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On Simple Ternary Γ-Semiring

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ABSTRACT
In this paper, the terms, simple ternary Γ-semiring, semi-simple, semisimple ternary Γ-semiring are introduced. It is proved that (1) If T is a left simple ternary Γ-semiringor a lateral simple ternary Γ-semiring or a right simple ternary Γ-semiring then T is a simple ternary Γ-semiring. (2) A ternary Γ-semiring T is simple ternary Γ-semiring if and only if TΓTΓaΓTΓT = T for all $a \in$ T. (3) A ternary Γ-semiring T is regular then every principal ternary Γ-ideal of T is generated by an idempotent. (4) An element a of a ternary Γ-semiring T is said to be semi simple if $a \in \langle a\Gamma\rangle^{n-1}\langle a\rangle$ i.e. $\langle a\Gamma\rangle^{n-1}\langle a\rangle = \langle a\rangle$ for all odd natural number n. (5) Let T be a ternary Γ-semiring and $a \in T$. If a is regular, then a is semisimple. (17) a be an element of a ternary Γ-semiringT and a is left regular or lateral regular or right regular, then a is semisimple. (18) Let a be an element of a ternary semiring T and if a is intra regular then a is semisimple.
Keywords: Simple ternary Γ-semiring, Idempotent, regular, principal ternary Γ-ideal, semi simple element.

I. INTRODUCTION

The concept of the theory of semi rings plays a large role in the development of Mathematics. The theory of semi rings is similar to ring theory. The earliest major contributions to the theory of semi rings are strongly motivated by comparisons with rings. Semiring theory can be considered as one of the most successful off-springs of ring theory in the sense that the ring theory gives a clue how to develop the ideal theory of semi rings.

Dheena and Elavarasan made a study on prime ideals, completely prime ideals, Semi prime ideals and completely Semiprime ideals in partially ordered Γ-semigroups. Madhusudhana Rao, Anjaneyulu [33, 34] study about the prime ideals, completely prime ideals, Semiprime ideals and completely Semiprime ideals, prime radicals and generalize all these results in general Γ-semigroups and duo Γ-semigroups. In this paper we study about the ternary Γ-ideals and characterise them in general ternary Γ-semi rings.

II. PRELIMINARIES

Definition 2.1[5]: Let T and Γ be two additive commutative semigroups. T is said to be a ***Ternary Γ-Semiring*** if there exist a mapping from T ×Γ× T ×Γ× T to T which maps $(x_1, \alpha, x_2, \beta, x_3) \to [x_1\alpha x_2\beta x_3]$ satisfying the conditions:

i)$[[a\alpha b\beta c]\gamma d\delta e] = [a\alpha[b\beta c\gamma d]\delta e] = [a\alpha b\beta[c\gamma d\delta e]]$
ii)$[(a + b)\alpha c\beta d] = [a\alpha c\beta d] + [b\alpha c\beta d]$
iii) $[a\alpha (b + c)\beta d] = [a\alpha b\beta d] + [a\alpha c\beta d]$
iv) $[a\alpha b\beta(c + d)] = [a\alpha b\beta c] + [a\alpha b\beta d]$ for all $a, b, c, d \in$ T and $\alpha, \beta, \gamma, \delta \in \Gamma$.

Obviously, every ternary semiring T is a ternary Γ-semiring. Let T be a ternary semi ring and Γ be a commutative ternary semi group. Define a mapping T ×Γ× T ×Γ× T ⟶ T by $a\alpha b\beta c = abc$ for all $a, b, c \in$ T and $\alpha, \beta \in \Gamma$. Then T is a ternary Γ-semiring.

Definition 2.2[5]: An element 0 of a ternary Γ-semiring T is said to be an ***absorbingzero*** of T provided $0 + x = x = x + 0$and $0\alpha a\beta b = a\alpha 0\beta b = a\alpha b\beta 0 = 0 \;\forall\; a, b, x \in$ T and $\alpha, \beta \in \Gamma$.

