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Dynamic Evolving Neuro-Fuzzy Inference System for Mortality Prediction

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ABSTRACT
In this paper we propose a dynamic evolving neuro-fuzzy inference system (DENFIS) to forecast mortality. DENFIS is an adaptive intelligent system suitable for dynamic time series prediction. An Evolving Cluster Method (ECM) drives the learning process. The typical fuzzy rules of the neuro- fuzzy systems are updated during the learning process and adjusted according to the features of the data. This makes possible to capture the changes in the mortality evolution at the basis of the so called longevity risk.
Keywords – adaptive intelligent system, DENFIS, ECM, longevity risk, mortality prediction.

I. INTRODUCTION

In the last century, global population have experienced improvements in standards of living and human mortality has declined globally. The decreasing trends in mortality represent risk for insurers, which price their products on the basis of the historical mortality tables. Also governments refer to mortality prediction when define health and pension policies. From the point of view of the life insurance business risk profile or pension systems stability, different risk sources have to be evaluated. In particular, the demographic risk can be split into two components: the insurance risk and the longevity risk. The former arises from accidental deviations of the number of the deaths from its expected values, and it is a pooling risk, i.e. it can be mitigated by increasing the number of policies. The longevity risk derives from improvements in mortality trend with systematic deviations of the number of the deaths from its expected values. In order to capture this trend, accurate mortality forecasting techniques have to be used. Recently many approaches have been proposed for forecasting mortality using stochastic model, such as the Lee-Carter (LC) model [1], a milestone in the literature. The main statistical tools of the LC model are the least square estimation through the Singular Value Decomposition of the matrix of the log age specific mortality rate and the Box and Jenkins modelling and forecasting for time series. The LC is fitted to historic data and used to forecast long term mortality. However, strong structural changes have occurred in mortality patterns. Many extensions have been proposed to overcome the limits of the model due to extrapolation based on past data. Recently, a different approach has been experimented. Neural network (NN) and fuzzy inference system (FIS) have been introduced in the context of mortality data by Atsalaki et. al [2]. They implement an Adaptive Neuro-Fuzzy Inference System (ANFIS) model based on a first order Sugeno-type FIS. They predict the yearly mortality in a one step ahead prediction scheme and use the method of trial and error to select the type of membership function that describe better the model. The least-squares method and the backpropagation gradient descent method are used for training the parameters of the Fuzzy Inference System (FIS). They show that the ANFIS model produces better results than the AR and ARIMA models for mortality projections. D'Amato et al. [3] produce a comparative analysis between classical stochastic models and ANFIS implementing them on Italian mortality dataset.

In the light of the complexity of mortality data, in this paper we propose to implement a Dynamic Evolving Neuro-Fuzzy Inference System (DENFIS) for longevity predictions. DENFIS is introduced by Kasabov et al [4] for adaptive learning of dynamic time series. It is an adaptive intelligent system because, depending on the position of the input vector in the input space, a fuzzy inference system is adjusted dynamically based on fuzzy rules that change during the learning phase. The Evolving Clustering Method (ECM) is used to subdivide the input set and determine the position of each data in the input set. [4] show that DENFIS effectively describes complex data and outperforms some existing methods. For this reason, we analyse the performance of DENFIS for mortality projections and compare results with ANFIS. The paper is organized as follows: in Section 2 we present neuro adaptive learning procedure; Section 3 is devoted to describe DENFIS; in Section 4 we show a comparative application to Italian mortality dataset; final remarks are offered in Section 5.

II. NEURO ADAPTIVE LEARNING

The neuro-adaptive learning method works similarly to that of the NN. Neuro-adaptive learning techniques provide a method for the fuzzy modelling procedure to learn information about a data set. Jang [5] introduce the ANFIS; the main difference between the FIS and ANFIS is that in the former case the rule are predetermined by the user's interpretation while in the latter case the procedure learn information from the data. In this way, Fuzzy Logic computes the membership function parameters that best allow the associated fuzzy inference system to track the given input/output data. Using a given input/output data set, ANFIS constructs a system whose membership function parameters are tuned using either a backpropagation algorithm alone or in combination with a least squares (LS) type of method. This adjustment allows your fuzzy systems to learn from the data they are modelling. The ANFIS can be implemented using a first order Takagi-Sugeno (TS) type FIS [6]. The TS model is described in Figure 1.

