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Analysis & Control of Inverted Pendulum System Using PID Controller

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ABSTRACT
This Analysis designs a two-loop proportional–integral–derivative (PID) controller for an inverted cart–pendulum system via pole placement technique, where the (dominant) closed-loop poles to be placed at the desired locations are obtained from an Linear quadratic regulator (LQR) design. It is seen that in addition to yielding better responses (because of additional integral action) than this LQR (equivalent to two-loop PD controller) design, the proposed PID controller is robust enough. The performance and of the PID compensation are verified through simulations as well as experiments.
Keywords: PID, inverted pendulum, pole placement, LQR design, robust

I. INTRODUCTION

The inverted cart–pendulum is an example of under actuated, non-minimum phase and highly unstable system. Therefore a controller design is difficult for such a system. The design becomes more difficult because of the physical constraints on track length, applied voltage and the pendulum angle.

It is well known that proportional–integral–derivative (PID) controllers are widely used in control systems The design of these controllers, however, are generally carried out using some tuning approaches such as trail error method, [1]. For the inverted cart–pendulum system, it is however seen that a design of LQ controller, which is equivalent to a two-loop PD controller here, exhibits poor real-time cart responses containing sustained oscillations. To achieve better response, this paper designs a two-loop PID controller using pole placement technique where the (dominant) closed-loop poles are placed at the same locations as obtained from the above LQR design. It is seen that while the robustness of LQR design is almost retained in the two-loop PID design, the real-time cart response is superior to LQR because of additional integral action. The performance and robustness of the proposed two loop PID design have been verified using simulations. In [2], a two-loop PID controller was designed for an inverted cart–pendulum system based on simultaneous tuning of the controllers using pole–placement technique This controller was, however, tested on SIMULINK environment only. In [3], a PD controller was designed and tested on experimental set-up, but, as expected, the real-time pendulum angle and cart responses exhibit oscillations. In [4], a linear state feedback controller was used to ensure infinite gain margin (GM). In this paper pole placement technique has been used to stabilize the MIP system. The stabilization of MIP system using pole placement method has been discussed in the literature [8-9].

In this paper PID controllers have been used to stabilize the MIP system. The tuning of the PID controllers has been done using pole placement technique. In this technique the dominant closed loop poles are placed at desired locations which are obtained from LQR technique. By placing these poles in the characteristic equation of the MIP system the parameters of the PID controllers are obtained.

II. STRUCTURE AND MATHEMATICAL MODEL OF INVERTED PENDULUM

The x inverted pendulum on a pivot driven by horizontal control force is shown in fig 1(a) the control based on the horizontal displacements x inverted pendulum are the total kinetic energy and potential energy . $K=1/2MX_2+1/2(X_P^2+Z^2)$, Z= mgzp Let l is the distance from the pivot to the maas center of the pendulum M,m are the piot and the pendulum (x,z) respectively(x,z) is the position of the pivot in the xoz coordinate is the speed in the xoz coordinate $(x_{p,}z_p)$ and angle Ө and F_x is the horizontal control force.

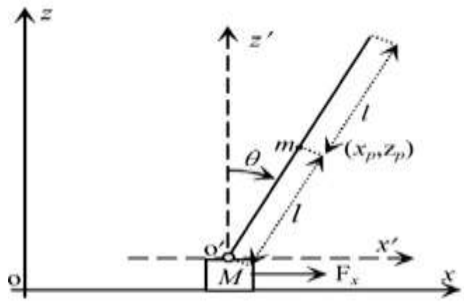

Figure -1

Inverted pendulum system free-body diagrams of the two elements

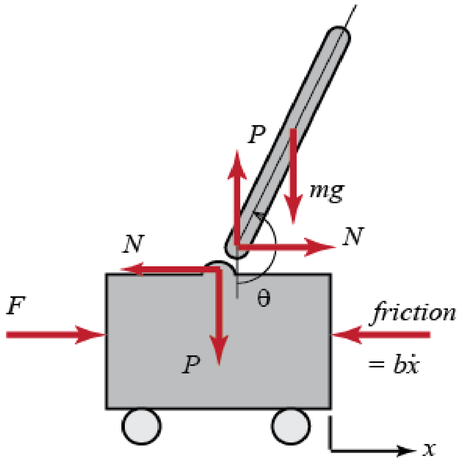

Figure -2

free-body diagram of the cart in the horizontal direction Summing the forces you get the following equation of motion Mx+bx+N=F (1)
forces in the free-body diagram of the pendulum in the horizontal direction, you get the following expression for the reaction force N.

N= mx + mlΘcosΘ-ml $Θ^2$sinΘ (2)

One of the two governing equations for this system is obtained if this equation is substituted into the first equation

(M+m)x +bx+ mlΘcosΘ-ml Θ sinΘ = F (3)

To get the second equation of motion for this system, sum the forces perpendicular to the pendulu

PSINΘ+NCOSΘ-mg SinΘ = ml^2Θ+mxcosΘ (4)

To get rid of the P and N terms in the equation above, sum the moments about the centroid of the pendulum to get the following equation

-PlSINΘ -NlcosΘ =IΘ (5)

Combining these last two expressions, the second governing equation is obtained

(I+ml^2) Θ+mglsinΘ = - mlxcosΘ (6)

Let ϕ represent the deviation of the pendulum's position from equilibrium, that is, $\theta = \pi + \phi$. Again presuming a small deviation (ϕ) from equilibrium, we can use the following small angle non linear function

cosΘ = cos(Π+φ) = - 1 (7)

SinΘ =Sin (Π+φ) = - φ (8)

$Θ^2$ = $φ^2$ = 0 (9)

After substiting the above approximations into our nonlinear governing equations, the two linearized equations of

Motion are obtained. Note u has been substituted for the input$F$

(I+ml^2) φ- mglsin φ = - mlx (10)

(M+m)x +bx-mlφ = u (11)

underline any of the headings, or add dashes, colons, etc.