Definition 2.3[5]: A ternary Γ-semiring T is said to be ***commutative ternary Γ-semiring*** provided $a\Gamma b\Gamma c = b\Gamma c\Gamma a = c\Gamma a\Gamma b = b\Gamma a\Gamma c = c\Gamma b\Gamma a = a\Gamma c\Gamma b$ for all $a, b, c \in$ T.

Definition 2.4[5]: Let T be ternary Γ-semiring. A non-empty subset 'S' is said to be a ***ternary Γ-subsemiring*** of T if S is an additive sub semigroup of T and $a\alpha b\beta c \in$ S for all $a,b,c \in$ S and $\alpha, \beta \in \Gamma$.

Definition 2.5[5]: A nonempty subset A of a ternary Γ-semiring T is said to be ***left ternary Γ-ideal*** of T if
(1) $a, b \in$ A implies $a + b \in$ A.
(2) $b, c \in$ T, $a \in$ A and $\alpha, \beta \in \Gamma$ implies $b\alpha c\beta a \in$ A.

Definition 2.6[5]: A nonempty subset of a ternary Γ-semiring T is said to be a ***lateral ternary Γ-ideal*** of T if
(1) $a, b \in$ A implies $a + b \in$ A.
(2) $b, c \in$ T, $\alpha, \beta \in \Gamma$, $a \in$ A implies $b\alpha a\beta c \in$ A.

Definition 2.7[5]: A nonempty subset A of a ternary Γ-semiring T is a ***right ternary Γ-ideal***of T if
(1) $a, b \in$ A implies $a + b \in$ A.
(2) $b, c \in$ T, $\alpha, \beta \in \Gamma$, $a \in$ A implies $a\alpha b\beta c \in$ A.

Definition 2.8[5]: A nonempty subset A of a ternary Γ-semiring T is a ***two sided ternary Γ-ideal***of T if
(1) $a, b \in$ A implies $a + b \in$ A
(2) $b, c \in$ T, $\alpha, \beta \in \Gamma$, $a \in$ A implies $b\alpha c\beta a \in$ A, $a\alpha b\beta c \in$ A.

Definition 2.9[5]: A nonempty subset A of a ternary Γ-semiring T is said to be ***ternary Γ-ideal***of T if
(1) $a, b \in$ A implies $a + b \in$ A
(2) $b, c \in$ T, $\alpha, \beta \in \Gamma$, $a \in$ A implies $b\alpha c\beta a \in$ A, $b\alpha a\beta c \in$ A, $a\alpha b\beta c \in$ A.

Definition 2.10[5]: A ternary Γ-ideal A of a ternary Γ-semiring T is said to be ***a principal ternary Γ-ideal*** provided A is a ternary Γ-ideal generated by $\{a\}$ for some $a \in$ T. It is denoted by J (a) (or) <a>.

III. SIMPLE TERNARY Γ-SEMIRING

We introduce the notion of left ternary Γ-semiring and characterize left simple ternary Γ-semiring.

Definition 3.1: A ternary Γ-semiring T is said to be ***left simpleternary Γ-semiring*** if T is its only left ternary Γ-ideal.

Theorem 3.2: A ternary Γ-semiring T is a left simple ternary Γ-semiring if and only if TΓTΓ*a* = T for all *a*$\in$ T.

Proof: Suppose that T is a left simple ternary Γ-semiring and $a \in$ T.
Let $s, v \in$ TΓTΓa; $t, u \in$ T, $\alpha, \beta \in \Gamma$.

$s, v \in$ TΓTΓ$a \Rightarrow s = \sum_{i=1}^{n} v_i\alpha_i w_i\beta_i a$,

$v = \sum_{j=1}^{n} v_j\alpha_j w_j\beta_j a$ where v_i ,v_j, w_i, $w_j \in$ T, $\alpha_i, \beta_i \in \Gamma$ $n \in z_0^{+}$.

