RESEARCH ARTICLE OPEN ACCESS

Minimization of total waiting time of jobs in n×2 specially structured Flow Shop Scheduling with set up time separated from processing time and each associated with probabilities

Dr. Deepak Gupta[1], Bharat Goyal[2]
[1]*Professor & Head Department of MathematicsMaharishiMarkandeshwar University Mullana*
[2]*Research Scholar Department of Mathematics Maharishi Markandeshwar Universit Mullana*

ABSTRACT
One of the most noteworthy issues in the setting up and function of programmed systems is scheduling but the creation of frequently admirable schedules has proven to be extremely complicated. In this paper an attempt is made to study specially structured n x2 flow shop scheduling in which set up time is detached from processing time both coupled by means of probabilities assuming that maximum of the equivalent processing time on first machine is less than or equal to the minimum of equivalent processing time on second machine. The problem conferred here is wider & basically more applicable and has noteworthy use when the goal is to minimize the total waiting time of jobs. The purpose of the study is to get most favorable progression of jobs in order to decrease the total waiting time of the jobs through iterative algorithm. The algorithm is made clear by numerical example.
***Keywords*:** Waiting time of jobs, Set up time, Flow shop Scheduling, Processing time.

I. INTRODUCTION

Scheduling might be defined as the problem of taking decision when to execute a given set of actions, subject to chronological constraints and possessions capacities, in order to optimize some function. A Flow shop problem survives when all the jobs share the same dealing out order on all the machines. Technological constraints insist that the jobs get ahead of between the machines in the same order. Hence there is ordinary sequence of the machines characterized by the technological constraints for each and every job in flow shop. The flow shop contains **n** different machines set in series on which a set of **m** jobs are to be executed. Each of the **m** jobs requires **n** operations and each function is to be performed on a separate machine. The flow of the occupation is unidirectional; thus each job must be processed through each machine in a prearranged order. The general **m** jobs, **n** machine flow shop scheduling is quite terrifying. Consider an arbitrary sequence of jobs on each machine, there are $(\mathbf{m!})^{\mathbf{n}}$ probable schedules which poses computational difficulties. With the aim to trim down the counting of probable schedules it is logical to assume that all machines practice jobs in the alike order. Efforts in the past have been made by researchers to reduce this number of possible schedules as much as achievable without compromising on optimality condition. Today's large-scale markets and immediate interactions mean that clients expect high-class goods and services at what time they require them, anywhere they require them. Organizations, whether public or private, have to make available these products and services as effectively and efficiently as possible.

The principle of optimality in the given flow shop scheduling problem is precised as minimization of waiting time of jobs. The waiting time of a job is defined as the subtraction of the completion time of job on the first machine from the starting time of job on second machine. There are some papers in the literature of scheduling theory which put stress on the waiting time for scheduling the jobs on the machines.

II. LITERATURE REVIEW

The Johnson's algorithm [1] is especially popular among analytical approaches that are used for solving n- jobs, 2- machines sequence problem to minimize the total elapsed time. Ignall and Schrage [2] introduced the permutation flow shop problem with branch and bound algorithms for makespan minimization. Lockett A.G. and Muhlemann A.P. [3], Crowin and Esogbue [4], Maggu & Dass [5] made attempts to extend the study by introducing various parameters. Yoshida & Hitomi [6] solved two stage production scheduling, the set up time being separated from processing time. Solution methods for flow shop scheduling using heuristics developed by Singh T.P. [7], Rajendran and Chaudhuri [10]. Singh T.P., Gupta D. [11] studied the problem related with group job restrictions in a flow shop which involves independent set- up time and transportation time. Singh V. [15] put his efforts to study three machine flow shop scheduling problems with total rental cost. Further Gupta D. [16] studied minimization of Rental Cost in n x2 Flow Shop Scheduling with job block concept and setup Time was separated from Processing Time and each coupled with probabilities. Gupta D. and et. al. [8], [9] studied optimal two and three stage open shop specially structured scheduling in which the processing times are coupled with probabilities with transportation time to minimize the rental cost,. Recently Gupta D. and Goyal B.[17], [18] considered the concept of reducing waiting time of jobs by considering processing times coupled with probabilities.