III. CONTROLLER DESIGN

Stabilizing the IP a two loop control scheme has been used for stabilizing the IP as shown in figure 2. PID1 has been used for controlling the position x while PID2 for controlling angleθ PID1 has been used for controlling the position x while PID2 for controlling angleθ.

The basic equation of a PID controller is given as

$$u(t) = K_p e(t) + K_i \int_0^t e(t)\,dt + K_d \frac{de(t)}{dt} \qquad (12)$$

Where e (t) is the error between the reference and the feedback of the system. In order to control the system two PID controllers are required one for controlling the position of the robot and the other for controlling the pendulum angle. The two loop PID controller is shown in figure 2. As shown in figure PID1 is used for position control while PID2 is used for stabilizing angle θ of the MIP. P1=X(s)/U(s) and P2= $\theta(s)/U(s)$ represents the two transfer functions. The tuning of the PID controllers has been done using pole placement technique. In this technique dominant closed loop poles are placed at desired locations which are obtained from LQR technique.

The characteristic equation for the control

$$1 - P1C1 + P2C2 = 0 \qquad (13)$$

Where $C1$ and $C2$ are given

$$C1 = \frac{K_d^1 s^2 + K_p^1 s + K_i^1}{s} \qquad (14)$$

$$C2 = \frac{K_d^2 s^2 + K_p^2 s + K_i^2}{s} \qquad (15)$$

Let the desired characteristic equation be

$$s^5 + p1s^4 + p2s^3 + p3s^2 + p4s + p5 \qquad (16)$$

The dominant poles of the two loop PID controller are obtained using LQR design.

The performance index is given as

$$j = \frac{1}{2}\int_0^\infty \{X^T QX + U^T RU\}\,dt \qquad (17)$$

Here Q is the state weighted matrix and R is the control weighted matrix the performance index j is minimized using Riccati equation given by

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \qquad (18)$$

State feedback gain vector K≜[K1,K2,K3,K4] is obtained by

$$K = -R^{-1}B^{T}P \qquad (19)$$

For simplicity the state weighted matrix is chosen as Q=diag {q1,q2,q3,q4} and the control weighted matrix is chosen as a scalar vector R=r. In this paper Q vector is q1=500q, q2=q3=20q, q4=q and r= 10^n. By trial and error It is found that for optimal results q=100 and n=4.

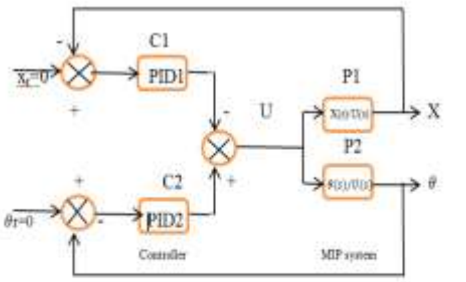

Figure 3 two loop PID controller for IP system

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Figure captions appear below the figure, are flush left, and are in lower case letters. When referring to a figure in the body of the text, the abbreviation "Fig." is used. Figures should be numbered in the order they appear in the text.

Table captions appear centered above the table in upper and lower case letters. When referring to a table in the text, no abbreviation is used and "Table" is capitalized.

IV. SIMULATION RESULTS

The state space obtained after putting these values is

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ (\dot{\phi}) \\ (\ddot{\phi}) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -0.1818 & 2.673 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -0.4545 & 31.18 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \phi \\ (\dot{\phi}) \end{bmatrix} + \begin{bmatrix} 0 \\ 1.818 \\ 0 \\ 4.545 \end{bmatrix} u$$

(16)

The closed loop poles are obtained by the Eigen values of A-BK. The four poles obtained are -5.7391,-5.451,-1.3209±1.209. The fifth pole is taken as six times the real part of the most dominant pole -7.92

The C matrix has 2 rows because both the cart's position and the pendulum's position are part of the output. Specifically, the cart's position is the first element of the output y and the pendulum's deviation from its equilibrium position is the second element of y

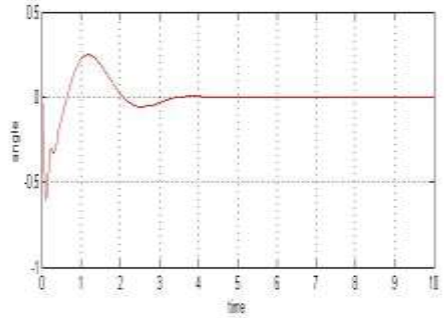

Figure 4 Angle vs. time curve

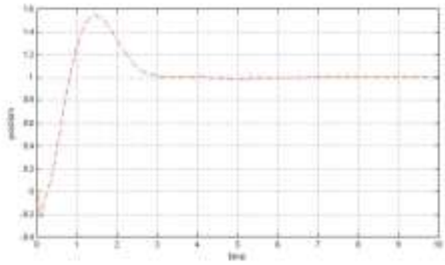

Figure 5 positions vs. time curve

Controller	Kp	Ki	Kd
PID1	52.35	102.93	10
PID2	18.27	13.91	3.224

Table 1 PID Parameter Using Pole Placement

V. CONCLUSION

A two loop PID control scheme has been for stabilization of IP system. The tuning of the PID controllers has been done using pole placement. Simulation results show effective stabilization of IP system with low rise time t_r and overshoot in both position and angle curves.

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