$s + v = \sum_{i=1}^{n} v_i\alpha_i w_i\beta_i a + \sum_{j=1}^{n} v_j\alpha_j w_j\beta_j a$ is a finite sum. Therefore, $s + v \in$ TΓTΓa and hence TΓTΓa is a subsemigroup of (T, +).

Now $u\alpha t\beta s = u\alpha t\beta(\sum_{i=1}^{n} v_i\alpha_i w_i\beta_i a)$

$= (\sum_{i=1}^{n} u\alpha t\beta v_i\alpha_i w_i\beta_i a) \in$ TΓTΓa

$\Rightarrow$ TΓTΓa is a left Γ-ideal of T. Since T is a left simple ternary Γ-semiring, TΓTΓa = T
Therefore, TΓTΓa = T for all $a \in$ T.
Conversely suppose that TΓTΓa = T for all $a \in$ T.
Let L be a left Γ-idealof T. Let $l \in$ L. Then $l \in$ T. By assumption TΓTΓl = T.

Let $t \in$ T. Then $t \in$ TΓTΓ$l \Rightarrow t = \sum_{i=1}^{n} u_i\alpha_i v_i\beta_i l$ for some $u_i, v_i \in$ T and $\alpha_i, \beta_i \in \Gamma$.
$l \in$ L; $u_i, v_i \in$ T, $\alpha_i, \beta_i \in \Gamma$ and L is a left Γ-ideal of T$\Rightarrow \sum_{i=1}^{n} u_i\alpha_i v_i\beta_i l \in$ L$\Rightarrow t \in$ L.
Therefore, T $\subseteq$ L. Clearly L $\subseteq$ T and hence L = T.
Therefore, T is the only left Γ-ideal of T. Hence T is left simple ternary Γ-semiring.

We now introduce a lateral simple ternary Γ-semiring and characterize lateral simple ternary Γ-semiring.

Definition 3.3: A ternary Γ-semiring T is said to be ***lateral simpleternary Γ-semiring*** if T is its only lateral ternary Γ-ideal.

Theorem 3,4: A ternary -semiring T is a lateral simple ternary -semiring if and only if TΓ*a*ΓT = TΓTΓ*a*ΓTΓT = T for all *a*$\in$ T.

Proof: Suppose that T is a lateral simple ternary Γ-semiring and $a \in$ T.
Let $s, v \in$ TΓaΓT; $t, u \in$ T and $\alpha, \beta \in \Gamma$.

$s, v \in$ TΓaΓT$\Rightarrow s = \sum_{i=}^{n} r_i\alpha_i a\beta_i t_i$ and

$v = \sum_{j=1}^{n} r_j\alpha_j a\beta_j t_j$ where $r_i, t_i, r_j, t_j \in T$ and $\alpha_i, \beta_i, \alpha_j, \beta_j \in \Gamma$.

Now $s + v = \sum_{i=}^{n} r_i\alpha_i a\beta_i t_i + \sum_{j=1}^{n} r_j\alpha_j a\beta_j t_j$ is a finite sum. Therefore, $s + v \in$ TΓaΓT and hence TΓaΓT is a additive subsemigroup of (T, +).

Now $u\alpha s\beta t = u\alpha(\sum_{i=}^{n} r_i\alpha_i a\beta_i t_i)\beta t$

$= \sum_{i=}^{n} u\alpha r_i\alpha_i a\beta_i t_i\beta t \in$ TΓTΓaΓTΓT = TΓaΓT

$\Rightarrow$ TΓaΓT is a lateral ternary Γ-ideal of T.Since T is a lateral simple ternary Γ-semiring, TΓaΓT = T.
Therefore, TΓaΓT = T for all $a \in$ T.
Conversely suppose that TΓaΓT = T for all $a \in$ T.
Let M be a lateral ternary Γ-ideal of T. Let $m \in$ M. Then $m \in$ T

