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A Study on Project Planning Using the Deterministic and Probabilistic Models by Network Scheduling Techniques

Rama.S[1], Sathya A[2], Shasikala A[3], and Cilfa Irene[4]

ABSTRACT:
Project planning is the important task in many areas like construction, resource allocation and many. A sequence of activities has to be performed to complete one task. Each activity has its unique processing time and all together to identify the critical activities which affect the completion of the project. In this paper the probabilistic and deterministic models to determine the project completion time and also the critical activities are considered. A case study on building construction project has been performed to demonstrate the application of the above said models. The two project scheduling namely PERT and CPM are used to determine numerically the different types of floating times of each activity and hence determined the critical path which plays an important role in the project completion time. Also a linear programing model has been developed to reduce the project completion time which optimize the resource allocation. To apply these techniques numerically the primary data from a housing project company in a metropolitan city has been taken, the network diagram of the activities involved in the building construction project has been drawn and the results are tabulated.

Keywords: Project scheduling, Probabilistic model, Deterministic model, CPM, PERT, Linear programming.

I. INTRODUCTION

A project consists of various activities which are mutually relative. There is a certain order to implement each activity before completing it. Some of the activities are independent to other activities and can be started right away while some other activities are dependent on completion of preceding activity or done simultaneously. Planning and scheduling are important aspects in project management. Network scheduling is an effective technique widely used in planning and scheduling of projects especially in construction projects. Nowadays housing construction project are common. Building a house within reasonable budget and in short time can be difficult without any planning. Good scheduling can effectively help in construction project management. Network scheduling will help construction companies (engineers) to finish their task in time and within reasonable budget. In network scheduling, PERT (Project Evaluation and Review Technique) and CPM (Critical Path Method) are valuable techniques used in planning, scheduling and controlling of construction projects. These techniques are used to calculate the earliest start, earliest finish, latest start, latest finish, and different types of float for each activity, hence thereafter to calculate the critical activities and critical paths. These techniques are used to provide analytical means for scheduling the activities by defining the project activities, their precedence relation-ships and their specific time requirements. The main advantage of Critical Path Analysis is to find the minimum duration of time required to complete the project (Adebowale et al. 2011). With the help of PERT and CPM it is possible to follow the actual progress of the project and evaluate the proposed alternatives with respect to cost and time. PERT method allows the calculation of the average time needed for completion of a project, identification of critical activities and estimations of probabilities of achieving the planned deadlines. (Gurau et al. 2012).

Dhirendra Rao (1978) explained the application of project scheduling in two main services of knowledge information center namely abstracting services and referencing services and concluded that balancing and completion of project on time would be optimized using these techniques. Gerald (1986) developed a mathematical model which optimizes the monitoring behavior of transcutaneous pCo2 which analyze the changes happening in the premature infants having respiratory diseases. Koteswara Rao et. al. (2008) developed a prototype tool to select an optimal policy obtained from fuzzy PERT network whereas the main constraints are considered viz., cost, manpower and unlisted points to crash the time.Fahimifard et al. (2009) applied project scheduling techniques in Agricultural Research Center of University of Zabol and reduced the cost and time estimation.Adebowaleet al. (2011) has estimated the cost and time duration to complete the project of a civil engineering company-ALMEGA, Nig. Ltd. Adegoke (2011) used time scheduling technique to solve parking space issue faced by various organization. Elambrouk (2011) used project scheduling technique to identify the critical activities and Linear Programming technique to crash the activities so as to optimize the cost and project completion time. Rotarescu (2011) applied the project scheduling management technique in human resource organization and produced a mathematical model to optimize the performance of the workers. Gurau et.al (2012) explained the uses of PERT and CPM to expand the furniture manufacturing company's favorable circumstances for sale on the internet and in examination of critical path. This paper concludes PERT/CPM of Win QSB helps us to choose the appropriate economic approach and a new view of facilitating decision for an enterprise. Rautela et al.(2012) applied project scheduling technique in large scale industries like shoe manufacturing industry and completion time has been calculated without delaying any activities.