The problem conferred here has noteworthy use of hypothetical results in process industries or in the conditions when the purpose is to minimize the total waiting time of jobs. The paper discussed here is an extension made by Gupta D. and Goyal B. [17] in the sense that we have taken into consideration the set up time of machine alienated from processing time.

III. PRACTICAL SITUATION

Manufacturing units/industries play a momentous role in the economic progress of a country. Flow shop scheduling occurs in various offices, service stations, banks, airports etc. In our routine working in industrial and manufacturing units diverse jobs are practiced on a variety of machines. In textile industry different types of fabric is produced using different types of yarn. Here, the maximum equivalent time taken in dying of yarn on first machine is always less than or equal to the minimum equivalent time taken in weaving of yarn on the second machine.

NOTATIONS

a_k	: Sequence obtained by applying the algorithm proposed.
P_j	: Time for processing of j^{th} job on machine P.
Q_j	: Time for processing of j^{th} job on machine Q.
P_j'	: Equivalent time for processing of j^{th} job on machine P.
Q_j'	: Equivalent time for processing of j^{th} job on machine Q.
S_j	: Set up time of j^{th} job on machine P.
T_j	: Set up time of j^{th} job on machine Q.
T_{aQ}	: The completion time of job a on machine Q.
W_β	: Waiting time of job β.
W_T	: Sum of waiting time of all the jobs.

IV. PROBLEM FORMULATION

Assume that two machines P and Q are processing n jobs in the order P Q , P_j and Q_j are the respective processing times with probabilities p_j and q_j respectively. S_j and T_j are the respective set up times of the j^{th} job on machines with probabilities s_j and t_j respectively. Our intention is to find an optimal sequence $\{a_k\}$ of jobs minimizing the total waiting time of all jobs. Equivalent processing times of j^{th} job on machine $P\ \&\ Q$ are defined as $P_j' = P_j * p_j - T_j * t_j$, $Q_j' = Q_j * q_j - S_j * s_j$ Satisfying processing times structural relationship Max $P_j' \leq$ Min Q_j'

Table 1: Matrix Form Of The Mathematical Model Of The Problem

Job	Machine P				Machine Q			
J	P_j	p_j	S_j	s_j	Q_j	q_j	T_j	t_j
1.	P_1	p_1	S_1	s_1	Q_1	q_1	T_1	t_1
2.	P_2	p_2	S_2	s_2	Q_2	q_2	T_2	t_2
3.	P_3	p_3	S_3	s_3	Q_3	q_3	T_3	t_3
.	. .	. .	. .	. .	. .	. .	. .	. .
m.	P_m	p_m	S_m	s_m	Q_m	q_m	T_m	t_m

ASSUMPTIONS

In the given flow shop scheduling the following assumptions are made

1) There are *m* number of jobs (I) and two machines (P & Q).
2) The order of sequence of operations in all machines is the same.

3) Jobs are not dependent on each other.
4) Machines break down interval, transportation time is not considered for calculating waiting time.

Lemma1. Assume that two machines P and Q are processing n jobs in the order P Q , P_j and Q_j are the respective processing times with probabilities p_j and q_j respectively. S_j and T_j are the respective set up times of the j^{th} job on machines with probabilities s_j and t_j respectively. Our intention is to find an optimal sequence $\{a_k\}$ of jobs minimizing the total waiting time of all jobs. Equivalent processing times of j^{th} job on machine $P \& Q$ are defined as

$P'_j = P_j * p_j - T_j * t_j$, $Q'_j = Q_j * q_j - S_j * s_j$ Satisfying processing times structural relationship Max $P'_j \le$ Min Q'_j then for the m job sequence S: $\sigma_1, \sigma_2, \sigma_3, \dots \dots \sigma_m$

$T_{\sigma_m Q} = P'_{\sigma_1} + Q'_{\sigma_1} + Q'_{\sigma_2} \dots + Q'_{\sigma_m}$ Where $T_{\sigma_m Q}$ is the completion time of job σ_m on machine Q.