By assumption T$\Gamma m\Gamma$T = T.Let $t\in$ T. Then $t\in$ T$\Gamma m\Gamma$T $\Rightarrow t = \sum_{i=}^{n} r_i\alpha_i m\beta_i t_i$ for some $r_i, t_i \in T$ and $\alpha_i, \beta_i \in \Gamma$.$m\in$ M; $r_i, t_i \in T$, $\alpha_i, \beta_i \in \Gamma$ and M is a lateral ternary Γ-ideal $\Rightarrow \sum_{i=}^{n} r_i\alpha_i m\beta_i t_i \in$ M $\Rightarrow t\in$ M. Therefore, T $\subseteq$ M. Clearly M $\subseteq$ T and hence M = T.Therefore, T is the only lateral ternary Γ-ideal of T. Hence T is lateral simple ternary Γ-semiring.

We now introduce a right simple ternary Γ-semiring and characterize right simple ternary Γ-semiring.

Definition 3.5: A ternary Γ-semiring T is said to be ***right simple ternary Γ-semiring*** if T is its only right ternary Γ-ideal.

Theorem 3.6: A ternary Γ-semiring T is a right simple ternary Γ-semiring if and only if aΓTΓT = T for all $a\in$ T.

Proof: Suppose that T is a right simple ternary Γ-semiring and $a\in$ T.
Let $s, v\in a$ΓTΓT; $t, u\in$ T, $\alpha, \beta\in\Gamma$.

$s, v \in a\Gamma T\Gamma T \Rightarrow s = \sum_{i=1}^{n} a\alpha_i v_i\beta_i w_i$,

$v = \sum_{j=1}^{n} a\alpha_j v_j\beta_j w_j$ where $v_i, v_j, w_i, w_j\in$ T,

$\alpha_i, \beta_i \in \Gamma$ $n\in z_0^+$

$s + v = \sum_{i=1}^{n} a\alpha_i v_i\beta_i w_i + \sum_{j=1}^{n} a\alpha_j v_j\beta_j w_j$ is a finite sum. Therefore, $s + v \in a$ΓTΓT and hence aΓTΓT is a subsemigroup of (T, +).

Now $s\alpha u\beta t = (\sum_{i=1}^{n} a\alpha_i v_i\beta_i w_i)\alpha u\beta t$

$= \sum_{i=1}^{n} a\alpha u\beta t\alpha_i v_i\beta_i w_i \in a\Gamma T\Gamma T$

$\Rightarrow a$ΓTΓT is a rightΓ-ideal of T. Since T is a right simple ternary Γ-semiring, aΓTΓT = T
Therefore, aΓTΓT = T for all $a\in$ T.
Conversely suppose that aΓTΓT = T for all $a \in$ T.
Let R be a rightΓ-idealof T. Let $r\in$ R. Then $r\in$ R. By assumption rΓTΓT = T.

Let $t \in$ T. Then $t\in r$ΓTΓT $\Rightarrow t = \sum_{i=1}^{n} r\alpha_i u_i\beta_i v_i$ for some $u_i, v_i\in$ T and $\alpha_i, \beta_i \in \Gamma$.
$r\in$ R; $u_i, v_i\in$ T, $\alpha_i, \beta_i \in \Gamma$ and R is a rightΓ-ideal of T$\Rightarrow \sum_{i=1}^{n} r\alpha_i u_i\beta_i v_i \in$ R$\Rightarrow t\in$ R.

Therefore, T $\subseteq$ R. Clearly R $\subseteq$ T and hence R = T. Therefore, T is the only rightΓ-ideal of T. Hence T is right simple ternary Γ-semiring.
We now introduce a simple ternary Γ-semiring and characterize simple ternary Γ-semirings.

Definition 3.7: A ternary Γ-semiring T is said to be ***simple ternary Γ-semiring*** if T is its only ternary Γ-ideal of T.