Elmabrouk (2012) used linear programming and CPM to develop the model which helps a project manager to determine the crash cost and crash time in construction. Paramveer Singh et al. (2013) used the project scheduling techniques in designing and replicating of the formation of human resources and proved that the project can be complete before the normal scheduled time which also would increase confidence and fulfillment among the workers. Rashmi Agarwal et al. (2013) explained planning and control techniques of operations research. Vikash Agarwal et al. (2013) had made a comparative study on time-cost trade off problems (TCTP) using CPM and PERT. Peng Wang (2013) derived Ant colony optimization algorithm (ACO) and Genetic Algorithm (GA) using PERT network diagram. Tamrakar (2013) implemented the critical path method in a project based company for analyzing the completion of an arbitrary project. Chatwal (2014) defined a simulated project scheduling technique for wide range of simplex problem which have large area of application. Aditi et al. (2014) modeled the construction project management by optimizing the time and cost of the project using critical path method. Shailla (2014) adapted a comparative study in hardware and software product research and development using CPM. Rajguru et al. (2016) proved mathematically that cost and time are the main aspects to be considered in the planning of every project by adopting the project scheduling techniques.

The main objective of this paper to minimize the schedule time and the cost for completion of the project with adequate resources. In this paper, the project scheduling techniques CPM and PERT are used efficiently to optimize the cost and the time required to construct a house in a metropolitan city. CPM is used to produce a graphical representation of the project and hence find the critical activities which have to be focused more on completion of project in time. These critical activities form the critical path. PERT is used to calculate the project completion time, by considering the probabilistic model of the construction project. To reduce the duration of any project, a method called project crashing is used. It is done by reducing the duration of each critical activity.

II. CONSTRUCTION OF NETWORK DIAGRAM

The sequences of activities involved in the projects are listed out to draw the network diagram. To draw the network diagram of the project the precedence relationship of the activities are to be determined. Network diagram is the graphical representation of activities that have to be attempted and completed to execute the project. Arrows and circles are the two symbols used to draw the network. Arrows are used to represent the activities. Each activity is preceded and succeeded by an event, represented by numbered circles and are referred as tail event and head event respectively. Preceding activity is an activity which must be finished before starting the next activity. Succeeding activity is an activity which must begin only after the completion of preceding activity. Any two events are joined only by one activity. If any two activities have same tail event or head event then a dummy activity would be introduced by dotted arrow line. Clearly it is an imaginary activity and is not a part of the project activities. In the network diagram the events are numbered from left to right by using Fulkerson's rule.

For numerical study a house construction project in a metropolitan city has been considered and the corresponding data produced in Table 1

Table.1 Data for construction of a house in metropolitan city

Activity code	Name of the Activity	Normal cost	Immediate predecessor	Estimated duration
A	Site clearing	3000	-	1
B	Excavation	10000	A	2
C	Pcc-Plain cement concrete for bed1	5740	B	1
D	Column footing and	102600	C	5

	column raising up to plinth level			
E	Foundation	60000	D	3
F	Basement	39900	E	2
G	Plinth Beam-RCC	65900	F	3
H	Column raising up to lintel level	45000	G	6
I	Super structure (wall)	90450	H	15
J	Electrical pipes and box fitting	150000	I	3
K	Roof-RCC	131600	J	7
L	Lintel and sunshade	20000	K	4
M	Door and window frame fixing	50000	L	3
N	Plumbing and sanitary pipes	50000	M	3
O	Plastering	108200	N	5
P	Hand rails fixing	80000	N,O	6
Q	Flooring	66000	I,J,P	10
R	Dadoing	28000	Q	3
S	Door and window shutter fixing	88778	N,M	3
T	Painting	95534	S	10
U	Interior fixtures	85000	Q	10
V	Electrification with all fitting	50000	N	5
W	Cladding	30000	O	5
X	Landscaping	20000	U	5

The precedence relationship for the activity is
A < B; B < C; C< D; D < E; E < F; F < G; G < H; H < I; I < J; J < K; K < L; L< M; M < N; N < O; O < P; I, J, P < Q; Q < R; N, M <S; S < T; Q<U; N<V;O< W; U < X (1)

Using the precedence relationship given in equation (1) the network diagram of the project has been prepared and given in Figure 1.