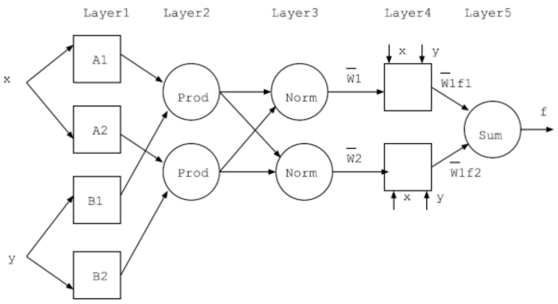

Fig1: Takagi- Sugeno Architecture

Let us assume that the FIS has two input *x* and *y* and one output z. A first order TS fuzzy model has the following rules:

Rule 1: if x is A_1 and y is B_1 then $f_1=p_1x+q_1y+r_1$

Rule 2: if x is A_2 and y is B_2 then $f_2=p_2x+q_2y+r_2$

The procedure follows the steps:

Let $O_{i,l}$ be the output of the node *i* in the layer *l*

1. Layer 1: Every node in this layer is an adaptive one with a node function

$$O_{1,i} = \mu_{A_i}(x) \text{ for } i = 1, 2, \text{ or}$$
$$O_{1,i} = \mu_{B_{i-2}}(x) \text{ for } i = 3, 4 \quad (1)$$

With typical membership function

$$\mu_A(x) = \frac{1}{1 + |\frac{x-c_i}{a_i}|^{2b_i}} \quad (2)$$

$a_i,b_i,c_i,$ are the premise parameters.

2. Layer 2: The output of each node is the product of all the incoming signals:

$$O_{2,i} = w_i = \mu_{A_i}(x) \cdot \mu_{B_i}(y), \quad i = 1, 2 \quad (3)$$

3. Layer 3: the outputs of this layer are the normalization of the incoming signals:

$$O_{3,i} = \overline{w}_i = \frac{w_i}{w_1+w_2}, \quad i = 1, 2 \quad (4)$$

4. Layer 4: each node in this layer is an adaptive node with a node function

$$O_{4,1} = \overline{w}_i f_i = \overline{w}_i(p_x + q_i y + r_i) \quad (5)$$

$p_i,q_i,r_i,$ are the consequent parameters.

5. Layer 5: the i^{th} output of this layer is computed as the summation of the all incoming signals $\sum_i \overline{w}_i f_i$

In the hybrid learning algorithm the consequent parameters are identified by the LS estimation while the premise parameters are updated by gradient descent.

III. DYNAMIC EVOLVING LEARNING

The DENFIS uses TS model where the fuzzy rule are created and updated through the LS estimation. An ECM is introduced to create a partition of the input space. In the clustering process a parameter Dthr is set; it is a threshold value that will affect the number of clusters created. A first sample of inputs from the training data is extracted; it represents the first cluster, whose radius is set equal to zero. Another sample is extracted: if the distance between its centre and that of the existing cluster is less than the value of parameter Dthr then the vector extracted is incorporated in the first cluster and the centre is updated and the radius increased; otherwise another cluster is created. A cluster will not be modified anymore when its radius becomes equal to Dthr. We refer to Song et al. [5] for a detailed description of the ECM algorithm.

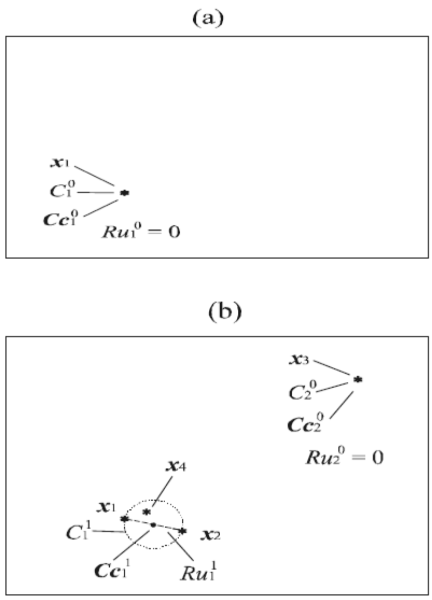

Fig 2: The ECM process

Once the clusters are created, the fuzzy rules of DENFIS are created and updated within the partitioned input space using a TS model. The steps of the DENFIS are the following:

1. Define the training data set
2. Apply the ECM to the training data set
3. For each cluster create the fuzzy rule through the triangular membership function

$$\mu(x) = mf(x, a, b, c) = \max(\min((x-a)/(b-a),(c-x)/c-b)),0) \quad (6)$$

where x is the input vector, b is the cluster centre, $a=b-d\times Dthr$, $c= b+d\times Dthr$, d is a parameter of the width of the triangular function.

4. The consequent parameters of the TS procedure are calculated through a weighted LSE. In particular, the weighs are represented by $1-d_j$ where d_j is the distance between the j-th sample and the corresponding cluster centre.
5. The fuzzy rules and the parameters are updated when a new cluster is created or the existing clusters are modified. When the ECM stops, the output of the system is generated according to the TS procedure.

