5b)$$

From the Gorter-Casimir result [13, 15],  $\frac{n_s}{n_n} = \left( \frac{T_c}{T} \right)^4 - 1$  and the London-penetration depth, we obtain

$$\lambda_L(T) = \frac{\lambda_L(0)}{\sqrt{1 - \left( \frac{T}{T_c} \right)^4}} \quad (6)$$

where the conductivity equation (5b) is temperature dependent. In the superconductor layer, with no external source current and charge, the Maxwell's equation becomes - 
$$\vec{\nabla}^2 \vec{B} + k_1^2 \vec{B} = 0 \quad (7)$$

By substituting equation (5), we have

$$k_1^2 = \left( \frac{\omega^2}{c^2} - \frac{1}{\lambda_L^2} \right) \quad (8)$$

Then, from Snell's law, the length of the tangential wave vector  $k_{1x}$  (parallel to the dielectric superconductor interface) is conserved. That is

$$k_{1y} = \frac{\omega}{c} \sin \theta = \beta, \quad \text{where } \theta \text{ is the angle of}$$

incident (relative to normal of interface) of the electromagnetic wave as vacuum. Then, we have the frequency dependent normal vector,

$$k_{1x} = \sqrt{\frac{\omega^2}{c^2} \cos^2 \theta - \frac{1}{\lambda_L^2}} = \frac{\omega}{c} n_1(\omega)$$

(9) where

$$n_1(\omega) = \sqrt{\cos^2 \theta - \frac{c^2}{\omega^2} \frac{1}{\lambda_L^2}}. \quad \text{In the}$$

dielectric layer, with no external source current and charge, the Maxwell's equation becomes 
$$\vec{\nabla}^2 \vec{B} + k_2^2 \vec{B} = 0 \quad (10)$$

From equation (10) and Snell's law, we have

$$k_2^2 = \epsilon \frac{\omega^2}{c^2} \text{ or}$$

$$k_{2x}^2 = \epsilon \frac{\omega^2}{c^2} - \frac{\omega^2}{c^2} \sin^2 \theta = \frac{\omega^2}{c^2} (\epsilon - \sin^2 \theta)$$

$$\text{or} \quad k_{2x} = \frac{\omega}{c} n_2 \quad (11)$$

where

$$n_2 = \sqrt{\epsilon - \sin^2 \theta}$$

According to transfer matrix method [16-17], the characteristic matrix corresponding to one period is as follows

$$m(d) = \begin{pmatrix} m_{1,1} & m_{1,2} \\ m_{2,1} & m_{2,2} \end{pmatrix} \quad (12)$$

Where

$$m_{1,1} = \cos(k_{1x} d_1) \cos(k_{2x} d_2) - \frac{p_2}{p_1} \sin(k_{1x} d_1) \sin(k_{2x} d_2)$$

$$m_{1,2} = \frac{i}{p_2} \cos(k_{1x} d_1) \sin(k_{2x} d_2) + \frac{i}{p_1} \sin(k_{1x} d_1) \cos(k_{2x} d_2)$$

$$m_{2,1} = ip_1 \sin(k_{1x} d_1) \cos(k_{2x} d_2) + ip_2 \cos(k_{1x} d_1) \sin(k_{2x} d_2)$$

$$m_{2,2} = \cos(k_{1x}d_1) \cos(k_{2x}d_2) - \frac{p_1}{p_2} \sin(k_{1x}d_1) \sin(k_{2x}d_2)$$

where

$$k_i = \frac{\omega}{c} n_i \cos \theta_i \text{ and } p_i = \frac{n_i}{Z_0} \cos \theta_i \text{ (TE), } p_i = \frac{1}{Z_0 n_i} \cos \theta_i \text{ (TM)}$$

with  $i = 1, 2$  and impedance  $Z_0 (= \sqrt{\mu_0} / \sqrt{\epsilon_0})$  of free space. The total characteristic matrix for the N-period structure can be obtained,

$$\mathbf{M}(\mathbf{Nd}) = [\mathbf{m}(\mathbf{d})]^N = \begin{pmatrix} \mathbf{M}_{1,1} & \mathbf{M}_{1,2} \\ \mathbf{M}_{2,1} & \mathbf{M}_{2,2} \end{pmatrix}$$

(13) where  $M_{1,1} = m_{11} U_{N-1} - U_{N-2}$ ,  $M_{1,2} = m_{12} U_{N-1}$ ,  $M_{2,1} = m_{21} U_{N-1}$ ,  $M_{2,2} = m_{22} U_{N-1} - U_{N-2}$ .  $m_{i,j}$  are the characteristic matrix of equation (13) and  $U_N$  is the Chebyshev polynomial of second kind defined by

$$U_N = \frac{\sin [(N+1) K(\omega) d]}{\sin (K(\omega) d)} \quad (14)$$

In order to calculate the transmission and reflectance for a periodic multilayered structure, we use the transfer matrix method. Using this, we can obtain the dispersion relation as

$$K(\omega) = \frac{1}{d} \cos^{-1} \left[ \cos(k_{1x}d_1) \cos(k_{2x}d_2) - \frac{1}{2} \left( \frac{p_1}{p_2} + \frac{p_2}{p_1} \right) \sin(k_{1x}d_1) \sin(k_{2x}d_2) \right] \quad (15)$$

The reflection and transmission coefficients can be determined and are given by

$$r = \frac{(\mathbf{M}_{1,1} + \mathbf{M}_{1,2} p_s) p_0 - (\mathbf{M}_{2,1} + \mathbf{M}_{2,2} p_s)}{(\mathbf{M}_{1,1} + \mathbf{M}_{1,2} p_s) p_0 + (\mathbf{M}_{2,1} + \mathbf{M}_{2,2} p_s)}$$

$$t = \frac{2 p_0}{(\mathbf{M}_{1,1} + \mathbf{M}_{1,2} p_s) p_0 + (\mathbf{M}_{2,1} + \mathbf{M}_{2,2} p_s)}$$

(16) Here  $p_0$  and  $p_s$  are for the first and last medium of the structure, which are given as

$$p_0 = \frac{n_0 \cos \theta_0}{Z_0}, p_s = \frac{n_s \cos \theta_s}{Z_0} \text{ (TE)}$$

and impedance  $Z_0 (= \sqrt{\mu_0} / \sqrt{\epsilon_0})$  of free space.

In this case,  $n_0 = n_s = 1.0$  for the free space. The reflectance (R) and transmittance (T) for

superconductor/dielectric photonic crystal are related

$$\text{by } \mathbf{R} = |\mathbf{r}|^2 \text{ and } \mathbf{T} = \frac{p_s}{p_0} |\mathbf{t}|^2 \quad (17)$$

The expressions of the reflectivity for N<sup>th</sup> layers of the structure is given

$$|\mathbf{R}_N|^2 = \frac{|\mathbf{M}_{2,1}|^2}{|\mathbf{M}_{2,1}|^2 + \left( \left| \frac{\sin K(\omega) d}{\sin NK(\omega) d} \right| \right)^2} \quad (18)$$

where N is the total number of unit cells (i.e. number of pairs of layers).

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