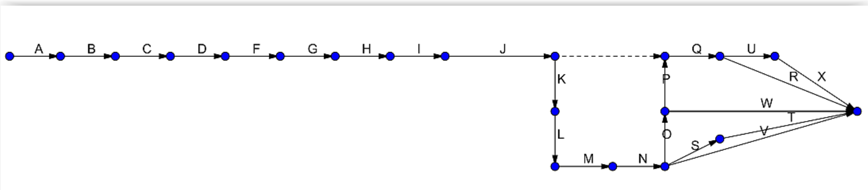

Figure 1. The network diagram of the construction project in a metropolitan city.

III. ESTIMATION OF ACTIVITY TIMES

3.1 Deterministic model:

CPM is a deterministic model whose results have some certainty. By using this deterministic model the different time estimates and floats for each activity have been determined by using to computation techniques namely forward pass computation and backward pass computation.

Using forward pass computation

$ES(1)=EC(1)=0$

$ES(j)= \underset{i\in P(j), i*}{Max} EC(i)$

$EC(j)=ES(j) + t_{i*j}$, for i=2:n (2)

Using backward pass computation

$LC(n)=EC(n)$

$LC(i)= \underset{j\in S(i), j*}{Min} LC(i)$

$LS(i)=LC(i)-t_{ij*}$, for i=n-1:-1:1 (3)

The total float is

$T F = LS - ES = LC - EC$ (4)

The total time completion of the project

$T_p=LC(n)$ (5)

Using the data given in table.1 the different time estimates and floats have been calculated and are tabulated in table 2. The symbols used are explained in Appendix A.

Table 2. Time estimates and total float using CPM method

Activity code	Time	Earliest start	Earliest Finish	Latest Start	Latest Finish	Total Float
A	1	0	1	0	1	0
B	2	1	3	1	3	0
C	1	3	4	3	4	0
D	5	4	9	4	9	0
E	3	9	12	9	12	0
F	2	12	14	12	14	0
G	3	14	17	14	17	0
H	6	17	23	17	23	0
I	15	23	38	23	38	0
J	3	38	41	38	41	0
K	7	41	48	41	48	0
L	4	48	52	48	52	0
M	3	52	55	52	55	0
N	3	55	58	55	58	0
O	5	58	63	58	63	0
P	6	63	69	63	69	0
Q	10	69	79	69	79	0
R	3	79	82	91	94	12
S	3	58	61	81	84	23
T	10	61	71	84	94	23
U	10	79	89	79	89	0
V	5	58	63	89	94	31
W	5	63	68	89	94	26
X	5	89	94	89	94	0

3.2 Probabilistic model:

The three time duration of each activity namely optimistic, most likely and pessimistic time are considered. The expected time and variance for each activity are given

$$t_e = (t_0+4t_m+t_p)/6 \qquad (6)$$

$$\text{Variance}=((t_p-t_0)/6)^2 \qquad (7)$$

Sum of variance of all critical activities will give variance of total time of the project. Table 3 gives the probabilistic data for the construction project and the estimated variance and average time.

Table.3 Estimated time for probabilistic model by PERT

Activity code	Immediate predecessor	t_0	t_m	t_p	t_e	Variance
A	-	1	1	1	1	0
B	A	1	2	3	2	1/9
C	B	1	0.5	3	1	1/9
D	C	3	5	7	5	4/9
E	D	2	3	4	3	1/9
F	E	1	2	3	2	1/9
G	F	2	3	4	3	1/9
H	G	1	6	11	6	25/9
I	H	6	16	20	15	49/9
J	I	1	3	5	3	4/9
K	J	4	7	10	7	1
L	K	2	4	6	4	4/9
M	L	1	3	5	3	4/9
N	M	2	3	4	3	1/9
O	N	3	5	7	5	4/9
P	N,O	4	5.5	10	6	1

Q	I,J,P	5	10	15	10	25/9
R	Q	1	3	5	3	4/9
S	N,M	1	3	5	3	4/9
T	S	8	10	12	3	4/9
U	Q	7	10	13	10	1
V	N	2	5	8	5	1
W	O	3	5	7	5	4/9
X	U	4	5	6	5	1/9