Proof. Applying mathematical Induction hypothesis on n:

Let the statement $a(m): T_{\sigma_m Q} = P'_{\sigma_1} + Q'_{\sigma_1} + Q'_{\sigma_2} \dots + Q'_{\sigma_m}$

$T_{\sigma_1 P} = P'_{\sigma_1}$

$T_{\sigma_1 Q} = P'_{\sigma_1} + Q'_{\sigma_1}$

Hence for m= 1 the statement $a(1)$ is true.

Let for m= k, the statement $a(k)$ be true, i.e.,

$T_{\sigma_k Q} = P'_{\sigma_1} + Q'_{\sigma_1} + Q'_{\sigma_2} \dots + Q'_{\sigma_k}$

Now,

$T_{\sigma_{k+1} Q} = \mathrm{Max}(T_{\sigma_{k+1} P}, T_{\sigma_k Q}) + Q'_{\sigma_{k+1}}$

As Max $P'_j \le$ Min Q'_j

Hence

$T_{\sigma_{k+1} Q} = P'_{\sigma_1} + Q'_{\sigma_1} + Q'_{\sigma_2} \dots + Q'_{\sigma_k} + Q'_{\sigma_{k+1}}$

Hence for $n = k + 1$ the statement $a(k + 1)$ holds true. Since $a(n)$ is true for $m = 1, m = k$, $m = k + 1$, and k being arbitrary. Hence $a(m): T_{\sigma_m Q} = P'_{\sigma_1} + Q'_{\sigma_1} + Q'_{\sigma_2} \dots + Q'_{\sigma_m}$ is true.

Lemma 2. With the same notations as that of Lemma1, for n- job sequence S: $\sigma_1, \sigma_2, \sigma_3, \dots \dots \sigma_k \dots \sigma_m$

$W_{\sigma_1} = 0$

$$W_{\sigma_k} = P'_{\sigma_1} + \sum_{r=1}^{k-1} x_{\sigma_r} - P'_{\sigma_k}$$

Where W_{σ_k} is the waiting time of job σ_k for the sequence $(\sigma_1, \sigma_2, \sigma_3, \dots \dots \dots \sigma_m)$ and

$x_{\sigma_r} = Q'_{\sigma_r} - P'_{\sigma_r}$, $\sigma_r \in (1, 2, 3, \dots., m)$

Proof. $W_{\sigma_1} = 0$

$W_{\sigma_k} = \mathrm{Max}(T_{\sigma_k P}, T_{\sigma_{k-1} Q}) - T_{\sigma_k P}$

$= P'_{\sigma_1} + Q'_{\sigma_1} + Q'_{\sigma_2} \dots + Q'_{\sigma_{k-1}} - P'_{\sigma_1} - P'_{\sigma_2} \dots - P'_{\sigma_k}$

$$= P'_{\sigma_1} + \sum_{r=1}^{k-1} (Q'_{\sigma_r} - P'_{\sigma_r}) - P'_{\sigma_k}$$

$$= P'_{\sigma_1} + \sum_{r=1}^{k-1} x_{\sigma_r} - P'_{\sigma_k}$$

Theorem 1. Assume that two machines P and Q are processing n jobs in the order P Q , P_j and Q_j are the respective processing times with probabilities p_j and q_j respectively. S_j and T_j are the respective set up times of the j^{th} job on machines with probabilities s_j and t_j respectively. Our intention is to find an optimal sequence $\{a_k\}$ of jobs minimizing the total waiting time of all jobs. Equivalent processing times of j^{th} job on machine $P \& Q$ are defined as

$P'_j = P_j * p_j - T_j * t_j$, $Q'_j = Q_j * q_j - S_j * s_j$ Satisfying processing times structural relationship