Theorem 3.8: If T is a left simple ternary Γ-semiring (or) a lateral simple ternary Γ-semiring (or) a right simple ternary Γ-semiring then T is a simple ternary Γ-semiring.

Proof: Suppose that T is a left simple ternary Γ-semiring. Then T is the only left ternary Γ-ideal of T. If A is a ternary Γ-ideal of T, then A is a left ternary Γ-ideal of T and hence A = T. Therefore, T itself is the only ternary Γ-ideal of T and hence T is a simple ternary Γ-semiring.

Suppose that T is a lateral simple ternary Γ-semiring. Then T is the only lateral ternary Γ-ideal of T. If A is a ternary Γ-ideal of T, then A is a lateral ternary Γ-ideal of T and hence A = T. Therefore, T itself is the only ternary Γ-ideal of T and hence T is a simple ternary Γ-semiring. Similarly, if T is right simple ternary Γ-semiring then T is simple ternary Γ-semiring.

Theorem 3.9: A ternary Γ-semiring T is simple ternary Γ-semiring if and only if TΓTΓaΓTΓT = T for all $a\in$ T.

Proof: Suppose that T is a simple ternary Γ-semiring and $a\in$ T. Let $s, t\in$ TΓTΓaΓTΓT; $t, u\in$ T

$s, t\in$ TΓTΓaΓTΓT$\Rightarrow s = \sum_{i=1}^{n} r_i\alpha_i s_i\beta_i a\gamma_i v_i\delta_i w_i$ and

$t = \sum_{j=1}^{n} r_j\alpha_j s_j\beta_j a\gamma_j v_j\delta_j w_j$

where $r_i, r_j, s_i, s_j, v_i, v_j, w_i, w_j\in$ T,
$\alpha_i, \beta_i, \gamma_i, \delta_i, \alpha_j, \beta_j, \gamma_j, \delta_j \in \Gamma$, $n\in z_0^+$.
Therefore, $s + t$

$= \sum_{i=1}^{n} r_i\alpha_i s_i\beta_i a\gamma_i v_i\delta_i w_i + \sum_{j=1}^{n} r_j\alpha_j s_j\beta_j a\gamma_j v_j\delta_j w_j$

is a finite sum and hence $s + t\in$TΓTΓaΓTΓT. Therefore, TΓTΓaΓTΓT is an additive sub semigroup of T.

Now $s\alpha u\beta t = (\sum_{i=1}^{n} r_i\alpha_i s_i\beta_i a\gamma_i v_i\delta_i w_i)\alpha u\beta t$ =

$\sum_{i=1}^{n} r_i\alpha_i s_i\beta_i(a\alpha u\beta t)\gamma_i v_i\delta_i w_i \in$ TΓTΓaΓTΓT and

$u\alpha t\beta s = u\alpha t\beta(\sum_{i=1}^{n} r_i\alpha_i s_i\beta_i a\gamma_i v_i\delta_i w_i)$

$= \sum_{i=1}^{n} r_i\alpha_i s_i\beta_i(u\alpha t\beta a)\gamma_i v_i\delta_i w_i \in T\Gamma T\Gamma a\Gamma T\Gamma T$

and $u\alpha s\beta t = u\alpha\ (\sum_{i=1}^{n} r_i\alpha_i s_i\beta_i a\gamma_i v_i\delta_i w_i)\beta t =$

$\sum_{i=1}^{n} r_i\alpha_i s_i\beta_i(u\alpha a\beta t)\gamma_i v_i\delta_i w_i \in T\Gamma T\Gamma a\Gamma T\Gamma T$.