IV. DETERMINATION OF CRITICAL ACTIVITIES AND CRITICAL PATH

Any activity whose total float is zero has been considered as critical activity. The path connecting the critical activities forms the critical path. These critical activities delay would affect the project on time completion. The activities which are having positive value as total float would not affect the project complete if the activities are delayed. For the construction project from Table 2 the critical activities and critical path are determined

Critical Activities: A B C D E F G H I J K L M N O P Q U and X

Critical Path:-1-2 2-3 3-4 4-5 5-6 6-7 7-8 8-9 9-10 10-11 11-12 12-13 13-1414-15 15-16 16-18 18-19 19-20 20-21.

The total estimation time of the project has been calculated as 94 days. The probability to delay the project for 4 more days has been estimated as 83%.

V. TIME-COST TRADE-OFF

Even though CPM/PERT minimizes the project duration and the cost, sometimes the project managers are compelled to complete the project before the deadline. This has been done by increasing the labor and material cost. Crashing is a method used to decrease the project duration by reducing the duration of critical activities from it's the normal duration. Cost Slope for each activity has been calculated as

Cost slope = (Crash cost – Normal Cost) /(Normal time – Crash time) (8)

Crash time is the minimum duration in which the particular activity should be finished. Table 4 gives the crash slope for each activity considered in the project.

Table 4 Crash slope for each activity

Activity code	Normal Time	Normal Cost	Crash Time	Crash cost	Δc	Δt	r=Δc/Δt
A	1	3000	½	4050	1050	½	2100
B	2	10000	1	15000	5000	1	5000
C	1	5740	½	8610	2870	½	5740
D	5	102600	3	138510	35910	2	17955
E	3	60000	2	72000	12000	1	12000
F	2	39900	1	53865	13965	1	13965
G	3	65900	2	82375	16475	1	16475
H	6	45000	5	51750	6750	1	6750
I	15	90450	13	113060	22610	2	11305
J	3	150000	2	202500	52500	1	52500
K	7	131600	5	177660	46060	2	23030
L	4	20000	2	31000	11000	2	5500
M	3	50000	2	62500	12500	1	12500
N	3	50000	2	65000	15000	1	15000
O	5	108200	3	146070	37870	2	18935
P	6	80000	5	100000	20000	1	20000
Q	10	66000	8	89100	23100	2	1550
R	3	28000	2	35000	7000	1	7000
S	3	88778	2	119848	31070	1	31070
T	10	95534	9	109864	14330	1	14330
U	10	85000	8	106250	21250	2	10625
V	5	50000	3	70000	20000	2	10000
W	5	30000	4	40500	10500	1	10500
X	5	20000	3	25000	5000	2	2500

To minimize this crash cost a linear programming model has been formulated. Let Z as the overall cost for crashing the activities and Xi, (i=A, B, C…X) are the decision variables, denote the activity duration. Yi-start time of i^{th} activity (i=B,C,D,E,F,G,I,J,K,L,M,N,O,P,Q,R,S,T,U,V,W, X). The problem has been formulated as

Objective function:

$$\text{Min } z= 2100 X_A + 5000 X_B + 5740 X_C + 17955 X_D + 12000 X_E + 13965 X_F + 16475 X_G + 6750 X_H + 11305 X_I + 52500 X_J + 23030 X_K + 5500 X_L + 12500 X_M + 15000 X_N + 18935 X_O + 20000 X_P + 11550 X_Q + 7000 X_R + 31070 X_S + 14330 X_T + 10625 X_U + 10000 X_V + 10500 X_W + 2000 X_X \quad (9)$$

Slope time constraints:

$$X_A\leq 0.5 ,\ X_B\leq 1 ,\ X_C\leq 0.5 ,\ X_D\leq 2 ,\ X_E\leq 1 ,\ X_F\leq 1 ,\ X_G\leq 1 ,\ X_H\leq 1 ,\ X_I\leq 2 ,\ X_J\leq 1 ,\ X_K\leq 2 ,\ X_L\leq 2 ,\ X_M\leq 1 ,\ X_N\leq 1 ,\ X_O\leq 2 ,\ X_P\leq 1 ,\ X_Q\leq 2 ,\ X_R\leq 1 ,\ X_S\leq 1 ,X_T\leq 1 ,\ X_U\leq 2 ,\ X_V\leq 2 ,\ X_W\leq 1 ,\ X_X\leq 2 \quad (10)$$