Max $P'_j \le$ Min Q'_j then for the m job sequence S: $\sigma_1, \sigma_2, \sigma_3, \dots \dots \dots \sigma_m$ the total waiting time W_T (say)

$$W_T = mP'_{\sigma_1} + \sum_{r=1}^{m-1} z_{\sigma_r} - \sum_{i=1}^{m} P'_i$$

$z_{\sigma_r} = (m - r)x_{\sigma_r}$; $\sigma_r \in (1, 2, 3, \dots, m)$

Proof. From Lemma 2 we have

$W_{\sigma_1} = 0$

For m = 2,

$W_{\sigma_2} = P'_{\sigma_1} + x_{\sigma_1} - P'_{\sigma_2}$

For m = 3,

$$W_{\sigma_3} = P'_{\sigma_1} + \sum_{r=1}^{2} x_{\sigma_r} - P'_{\sigma_3}$$

Continuing in this way

For m = n,

$$W_{\sigma_m} = P'_{\sigma_1} + \sum_{r=1}^{m-1} x_{\sigma_r} - P'_{\sigma_m}$$

Hence total waiting time

$$W_T = \sum_{i=1}^{m} W_{\sigma_i}$$

$$W_T = mP'_{\sigma_1} + \sum_{r=1}^{m-1} z_{\sigma_r} - \sum_{i=1}^{m} P'_i$$

Where $z_{\sigma_r} = (m-r)x_{\sigma_r}$

ALGORITHM

Step 1: Equivalent processing times P'_j and Q'_j on machine P & Q respectively be calculated in first step as defined in the lemma 1.

Step 2: Calculate the entries for the following table

Table 2

Job	**Machine P**	**Machine Q**		$\mathbf{z_{jr} = (m-r)x_j}$				
I	$\mathbf{P'_j}$	$\mathbf{Q'_j}$	x_j	r = 1	r = 2	r = 3	...	r = m-1
1.	P'_1	Q'_1	x_1	z_{11}	z_{12}	z_{13}	$\cdots$	$z_{1\,m-1}$
2.	P'_2	Q'_2	x_2	z_{21}	z_{22}	z_{23}	$\cdots$	$z_{2\,m-1}$
3.	P'_3	Q'_3	x_3	z_{31}	z_{32}	z_{33}	$\cdots$	$z_{3\,m-1}$
.	.	.	.	.	.	.	.	.
m.	P'_m	Q'_m	x_m	z_{m1}	z_{m2}	z_{m3}	$\cdots$	$z_{m\,m-1}$

Step 3: Assemble the jobs in increasing order of x_j.

Assuming the sequence found be $(\sigma_1, \sigma_2, \sigma_3, \dots \dots \dots \sigma_m)$

Step 4: Locate min{ P'_j }

For the following two possibilities

$P'_{\sigma_1} = \min\{P'_j\}$ Schedule according to step 3 is the required optimal sequence

$P'_{\sigma_1} \neq \min\{P'_j\}$ move on to step 5

Step 5: Consider the different sequence of jobs $a_1, a_2, a_3, \dots \dots, a_m$. Where a_1 is the sequence obtained in step 3, Sequence $a_k (k = 2,3, \dots \dots, m)$ can be obtained by placing k^{th} job in the sequence a_1 to the first position and rest of the sequence remaining same.

Step 6: Compute the total waiting time W_T for all the sequences $a_1, a_2, a_3, \dots \dots, a_m$ using the following formula:

$$W_T = mP'_b + \sum_{r=1}^{m-1} z_{ar} - \sum_{i=1}^{m} P'_i$$

P'_b = Equivalent processing time of the first job on machine P in sequence S_j

$z_{ar} = (m-r)x_{ar}$; $a = \sigma_1, \sigma_2, \sigma_3, \dots \dots \dots \sigma_m$

The sequence with minimum total waiting time is the required optimal sequence.