Therefore, TΓTΓaΓTΓTis a ternary Γ-ideal of T. Since T is a simple ternary Γ-semiring, TΓTΓaΓTΓT = T. Therefore, TΓTΓaΓTΓT = T for all $a \in$ T.
Conversely suppose that TΓTΓaΓTΓT = T for all $a \in$ T. Let I be a ternary Γ-ideal of T. Let $l \in$ I. Then $l \in$ T. By assumption TΓTΓlΓTΓT = T.
Let $t \in$ T. Then $t \in$ TΓTΓlΓTΓT

$\Rightarrow t = \sum_{i=1}^{n} r_i\alpha_i s_i\beta_i l\gamma_i v_i\delta_i w_i$ for some r_i, s_i, u_i, $v_i \in$ T and $\alpha_i, \beta_i, \gamma_i, \delta_i \in \Gamma$.

$l \in$ I; r_i, s_i, u_i, $v_i \in$ T, $\alpha_i, \beta_i, \gamma_i, \delta_i \in \Gamma$ and I is a ternary Γ-ideal of T $\Rightarrow \sum_{i=1}^{n} r_i\alpha_i s_i\beta_i l\gamma_i v_i\delta_i w_i \in$ I $\Rightarrow$ $t \in$ I.

Hence, T $\subseteq$ I. Clearly I $\subseteq$ T and hence I = T.
Therefore, T is the only ternary Γ-ideal of T. Hence T is a simple ternary Γ-semiring.

Definition 3.10: An element a of a ternary Γ-semiring T is said to be ***additive idempotent*** element provided $a + a = a$.

Note 3.11: The set of all additive idempotent of a ternary Γ-semiring T is denoted by E^+(T).

Example 3.12: R∪{−∞} is a commutative, additive idempotent ternary Γ-semiring with the addition and multiplication operations where Γ = R defined as:
$a \oplus b$ = max(a,b) and $a \otimes b \otimes c$ = a + b + c where + is the ordinary addition as ternary Γ-semiring multiplication. Clearly, −∞ is the zero element, and 0 is the unity.

Definition 3.13: An element a of a ternary Γ-semiring T is said to be an ***α-idempotent*** element provided $a\alpha a\alpha a = a$.

Note 3.14: The set of all idempotent elements in a ternary Γ-semiring T is denoted by E_α(T).

Example 3.15: Every identity, zero elements are α-idempotent elements.

Definition 3.16: An element a of a ternary Γ-semiring T is said to be an ***(α, β)-idempotent*** element provided $a\alpha a\beta a = a$ for all $\alpha, \beta \in \Gamma$.

Note 3.17: In a ternary Γ-semiring T, ***a*** is an idempotent iff a is an (α, β)-idempotent for all α, $\beta \in \Gamma$.

Note 3.18: If an element ***a*** of a ternary Γ- semiring T is *an* ***idempotent,*** then $a\Gamma a\Gamma a = a$.
In the following we introduce proper idempotent element ternary Γ-semiring.

Definition 3.19: An element a of a ternary Γ-semiring T is said to be a ***proper idempotent*** element provided a is an idempotent which is not the identity of T if identity exists.

Definition 3.20: A ternary Γ-semiring T is said to be an ***idempotent ternary Γ-semiring*** provided every element of T is an α –idempotent for some $\alpha \in \Gamma$.

Definition 3.21:A ternary Γ-semiring T is said to be a ***strongly idempotent Γ- semiring*** provided every element in T is an idempotent.

Definition 3.22: An element a of a ternary Γ-semiring T is said to be ***ternary multiplicatively sub-idempotent*** provided $a + a\Gamma a\Gamma a = a$.

Definition 3.23: A ternary Γ-semiring T is said to be a ***sub-idempotent ternary Γ-semiring*** provided each of its element is sub-idempotent.

Definition 3.24: An element x of a ternary Γ-semiring T is said to be***additively regular*** provided there exist $y \in$T such that $x + y + x = x$.

Definition 3.25: An element a of a ternary Γ-semiring T is said to be ***ternary multiplicatively regular*** if there exist x, $y \in$ T and $\alpha, \beta, \gamma, \delta \in \Gamma$ such that $a\alpha x\beta a\gamma y\delta a = a$.

Definition 2.26: A ternary Γ-semiring T is said to be ***regular ternary Γ-semiring*** provided every element is regular.