Start time constraints:

$Y_B+X_A\geq 1, Y_C-Y_B+X_B\geq 2, Y_D-Y_C+X_C\geq 1,$
$Y_E-Y_D+X_D\geq 5, Y_F-Y_E+X_E\geq 3, Y_G-Y_F+X_F\geq 2,$
$Y_H-Y_G+X_G\geq 6, Y_I-Y_H+X_H\geq 6, Y_J-Y_I+X_I\geq 15,$
$Y_K-Y_J+X_J\geq 3, Y_L-Y_K+X_K\geq 7, Y_M-Y_L+X_L\geq 4,$
$Y_N-Y_M+X_M\geq 3, Y_O-Y_N+X_N\geq 3, Y_P-Y_O+X_O\geq 5,$
$Y_P-Y_N+X_N\geq 3 ,Y_Q-Y_I+X_I\geq 15, Y_Q-Y_J+X_J\geq 3 ,$
$Y_Q-Y_P+X_P\geq 6 ,Y_R-Y_Q+X_Q\geq 10, Y_S-Y_N+X_N\geq 3,$
$Y_S-Y_M+X_M\geq 3, Y_T-Y_S+X_S\geq 3, Y_U-Y_Q+X_Q\geq 10,$
$Y_V-X_N+Y_N\geq 3,Y_W-Y_O+X_O\geq 5,Y_{X-U}+X_U\geq 10,$
$Y_{Finish}-Y_R+X_R\geq 3,Y_{Finish}-Y_T+X_T\geq 10,Y_{Finish-V}+X_V\geq 5,$
$Y_{Finish}-Y_W+X_W\geq 5 ,Y_{Finish}-Y_X+X_X\geq 5 , Y_{Finish}\leq 9$ (11)

Non-negative constraint for decision variable

$X_A, X_B , X_C , X_D , X_E , X_F, X_G, X_H, X_I, X_J,, X_K, X_L, X_M, X_N, X_O, X_P, X_Q, X_R, X_S, X_T, X_U, X_V, X_W, X_X\geq 0$ (12)

Non-negative constraint for start time variables

$Y_A ,Y_B , Y_C , Y_D , Y_E , Y_F , Y_G , Y_H , Y_I , Y_K , Y_L , Y_M , Y_N , Y_O , Y_P , Y_Q , Y_R , Y_S, Y_T , Y_U , Y_V , Y_W , Y_X,Y_{Finish} \geq 0$ (13)

Here the target of the project completion time has been taken as 90days. By crashing of 4 days from the duration of the project the target would be achieved. By solving the above LPP model, the crash cost and a crash time for each activity has been estimated and minimizes the total crash cost of the project.

VI. CONCLUSION

By using deterministic and probabilistic technique in construction project, the project duration and cost are optimized. Initially the project duration was 120 days. After applying these techniques the project duration has been reduced to 94days. Using linear programming model the project duration of 94 days has been crashed to 90 days. This paper concludes that CPM and PERT techniques are useful for the optimizing time schedule of any construction project. The complexity of this model increases when more activities are considered and their precedence relationship is complicated. While preparing the network diagram less number of dummy activities increase the computation time and also the probabilistic model is mostly preferable by all the construction based companies as the activity durations are mainly depend on the workers, the raw materials arrival and other circumstances.

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Appendix A

n-last node
P(j)={immediate predecessors of node j}
S(i)={immediate successors of node i}
t_{ij}-duration time of the activity (i,j)
ES(j)-earliest start of the activity (k,j)
EC(j)-earliest completion of the activity (k,j)
LS(i)-Latest start of the activity (i,k)
LC(i)-Latest completion of the activity (i,k)
TF-Total float
IF-independent float
FF-free float
T_p-total project completion time
t_0- optimistic time
t_m-most likely time
t_p - pessimistic time
t_e-expected time