V. NUMERICAL ILLUSTRATION

Assume 5 jobs 1, 2, 3, 4, 5 with processing times P_j and Q_j and respective probabilities p_j and q_j and set up times S_j and T_j with respective probabilities s_j and t_j are made to be processed on the machines P & Q.

Table 3: Processing Time Matrix

Job	**Machine P**				**Machine Q**			
J	P_j	p_j	S_j	s_j	Q_j	q_j	T_j	t_j
1.	5	0.1	2	0.1	10	0.1	3	0.1
2.	7	0.2	3	0.5	5	0.4	3	0.3
3.	6	0.3	3	0.2	6	0.2	5	0.3
4.	4	0.2	9	0.1	8	0.2	7	0.1
5.	8	0.2	2	0.1	9	0.1	7	0.2

Our purpose is to attain optimal string, minimizing the total waiting time for the jobs.
Solution
As per step 1- Equivalent processing time P_i' & Q_i' on machine P & Q given in the following table

`**Table 4:** Equivalent Processing Time Matrix

Job	**Machine P**	**Machine Q**
I	$\mathbf{P_j'}$	$\mathbf{Q_j'}$
1.	0.2	0.8
2.	0.5	0.5
3.	0.3	0.6
4.	0.1	0.5
5.	0.2	0.7

Max $P_j' = 0.5 \leq$ Min $Q_j' = 0.5$
As per step 2- Obtaining the values for TABLE 2

Table 5

Job	**Machine P**	**Machine Q**		$\mathbf{z_{jr} = (5 - r)x_j}$			
I	$\mathbf{P_j'}$	$\mathbf{Q_j'}$	$\mathbf{x_j = Q_j' - P_j'}$	$\mathbf{r = 1}$	$\mathbf{r = 2}$	$\mathbf{r = 3}$	$\mathbf{r = 4}$
1.	0.2	0.8	0.6	2.4	1.8	1.2	0.6
2.	0.5	0.5	0	0	0	0	0
3.	0.3	0.6	0.3	1.2	0.9	0.6	0.3
4.	0.1	0.5	0.4	1.6	1.2	0.8	0.4
5.	0.2	0.7	0.5	2.0	1.5	1.0	0.5

As per step 3-.The sequence thus found be 2, 3, 4, 5, 1.
As per step 4- $\text{Min}\{P_j'\} = 0.1 \neq P_1'$
As per step 5- Different sequence of jobs can be considered as:
$a_1: 2, 3, 4, 5, 1$; $a_2: 3, 2, 4, 5, 1$; $a_3: 4, 2, 3, 5, 1$; $a_4: 5, 2, 3, 4, 1$; $a_5: 1, 2, 3, 4, 5$
As per step 6- The total waiting time for the sequences obtained in step 5 can be calculated
Here, $\sum_{i=1}^{5} P_i' = 1.3$
For the sequence $a_1: 2, 3, 4, 5, 1$
Total waiting time $W_T = 3.4$
For the sequence $a_2: 3, 2, 4, 5, 1$
Total waiting time $W_T = 2.7$
For the sequence $a_3: 4, 2, 3, 5, 1$
Total waiting time $W_T = 1.9$
For the sequence $a_4: 5, 2, 3, 4, 1$
Total waiting time $W_T = 2.7$
For the sequence $a_5: 1, 2, 3, 4, 5$
Total waiting time $W_T = 3.1$
Hence schedule $a_3: 4, 2, 3, 5, 1$ is the required schedule with minimum total waiting time.

VI. CONCLUSION

The present study deals with the flow shop scheduling problem with the main idea to reduce the total waiting time of jobs. However it may add to the other costs lime machine idle cost or penalty cost of the jobs, yet the idea of minimizing the waiting time may be an economical aspect from Factory /Industry manager's observation point when he has minimum time contract with a commercial party to complete the jobs. Extension to the present paper can be made by introducing various parameters like transportation time, break down interval etc.

REFERENCES

[1] Johnson [1954], *Optimal two and three stage production schedule with set up times included*. Nay Res Log Quart Vol 1 pp 61-68.