Example 2.27: Let T = {0, a, b} and Γ = {α} be any nonempty set. If we define a binary operation + and ternary multiplication on T as the following Cayley table, then T is a regular ternary Γ-semiring.

α	0	a	b
0	0	0	0
a	0	a	a
b	0	b	b

+	0	a	b
0	0	a	b
a	a	b	a
b	b	a	a

Theorem 2.28: A ternary Γ-semiring T is regular then every principal ternary Γ-ideal of T is generated by an idempotent.

Proof: Suppose T is a regular ternary Γ-semiring. Let <a> be a principal ternary Γ-ideal of T. Since T is regular, ∃x, $y \in$ T, and for all α, β, γ, $\delta \in \Gamma \ni$

$a\alpha x\beta a\gamma y\delta a = a$. Let $a\alpha x\beta a\gamma y = e$. Then $e\delta e\delta e = (a\alpha x\beta a\gamma y)\delta(a\alpha x\beta a\gamma y)\delta(a\alpha x\beta a\gamma y)$
$= (a\alpha x\beta a\gamma y\delta a)\alpha x\beta\,(a\gamma y\delta a\alpha x\beta a)\gamma y = a\alpha x\beta a\gamma y = e$.
Thus e is an idempotent of T.
Now $a = a\alpha x\beta a\gamma y\delta a = a\alpha x\beta a\gamma y\delta a\alpha x\beta a\gamma y\delta a = e\delta e\delta a \in \langle e\rangle \Rightarrow \langle a\rangle\subseteq\langle e\rangle$.
Now $e = a\alpha x\beta a\gamma y\in\langle a\rangle\Rightarrow\langle e\rangle\subseteq\langle a\rangle$. Therefore $\langle a\rangle = \langle e\rangle$ and hence every principal ternary Γ-ideal generated by an idempotent.

We now introduce a semi simple element of a ternary Γ-semiring and a semi simple ternary Γ-semiring.

Definition 3.29: An element a of a ternary Γ-semiring T is said to be ***semisimple*** if $a\in \langle a\rangle\Gamma\langle a\rangle\Gamma\langle a\rangle$ i.e. $\langle a\rangle\Gamma\langle a\rangle\Gamma\langle a\rangle = \langle a\rangle$.

Theorem 3.30: An element a of a ternary Γ-semiring T is said to be ***semisimple*** if $a\in \langle a\Gamma\rangle^{n-1}\langle a\rangle$ i.e. $\langle a\Gamma\rangle^{n-1}\langle a\rangle = \langle a\rangle$ for all odd natural number n.

Proof: Suppose that a is semi simple element of T. Then $\langle a\rangle\Gamma\langle a\rangle\Gamma\langle a\rangle = \langle a\rangle$.
Let $a\in$ T and n is odd natural number.
If $n = 1$ then clearly $a\in\langle a\rangle$.
If $n = 3$ and a is semi simple, then $\langle a\rangle\Gamma\langle a\rangle\Gamma\langle a\rangle = \langle a\rangle$.
If $n = 5$ then $\langle a\Gamma\rangle^{5-1}\langle a\rangle = \langle a\rangle\Gamma\langle a\rangle\Gamma\langle a\rangle\Gamma\langle a\rangle\Gamma\langle a\rangle = \langle a\rangle\Gamma\langle a\rangle\Gamma\langle a\rangle = \langle a\rangle$.
Therefore, by induction of n is an odd natural number, we have $\langle a\Gamma\rangle^{n-1}\langle a\rangle = \langle a\rangle$.
The converse part is trivial.

Definition 3.31: A ternary Γ-semiring T is called ***semi simple ternary Γ-semiring*** provided every element in T is semisimple.

Theorem 3.32: Let T be a ternary □-semiring and $a \in T$. If *a* is regular, then *a* is semisimple.