IV. MORTALITY FORECAST WITH THE ANFIS AND DENFIS MODELS

In this work we propose to apply DENFIS to mortality forecasts. Just few contributions apply the NN and FIS in the context of mortality data: [2] and [3] implement the ANFIS . [2] present and ANFIS based on a first order Sugeno-type FIS and predict the yearly mortality in a one step ahead prediction scheme. They chose as the inputs of the model the mortality one year and two years before to predict the mortality of the next year. The choice of the number of input is arbitrary and the lag period is not tested. However, Wei et al. [7] propose an integrated autoregressive AR-ANFIS model to test the lag period of the variable of interest and define the number of inputs of the ANFIS to produce accurate forecasts of the output. Their experimental shows that the integrated model outperforms the ANFIS model.

In the light of this contributions, we introduce an integrated AR-DENFIS model. In order to define the number of inputs of the FIS in a mortality dataset, we firstly apply an AR scheme and then we compare the results of mortality forecasts obtained by the ANFIS and DENFIS. The data used are taken from the Human Mortality Database [8]. We work on the mortality rates m_t for the Italian males aged 50, collected from t=1940 up to t=2012. The data, considered by single calendar year, are split into training dataset from 1940 up to 1993 and test dataset from 1994 up to 2012. The AR is fitted to the whole time serie and the order equal to 2 is chosen minimizing the Akaike Information Criterion; consequently, in our FIS introduce two input variable x_1 and x_2 (mortality one and two years before) and one output y (mortality one step ahead).

Training dataset

x_1	x_2	y
m_{1940}	m_{1941}	m_{1942}
m_{1941}	m_{1942}	m_{1943}
… m_{1991}	… m_{1992}	… m_{1993}

(a)

Test dataset

x_1	x_2	y
m_{1992}	m_{1993}	m_{1994}
m_{1993}	m_{1994}	m_{1995}
… m_{2010}	… m_{2011}	… m_{2012}

(b)

Table 1: The training and test datasets

Firstly, we implement an ANFIS on the training dataset, using a standard first order TS: the type of membership functions is Gaussian, the number of layer is 5, the backpropagation descendent method is used for training the parameters of the membership functions, the linear equation of the consequent part are determined by LSE. The maximum number of iteration is set equal to 10, the step size of the gradient descent is set equal to 0.01. Once the ANFIS is created, the mortality rate is projected from 1994 up to 2012 using the fuzzy rules generated and the results are compared with the realized mortality with a backtesting procedure. Successively, we repeat the same steps and implement a DENFIS on the training dataset, setting the value of Dthr equal to 0.1, the maximum number of iteration equal to 10, the parameter d equal to 2, the step size of the gradient descent equal to 0.01. Once the DENFIS is created, the mortality rate is projected on the testing period and the results are compared with the realized mortality. Finally, the MSE of both models are calculated. The results are shown in Tables 2 and 3.

t	Realized	ANFIS	DENFIS
1994	0.00461	0.006748497	0.004458675
1995	0.00413	0.006911526	0.004593811
1996	0.00409	0.007093047	0.004354364
1997	0.00388	0.006623053	0.004143083
1998	0.00390	0.006627108	0.004003856
1999	0.00371	0.006408018	0.003933489
2000	0.00359	0.006473568	0.003829500
2001	0.00359	0.006307787	0.003684567
2002	0.00316	0.006182709	0.003637632
2003	0.00334	0.006315005	0.003384584
2004	0.00312	0.005812602	0.002780000
2005	0.00305	0.006064491	0.002780000
2006	0.00297	0.005859077	0.002780000
2007	0.00304	0.005814189	0.002780000

2008	0.00294	0.005709545	0.002780000
2009	0.00292	0.005814263	0.002780000
2010	0.00278	0.005719193	0.002780000
2011	0.00288	0.005748325	0.002780000
2012	0.00286	0.005572785	0.002780000

TABLE 2: THE MORTALITY RATES REALIZED VS PROJECTED THROUGH ANFIS AND DENFIS

	ANFIS	**DENFIS**
MSE	7.890026e-06	5.726724e-08

TABLE 3: RMSE IN THE ANFIS AND DENFIS

The backtesting procedure highlights the improvements in mortality forecasts moving from ANFIS to DENFIS: the mean square error decreases and the projected mortality trend appears more similar to the realized trend. In particular, the DENFIS catches the improvements in mortality realized in the last years better than the ANFIS. This feature makes it attractive to handle with the longevity risk.

V. FINAL REMARKS

Neural network and fuzzy inference have been introduced recently in the context of mortality and just few contributions have been proposed in the literature, limited to the application of the ANFIS. In this paper we have introduced an integrated AR-DENFIS procedure to forecasts mortality. The results show an improvement in mortality projections: the dynamic clusterization and learning algorithm at the basis of the DENFIS allow to capture better the complexity of the mortality trend. This is due to the fact that the new informations about the mortality evolution occurred in the recent years permits to update the rules of the FIS as they come in the system. This feature makes the application of the DENFIS attractive in the mortality projections for insurers and governments that have to handle with accurate measurement of the longevity risk.

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