[2] Ignall E. and Schrage L.E., [1965], *Application of branch and bound techniques to some flow shop problems*. Operation Research 13, 400-412.

[3] Lockett A.G. and Muhlemann A.P., [1972], *Technical notes: a scheduling Problem involving sequence dependent changeover times*. Operation Research 20, 895- 902.

[4] Corwin B. D. and Esogbue A. O. [1974], *Two machine flow shop scheduling problems with sequence dependent setup times: A dynamic programming approach*. Naval Research Logistics Quarterly, 21: 515.524. doi: 10.1002/nav.3800210311.

[5] Maggu P.L. and Dass G. [1977], *equivalent jobs for job block in job sequencing*, Operations Research, Vol 14 No. 4, pp 277-281.

[6] Yoshida & Hitomi [1979], *Optimal two stage Production Scheduling with set- up time separated*, AIIE Transactions, Vol. II, pp 261-26.

[7] Singh T.P. [1985], *on n×2 shop problem involving job block. Transportation times and Break-down Machine times*, PAMS Vol.XXI No.1-2.

[8] Gupta D., Bala S. and Singla P.[2012] *Optimal Two Stage Open Shop Specially Structured Scheduling To Minimize the Rental Cost, processing time Associated with Probabilities including transportation time*. IOSR Journal of Mathematics, Vol. 3, No. 3, pp 01-06.

[9] Gupta D., Bala S., Singla P. and Sharma S. [2015] *3- stage Specially Structured Flow Shop Scheduling to Minimize the Rental Cost Including Transportation Time, Job Weightage and Job Block Criteria*. European Journal of Business and Management, Vol. 7, No.4.

[10] Rajendran C. and Chaudhuri D. [1992], *an efficient heuristic approach to the scheduling of jobs in a flow-shop*. European Journal of Operational Research 61, 318. 325.

[11] Singh T.P. & Gupta D. [2004], *Optimal two stage production schedule with group jobs restrictions having set up times separated from processing time associated with probabilities*, Presented in International Conference on Industrial & Applied Mathematics at India International Centre, New Delhi, 4-6 Dec.(2004) and published in the journal reflections des ERA JMS VOL 1 FEB 2006, pp 53-70.

[12] Singh T.P., Gupta D. and Kumar R. [2006], *Optimal two stage production schedule with Group job-restrictions having set up times separated from processing time associated with probabilities*. Reflections des ERA, (JMS) Vol. I pp 53-70.

[13] Singh T.P., Gupta D. & Kumar R. [2006], *Bi-criteria in scheduling under specified rental policy, processing time associated with probabalities including job block concept*, presented at National Conference on information technology at NCCEI, March 18-20.

[14] Narrain L., Gupta D. & Kumar R.[2006], *Minimization rental cost under specified rental policy in two stage flow shop the processing times associated with probabilities including job block criteria*, Reflections des ERA, (JMS) Vol. 2 pp 107-120.

[15] Singh Vijay [2011],*Three machines flow shop scheduling problems with total rental cost*, International referred journal, Jan 2011, Vol-II pp79-80

[16] Gupta D. [2011], *Minimizing rental cost under specified rental policy in two stage flowshop, the processing time associated with probabilities including break down interval and job block criteria*, European Journal of Business and Management (USA), Vol.3 No.2 pp 85-103.

[17] Gupta D. & Goyal B.[2016] ,*Optimal Scheduling For Total Waiting Time Of Jobs In Specially Structured Two Stage Flow Shop Problem Processing Times Associated With Probabilities*, Aryabhatta Journal of Mathematics & Informatics, Jan- June 2016, Vol.8 No. 1 pp 45-52.

[18] Gupta D. & Goyal B.[2016] , *Job block concept in two stage specially structured Flow shop scheduling to minimize the total waiting time of jobs*, International Journal of

Latest Trends in Engineering and Technology, September 2016, Vol.(7) Issue(3), pp. 287-295