Proof: Suppose that a is regular. Then
$a = a\alpha x\beta a\gamma y\delta a$ for some $x, y\in$ T, $\alpha, \beta, \gamma, \delta\in\Gamma$
$\Rightarrow a\in \langle a\rangle\Gamma\langle a\rangle\Gamma\langle a\rangle$. Therefore, a is semisimple.

Theorem 3.33: Let *a* be an element of a ternary □-semiring T. If *a* is left regular, then *a* is semisimple.

Proof: Suppose a is left regular. Then $a \in a\Gamma a\Gamma a\Gamma x\Gamma y$ for some $x, y\in$ T $\Rightarrow a\in \langle a\rangle\Gamma\langle a\rangle\Gamma\langle a\rangle$. Therefore, a is semisimple.

Theorem 3.34: Let *a* be an element of a ternary □-semiring T. If *a* is right regular, then *a* is semisimple.

Proof: Suppose a is right regular, then $a \in x\Gamma y\Gamma a\Gamma a\Gamma a$ for some $x, y \in$ T $\Rightarrow a\in \langle a\rangle\Gamma\langle a\rangle\Gamma\langle a\rangle$. Therefore, a is semisimple.

Theorem 3.35: Let *a* be an element of a ternary □-semiring T. If *a* is lateral regular, then *a* is semi simple.

Proof: Suppose a is lateral regular. Then $a \in x\Gamma a\Gamma a\Gamma a\Gamma y$ for some $x, y\in$ T $\Rightarrow a\in \langle a\rangle\Gamma\langle a\rangle\Gamma\langle a\rangle$. Therefore, a is semisimple.

Theorem 3.36: Let *a* be an element of a ternary □-semiring T. If *a* is intraregular, then *a* is semisimple.

Proof: Suppose a is intra regular.
Then $a \in x\Gamma a\Gamma a\Gamma a\Gamma a\Gamma a\Gamma y$ for some $x, y \in$ T
$\Rightarrow a\in \langle a\rangle\Gamma\langle a\rangle\Gamma\langle a\rangle$. Therefore, a is semisimple.

Theorem 3.37: Let *a* be an element of a ternary □-semiring T. If *a* is completely regular, then *a* is semisimple.

Proof: Suppose that a is completely regular. Then $a = a\alpha x\beta a\gamma y\delta a$ for some $x, y\in$ T, $\alpha, \beta, \gamma, \delta \in \Gamma$
$\Rightarrow a\in \langle a\rangle\Gamma\langle a\rangle\Gamma\langle a\rangle$. Therefore, a is semisimple.
We now introduce the terms Archimedean ternary Γ-semiring and strongly Archimedean ternary Γ-semiring.

Definition 3.38: A ternary Γ-semiring T is said to be an ***Archimedean ternary Γ-semiring*** provided for any $a, b\in$ T there exists an odd natural number n such that $(a\Gamma)^{n-1}a\subseteq\langle b\rangle$.

Definition 3.39: A ternary Γ-semiring T is said to be a ***strongly Archimedean ternary Γ-semiring*** provided for any $a, b \in$ T, there exist an odd natural number n such that $(\langle a\rangle\Gamma)^{n-1}\langle a\rangle\subseteq\langle b\rangle$.

Theorem 3.40: Every strongly Archimedean ternary □-semiring is an Archimedean ternary □-semiring.

Proof: Suppose that T is strongly Archimedean ternary Γ-semiring. Let $a, b\in$ T. Since T is strongly Archimedean ternary Γ-semiring, there is an odd natural number n such that $(\langle a\rangle\Gamma)^{n-1}\langle a\rangle\subseteq\langle b\rangle$. Therefore $(a\Gamma)^{n-1}a\subseteq(\langle a\rangle\Gamma)^{n-1}\langle a\rangle\subseteq\langle b\rangle$. Therefore, T is an Archimedean ternary Γ-semiring.

IV. CONCLUSION

In this paper, efforts are made to introduce and characterize a simple ternary Γ-semiring.

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